(8 pages) Reg. No. : ....

Code No. : 7751 **Sub. Code : WMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics – Core

ALGEBRAIC STRUCTURES

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($15 \times 1 = 15$ marks)

Answer ALL questions.

Choose the correct answer:

1. Let A, B be subgroups of G if $x, y \in G$ than $x \sim y$ if $y =$ ________ for same $a \in A, b \in b$.

(a) ab (b) axb

(c) $ax^{-1}b$ (d) axa^{-1}

2. If $a \in G$ then the normalizer of a in G is the set $N(a) = \{x \in G / \_\_\_\_\}$.

(a) $xa = ax$ (b) $x = ax$

(c) axa^{-1} (d) $ax = x^{-1}$

3. If $o(G) = p^2$ where p is a prime number, the G is ________.

(a) Conjugate (b) Equivalence

(c) Inverse (d) Abelian

4. If A and B are groups then $A \times B$ is ________ to $B \times A$

(a) Isomorphic (b) Equal

(c) Not isomorphic (d) Unequal

5. Every abelian group G is a ________ over the ring of integers.

(a) Index (b) Normal

(c) Module (d) Conjugate

6. A group G is ________ if there exists a finite chain of subgroups $G = N_0 \supset N_1 \supset \ldots \supset N_k = (e)$ where each N_i is a normal subgroups of N_{i-1} and N_{i-1}/N_i is abelian.

(a) Solvable (b) Normal

(c) Index (d) Module

7. The subspace W of V is ________ under $T \in A(V)$ if $WT \subset W$

(a) Index (b) Invariant

(c) Module (d) Refluxive

8. If $T \in A(V)$ is nilpotent then K is called the index of T if ________ but $T^{k-1} \neq 0$

(a) $T = 0$ (b) $T^{k+1} = 0$

(c) $T^k = 0$ (d) $T^{k-1} = c$

9. The matrix of order $t \times t$ all of whose entries are o except on the superdiagonal where they are all is denoted by ________.

(a) 1×1 (b) M_t

(c) M_1 (d) M_2

10. The polynomials $q_1^{(x)^{l_{11}}}, q_1(x)^{l_{12}} \ldots q_k(x)^{l_{k1}} \ldots q_k(x)^{l_{kr_k}}$ in $F(x)$ are ________ of T.

(a) Elementary divisors

(b) Rational form

(c) Transform

(d) Minimal polynomial

11. Every linear transformation $T \in A_F(V)$ satisfies its ________.

(a) Roots

(b) Characteristic polynomial

(c) Cannonical form

(d) Divisor

12. If V is cyclic relative to T and if the minimal polynomial of T is $p(x)$ then for some basis of V the matrix of T is ________.

(a) Unitary (b) Singular

(c) Square (d) $C(P(x))$

13. The ________ of the elements on the main diagonal of A is trace of A

(a) Sum (b) Product

(c) Difference (d) Zero

14. The matrix A is said to be a symmetric matrix if ________.

(a) $A' < A$ (b) $A' > A$

(c) $A' = A$ (d) $A'A = 1$

15. If $T \in A(V)$ then the ________ of T is $(uT, v) = (u, vT^*); u, v \in V$

(a) Hermition adjoint
(b) Unitary
(c) Normal
(d) Inverse

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Verify that $N(a)$ is a subgroup of G.

Or

(b) Prove that $n(k) = 1 + p + \ldots + p^{k-1}$.

17. (a) If G and G' are isomorphic abelian groups then prove that for every integer s, $G(s)$ and $G'(s)$ are isomorphic.

Or

(b) Prove that G is solvable if and only if $G^{(k)} = e$ for some $k \in z$.

18. (a) If M of dimension m is cyclic with respect to T then prove that the dimension of MT^k is $m - k$ for all $k \leq m$.

Or

(b) If $u \in V_1$ is such that $uT^{n_1-k} = 0, 0 < k \leq n_1$ then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

19. (a) Prove that is S and T are nilpotent then so is ST, S + T.

Or

(b) Suppose the two matrices A, B in F_n are similar in k_n where K is an extension of F then prove A, B are in similar in F_n.

20. (a) For $A, B \in F_n$ and $\lambda \in F$, prove $tr(\lambda A) = \lambda tr A, tr(AB) = tr(BA)$.

Or

(b) If $(vT, vT) = (u, v)$ for all $v \in V$ then prove T is unitary.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Prove that the number of conjugate classes in S_n in $p(n)$, the number of partitions of n.

Or

(b) If p is a prime number and $p^n / o(G)$, then prove that G has a subgroup of order p^n.

22. (a) Prove that S_n is not solvable for $n \geq 5$.

Or

(b) Show that every finite abelian group is the direct product of cyclic group.

23. (a) If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

Or

(b) Prove that there exists a subgroup W of V invariant under T, such that $V = V_1 \oplus W$.

24. (a) For each $i = 1, 2 \ldots k, V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus \ldots \oplus V_k$ prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

Or

(b) Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

25. (a) If $T \in A(V)$ then prove the following.

(i) $T^* \in A(V)$

(ii) $(T^*)^* = T$

(iii) $(S + T)^* = S^* + T^*$

(iv) $(\lambda S)^* = \bar{\lambda} S^*$

(v) $(ST)^* = T^* S^*$

For all $S, T \in A(V)$ and all $\lambda \in F$

Or

(b) Define the transpose of the matrix A and prove the following for all $A, B \in F_n$.

(i) $(A')' = A$

(ii) $(A + B)' = A' + B'$

(iii) $(AB)' = B'A'$

————————