(7 pages) Reg. No. : ....................

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M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Second Semester

Mathematics — Elective

# CLASSICAL MECHANICS

(For those who joined in July 2021-2022)

Time : Three hours Maximum : 75 marks

## PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The angular momentum of a particle about point $O$ is

   (a) $\bar{r}\cdot\bar{p}$ (b) $\bar{r}\times\bar{p}$

   (c) $\bar{p}\times\bar{r}$ (d) $\bar{p}\cdot\bar{r}$

2. A system of 50 particles, free from constraints has ———— degrees of freedom.

   (a) 50 (b) 100

   (c) 150 (d) 25

3. The arbitrary virtual displacement $\delta r_i$ can be connected with the virtual displacements $\delta q_j$ by

   (a) $\delta r_i = \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j$ (b) $\delta r_i = \sum_j \delta q_j$

   (c) $\delta r_i = \sum_j \frac{\partial r_i}{\partial q_j}$ (d) $\delta r_i = \sum_j \frac{\partial q_j}{\partial r_i}\delta q_j$

4. The Lagrange's equations are

   (a) $\frac{\partial L}{\partial q_j^{\bullet}} - \frac{\partial L}{\partial q_j} = 0$ (b) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_j^{\bullet}}\right) + \frac{\partial L}{\partial q_j} = 0$

   (c) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_j^{\bullet}}\right) - \frac{\partial L}{\partial q_j} = 0$ (d) $\frac{-\partial L}{\partial q_j^{\bullet}} + \frac{d}{dt}\left(\frac{\partial L}{\partial q_j}\right) = 0$

5. Curves that give the shortest distance between two points on a given surface are called

   (a) Straight lines

   (b) Geodesics

   (c) Planes

   (d) Minimum surface



6. A holonomic equation of constraint $f(q_1, q_2, \ldots, q_n, t) = 0$ is equivalent to a differential equation

   (a) $\sum_k \frac{\partial f}{\partial q_K} dq_K = 0$

   (b) $\sum_k \frac{\partial f}{\partial q_K} dq_K + \frac{\partial f}{\partial t} dt = 0$

   (c) $\sum_k \frac{\partial f}{\partial q_K} dt + \frac{\partial f}{\partial t} dt = 0$

   (d) $\sum_k \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial q_k} = 0$

7. The central force motion of two bodies about their center of mass can always be reduced to an equivalent

   (a) two body problem (b) one body problem

   (c) three body problem (d) half body problem

8. Central force motion is always motion in a

   (a) straight line (b) space

   (c) sphere (d) plane

9. The Runge-Lenz Vector $A$ is defined by

   (a) $p \times L + mk\frac{\bar{r}}{r}$ (b) $p \cdot L - mk\frac{\bar{r}}{r}$

   (c) $p \times L - mk\frac{\bar{r}}{r}$ (d) $p \times L - mk\bar{r}$



10. The Kepler's equation is

    (a) $\omega t = \psi - e\sin\psi$ (b) $\omega t = \psi + e\sin\psi$

    (c) $\omega t = \psi - e\cos\psi$ (d) $\omega t = \psi + e\cos\psi$

## PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define the linear momentum of a particle and derive the conservation theorem for the Linear Momentum of a particle.

    Or

    (b) Explain holonomic constraints with an example.

12. (a) Derive the Lagrange's equation of motion of a bead sliding on a uniformly rotating wire in a force-free space.

    Or

    (b) Show that the kinetic energy of a system can always be written as the sum of three homogeneous functions of the generalized velocities $T = T_0 + T_1 + T_2$.



[P.T.O.]

13. (a) Show that the geodesics of a spherical surface are great circles.

Or

(b) Obtain the shortest distance between two points in a plane.

14. (a) Show that the central force motion of two bodies about their centre of mass can always be reduced to an equivalent one-body problem.

Or

(b) State and prove Virial theorem.

15. (a) Obtain the differential equation for the orbit if the potential $V$ is fonown.

Or

(b) State and prove Kepler's third law.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) State and prove the conservation theorem for the angular momentum of a particle.

(ii) Show that the work done by the external force upon a particle is the change in the kinetic energy.

Or

(b) State and prove the conservation theorem for total angular momentum.



17. (a) State D'Alembert's principle and if $T$ is the system kinetic energy D'Alembert's principle becomes $\sum_j \left[\left\{\frac{d}{dt}\left(\frac{\partial T}{\partial q_j^{\bullet}}\right) - \frac{\partial T}{\partial q_j}\right\} - Q_j\right]\delta q_j = 0$.

Or

(b) Define Rayleigh's dissipation function $\mathcal{F}$ and show that the Lagrange equations become $\frac{d}{dt}\left(\frac{\partial L}{\partial q_j^{\bullet}}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial \mathcal{F}}{\partial q_j^{\bullet}} = 0$.

18. (a) Derive the Euler-Lagrange differential equations.

Or

(b) Derive Lagrange's equations for non holonomic systems.

19. (a) Two particles move about each other in circular orbits under the influence of gravitational forces, with a period $\tau$. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$.

Or

(b) State and prove Kepler's second law of planetary motion.



20. (a) Obtain the equations of motion for the particle moving under the influence of a central force $f = -k/r^2$.

Or

(b) Prove the existence of the Laplace-Runge-Lenz vector and show that their vector provides another way of deriving the orbit equation for the Kepler problem.

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