(6 pages) **Reg. No. :** ....

Code No. : 7372 **Sub. Code : ZMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

Mathematics — Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2021–2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The arc rate at which the tangent changes direction as P moves along the curve is ————.
   (a) Curvature (b) Evolute
   (c) Torsion (d) Tangent plane

2. The limiting position as Q → P of the plane which contains the tangent line at P and the point Q is called the ————.
   (a) Osculating plane (b) Normal line
   (c) Tangent line (d) Osculating line

3. ———— is a space curve which lies on a cylinder and cuts the generators at a constant angle.
   (a) Cylindrical helix (b) Conic
   (c) Circular helix (d) Helix

4. The involutes of a circular helix are ————.
   (a) Plane curves (b) Evolute
   (c) Locus (d) Curvature

5. A direction in the tangent plane at P is described by the components of unit vector in the direction. These components are called ————.
   (a) Directional coefficient
   (b) Directional derivatives
   (c) Polar coordinates
   (d) Cartesian coordinates

6. The transformation is said to be ———— if φ and ψ are single valued and have non vanishing jacobian.
   (a) Proper (b) Geodesic
   (c) Curvature (d) Torsion

7. The orthogonal trajectories of the sections on paraboloid by the hyperbolic cylinder is ————.
   (a) $xy =$ constant (b) $xy = y - x$
   (c) $xy = yx$ (d) $xy = y + x$

8. If a particle is constrained to move on a smooth surface and no force except normal reaction, then the path is ————.
   (a) Geodesic (b) Curvature
   (c) Torsion (d) Evolute

9. The geodesic curvature of a geodesic is ————.
   (a) Zero (b) One
   (c) Constant (d) Infinity

10. ———— is a curve whose direction at every point is asymptotic.
    (a) Asymptotic line (b) Normal line
    (c) Tangent line (d) Osculating line

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the length of the common perpendicular 'd' of tangents at two near points distance 's' apart is approximately given by $d = \frac{k\tau s^3}{12}$.

    Or

    (b) Calculate the curvature and Torsion of the cubic curve given by $r = (u, u^2, u^3)$.

12. (a) Prove that a helix of constant curvature is necessarily a circular helix.

    Or

    (b) Derive the locus of center of spherical curvature.

13. (a) Find the coefficients of the direction which makes an angle $\frac{1}{2}\pi$ with the direction whose coefficients are (1,m).

    Or

    (b) Find the angle between intersecting curves on the surface with reference to the parametric curve.

14. (a) Prove that the curves of the family $\frac{v^3}{u^2} =$ constant are geodesics on a surface with the metric $v^2 du^2 - 2uv\,du\,dv + 2u^2\,dv^2$.

    Or

    (b) Explain the normal property of geodesics.

15. (a) Prove that if the orthogonal trajectories of the curves $v =$ constant are geodesics, then $\frac{H^2}{E}$ is independent of u.

    Or

    (b) Explain Dupin indicatrix.

[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive Serret-Frenet formulae for the space curve in terms of Darboux vector.

Or

(b) Obtain curvature and Torsion of a curve given as the intersection of two quadratic surfaces.

17. (a) Explain the Osculating sphere.

Or

(b) Prove that the characteristic property of helices is that the ratio of curvature to torsion is constant at all points.

18. (a) (i) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

(ii) Define tangent plane and normal plane in terms of parameters.

Or

(b) Obtain the Geometrical interpretation of metric.

19. (a) Prove the necessary and sufficient condition for the curve $v = c$ to be a geodesic.

Or

(b) If g(t) is continuous for $0 < t < 1$ and if $\int_0^1 v(t)\, g(t)\, dt = 0$ for all admissible functions $v(t)$ as defined above, then prove that $g(t) = 0$.

20. (a) State and prove Gauss Bonnet theorem.

Or

(b) Derive second fundamental form.