Code No.: 7756

Time: Three hours

post)	leg. No.:	2
le No.: 7756	Sub. Code: WMAE 13	
	REE EXAMINATION, BER 2023	3.
First S	Semester	3.
Math	ematics	
Elective 1 - ALGEBRA	AIC NUMBER THEORY	
(For those who joined	l in July 2023 onwards)	4.
: Three hours	Maximum : 75 marks	
PART A (18	5 × 1 = 15 marks)	
Answer Al Choose the correct ar	LL questions.	5,
the equation $ax + i$	zero and let $g = g.c.d.(a.b)$ in $by = c$. If $g \mid c$ then it has	
(a) finite	ns.	6.
(b) no (c) every pair of into	ogers is a	
(d) infinitely many		

(a) (2,4,4,4,)	(b)	(1,2,2,2,2,)
(c) (1,1,2,1,2,1,)	(d)	(0,2,2,2,2)
The infinite simple c	ontinuc	ed fraction of $\sqrt{2}$ -
(a) \(\langle 2, 4, 4, 4, \ldots \rangle \)	(b)	(1,2,2,2,2,)
(e) (1,1,2,1,2,1,)	(d)	(0, 2, 2, 2, 2,)
The value of the infir (1,1,1,1,1,1,) is		
(a) $(2-\sqrt{5})/2$, , ,	$(1+\sqrt{5})/2$
(c) $(1-\sqrt{5})/2$	(d)	$(2+\sqrt{5})/2$
(6) (1 (1), 2		
		ξ is
	ation to	divergent
Every good approxim	ation to	
Every good approxim (a) convergent	ation to (b) (d)	divergent fair approximatio
Every good approxim (a) convergent (c) approximation	ation to (b) (d) converg	divergent fair approximatio

O	ne	O.										
10	1)	(2,0))	-		(b)	(4,0	0)				
		(0,4				(b)						
	n tl	10 0	quat	tion :	$x^2 + y^2$	= z ³ ,	the	ger	era	al va	lue o	ſz
		r2	+ 82			(b)	r2	- s2				
		2rs				(d)	no	ne o	f th	ese		
					$x^2 + y^2$ $x, y, z) =$		ha	e in	ıfın 	itely	ma	ny
(1	a)	2				(b)	3					
(6	c)	0				(d)	1					
Ί	he	e	quati	ion	$x^4 + y^4$	= z ²	ha	s n	0	solu	tion	in
(:	a)	int	oger:	s s		(b)	po	sitiv	e i	ntege	ers	
(c)	ne	gativ	e int	egers	(d)	ra	tion	als			
					Pag	e 2		Co	de	No.	: 77	56
2	int	ege	r, the	en be	braic r x is an		er a	nd b	is-	any	ratio	ona
	int (a) (b)	Al Ri	r, the gebr ation	en <i>be</i> aic in al in			er a	nd b	o is	any	ratio	ona
	int (a) (b) (c)	Al Ri Gi	r, the gebr ation aussi	en <i>be</i> aic in al in	α is an ntegers teger nteger		er a	nd b	o is	any	ratio	ona
	inte (a) (b) (c) (d)	Al Ri Gi	r, the gebr ation aussi	en be aic in al in in of the	α is an ntegers teger nteger				•			onal
	inte (a) (b) (c) (d)	Al Ra Ga No	r, the gebr ation aussi one c	en be aic in al in in in of the ce field	a is an ateger teger ateger	(n) is	call		- eal			onal
	inte (a) (b) (c) (d)	Al Ri Gi No Juno	r, the gebr ation aussi one c	en be aic in al in in in of the ce field	α is an ateger ateger ateger ese	(b)	call	ed r	eal			ona)
	(a) (b) (c) (d) A (a) (a) (c)	Al Ra Ga No nuan m	r, the gebration aussione codration > 0	en be aic in al in ian in of the	α is an integers teger integer ese	(d	call) n	ed r u < 0	eal			ona
	(a) (b) (c) (d) A (a) (a) (c)	Al Re Ge No m	r, the gebration aussione codration > 0	en be aic in al in in in in the c fiel $Q(\sqrt{-}$	α is an ateger ateger ateger ese	(d	call) n	ed r u < 0	eal			onal
	(a) (b) (c) (d) A (a) (c) Th	Al Re Grand Market Mark	r, the gebruition aussi drati > 0	en be aic in al in in of the c field $\sqrt{-}$	α is an integers teger integer ese	(d	call) n	ed r: < 0	- eal	if —		ona)
	(a) (b) (c) (d) A (a) (c) Th (a)	All Re Ga No m m m E ha	r, the result of	en be aic in al in in of the c field $\sqrt{-}$	α is an integers teger integer ese id $Q(\sqrt{n})$ is — ique fa	(d	call) n	ed r: < 0	- eal	if —		ona)
	(a) (b) (c) (d) A (a) (c) Th (a) (b)	All Re Grand Market In the Control of the Control o	r, the result of	en be as it is all in it is al	α is an integers teger integer ese id $Q(\sqrt{n})$ is — ique fa d (b)	(d	call) n	ed r: < 0	- eal	if —		onal
	(a) (b) (c) (d) A (a) (c) Th (a) (b) (c) (d)	All Respondence of the best of	r, the r,	saic in all in all in it in i	α is an integers teger integer ese id $Q(\sqrt{n})$ is — ique fa d (b)	n) is (b) (d	call) n	ed r: < 0	- eal	if —		onal
	(a) (b) (c) (d) A (a) (c) Th (a) (b) (c) (d)	All Reger All Re	r, the r,	en be aic in all in all in it	α is an atteger atteger atteger at $Q(\sqrt{a})$ is — ique fand $Q(b)$ as a	n) is (b) (d	call) n	ed r: < 0	- eal	if —		onal
	inte (a) (b) (c) (d) A (a) (c) Th (a) (b) (d) In	All Reger All Re	r, the ration and ration are considered as the constant of the constant $(\sqrt{6})$; rime	en be aic in all in all in it	a is an atteger atteger at Q(\sqrt{i}) is — ique fa d (b)	n) is (b) (d	call) n	ed r: < 0	- eal	if —		ona
	inte (a) (b) (c) (d) A (a) (b) (d) In (a) (b) (c)	All Regord	r, the ration and ration are considered as the constant of the constant $(\sqrt{6})$; rime	en be aic in all in it	a is an atteger atteger at Q(\sqrt{i}) is — ique fa d (b)	n) is (b) (d	call) n	ed r: < 0	- eal	if —		onal

In the equation ax + by = c, if a = b = c = 0, then it

has solutions.

(c) every pair of integers is a

(d) infinitely many

(a) finite

(b) no

PART B — $(5 \times 4 = 20 \text{ marks})$

Answer ALL questions, choosing either (a) or (b).

16. (a) Let a, b, c be any positive integer. Prove that there is no solution of ax + by = c in positive integers, if a + b > c.

Or

- (b) Let ax + by = c has two solutions, $(x_0, y_0)(x_1, y_1)$ with $x_1 = 1 + x_0$ and let (a,b) = 1. Then prove that $b = \pm 1$
- 17. (a) Show that the equation $x^2 + y^2 = 9z + 3$ has no integral solution.

Oı

- (b) Find all the solutions for the equation $x^2 + y^2 = z^2$ if 0 < z < 30.
- 18. (a) Evaluate the infinite continued fraction $\langle 2, 2, 2,.... \rangle$

Or

(b) Prove that two distinct infinite simple continued fractions converges to different values.

Page 5 Code No.: 7756

- (b) (i) Show that the equation $15x^2 7y^2 = 9$ has no solutions in integers.
 - (ii) Show that the equation $x^3 + 2y^3 + 4z^3 = 9w^2$ has no nontrivial solutions.
- 23. (a) Evaluate the infinite continued fractions $\langle 1,2,1,2,...... \rangle$ and $\langle 1,3,1,2,1,2...... \rangle$

Oi

- (b) Expand the irrational numbers $\sqrt{2}/2$, $\sqrt{3}$ as infinite simple continued fractions.
- 24. (a) Prove that the integers of any algebraic number field form a ring.

O

- (b) Prove that the units of the rational number field Q are ± 1 and that the integers α and β are associates in this field if and only if $\alpha = \pm \beta$.
- 25. (a) Prove that $Q(\sqrt{-7})$ have the unique factorization property.

Or

(b) Prove that every Euclidean quadratic field has the unique factorization property.

Page 7 Code No.: 7756

19. (a) Prove that the reciprocal of a unit is unit.

Or

- (b) Show that the convergents h_n/k_n are successively closer to ξ .
- 20. (a) Prove that the units of $Q(\sqrt{2})$ are $\pm (1 + \sqrt{2})^n$ where *n* ranges over all integers.

Or

(b) Prove that $Q(\sqrt{-1})$ has the unique factorization property.

PART C —
$$(5 \times 8 = 40 \text{ marks})$$

Answer ALL questions, choosing either (a) or (b)

21. (a) Find all solutions of 999x - 49y = 5000

Or

- (b) Find all solutions in integers of the Simultaneous Equations 20x + 44y + 50z = 10, 17x + 13y + 11z = 19.
- 22. (a) Prove that the equation $x^4 + y^4 = z^2$ has no solution in positive integers.

Or

Page 6 Code No.: 7756