 Reg. No. : ..............................

**Code No. : 7753** **Sub. Code : WMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics — Core

## ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. Two functions $\varphi_1, \varphi_2$ defined on an interval $I$ are said to be ————— on $I$ if there exists two constants $c_1, c_2$ not both zero such that $c_1\varphi_1(x) + c_2\varphi_2(x) = 0$ for all $x$ in $I$.
   (a) linearly independent
   (b) linearly dependent
   (c) both (a) and (b)
   (d) neither (a) nor (b)

2. The value of the Wronskian is ————— if $\varphi_1 = x^2$ and $\varphi_2 = 5x^2$.
   (a) 1 (b) 0
   (c) –1 (d) $\infty$

3. The characteristic polynomial of $y'' + \omega^2 y = 0$ is —————.
   (a) $r^2 + \omega^2 = 0$ (b) $r^2 + r$
   (c) $r^2 + \omega^2 r$ (d) $r^2 + r + \omega^2$

4. The roots of the characteristic polynomial of the equation $y''' - 3y' + 2y = 0$
   (a) 1, 1, –1 (b) 1, 1, 2
   (c) 1, 1, –2 (d) 1, –1, –2

5. The real valued solution of $y'' - y = 0$ is —————.
   (a) $c_1 \cos x + c_2 \sin x$ (b) $c_1 e^{x} + c_2 e^{-x}$
   (c) $c_1 + c_2 x$ (d) $c_1 x + c_2 x^{-1}$

6. The particular solution of the equation $y'' + 4y = \cos x$ is —————.
   (a) $\frac{1}{3}\sin x$ (b) $\frac{1}{3}x$
   (c) $\frac{1}{3}$ (d) $\frac{1}{3}\cos x$



7. The linear differential equation $L(y) = b(x)$ is said to be homogeneous if $b(x)$ —————.
   (a) $\neq 0$ (b) $= 1$
   (c) $= 0$ (d) $> 1$

8. The value of $P_n(-1)$ is —————.
   (a) $(-1)^n$ (b) 1
   (c) 0 (d) $1^n$

9. The value of the Legendre polynomial $P_2(x)$ is —————.
   (a) 1 (b) $x$
   (c) $x^2$ (d) $\frac{3}{2}x^2 - \frac{1}{2}$

10. The singular point and its nature of the equation $x^2 y'' + (x + x^2)y' - y = 0$ is
    (a) $x = 0$, regular (b) $x = 1$, regular
    (c) $x = 0$, irregular (d) $x = 1$, irregular

11. The origin $x_0 = 0$ is ————— for the equation $x^2 y'' - y' - \frac{3}{4}y = 0$.
    (a) singular point (b) regular singular
    (c) irregular (d) analytic



12. Bessel equation has the ————— as a regular singular point.
    (a) Origin (b) $x = 1$
    (c) $x = \alpha$ (d) $x = -\alpha$

13. The solution of $y' = y^2$ with $\varphi(1) = -1$ is —————.
    (a) $-\frac{1}{x}$ (b) $x$
    (c) $x^2$ (d) 0

14. The Lipschitz constant for the function $f(x, y) = x^2 \cos^2 y + y \sin^2 x$ on $s : |x| \le 1,\ |y| < \infty$ is —————.
    (a) 2 (b) 1
    (c) 3 (d) –1

15. The equation $2xy\,dx + (x^2 + 3y^2)dy = 0$ is —————.
    (a) not exact (b) exact
    (c) neither (a) nor (b) (d) both (a) and (b)

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Compute the solution of the initial value problem $y'' - 2y' - 3y = 0$; $y(0) = 0$, $y'(0) = 1$.

    Or

    (b) Verify whether the functions $\varphi_1, \varphi_2$ given by $\varphi_1(x) = \sin x$, $\varphi_2(x) = e^{ix}$ are linearly independent or not.

17. (a) Find the solution of $y'' - y' = x$.

Or

(b) State and prove uniqueness theorem.

18. (a) Verify that $\varphi_1(x) = x$ satisfy the equation $x^2y'' - xy' + y = 0$.

Or

(b) Show that there exist $n$ linearly independent solutions of $L(Y) = 0$ on I.

19. (a) Compute the indicial equation and the roots of $x^2y'' + (x + x^2)y' - y = 0$.

Or

(b) Prove that $J_0'(x) = -J_1(x)$.

20. (a) Give an example of a function satisfying Lipschitz condition.

Or

(b) Check the exactness of the equation $y' = \dfrac{3x^2 - 2xy}{x^2 - 2y}$.



PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Find all solutions of $y'' - 7y' + 6y = \sin x$.

Or

(b) State and prove existence theorem for second order equations.

22. (a) Compute the solutions $\psi$ of $y''' + y'' + y' + y = 1$.

Or

(b) Let $\varphi$ be any solution of $L(y) = y^{(n)} + a_1y^{(n-1)} + \ldots a_ny = 0$ on I containing $x_0$. Then prove that for all $x$ in $I$

$$\|\varphi(x_0)\|e^{-k|x-x_0|} \le \|\varphi(x)\| \le \|\varphi(x_0)\|e^{k|x-x_0|}$$ where $k = 1 + |a_1| + \ldots + |a_n|$.

23. (a) Verify that the function $\varphi_1(x) = e^x (x > 0)$ satisfy the equation $xy'' - (x+1)y' + y = 0$ and find a second independent solution.

Or

(b) If $\varphi_1 \ldots \varphi_n$ are $n$ solutions of $L(y) = 0$ on I, prove that they are linearly independent if and only if $W(\varphi_1 \ldots \varphi_n) \neq 0$ for all $x$ in $I$.



24. (a) Find a solution $\varphi$ of the form $\varphi(x) = |x-1|^r \sum_0^\infty c_k (x-1)^k$ for the Legendre's equation $(1 - x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$.

Or

(b) Solve the Euler equation of $n$th order $x^n y^{(n)} + a_1x^{n-1}y^{(n-1)} + \ldots + a_ny = 0$.

25. (a) Let $M$ and $N$ be two real valued functions which have continuous partial derivatives on $R : |x - x_0| \le a$, $|y - y_0| \le b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact if and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ in R.

Or

(b) Consider the initial value problem $y' = 3y + 1$, $y(0) = 2$. Compute the first four approximations $\varphi_0, \varphi_1, \varphi_2, \varphi_3$. Compute the solution by direct method and compare.

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