(7 pages) Reg. No. : ....

Code No. : 5384 **Sub. Code : ZMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A function u is harmonic if it satisfies

   (a) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ (b) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$

   (c) $\frac{\partial^2 u}{\partial x^2}=-\frac{\partial^2 u}{\partial y^2}$ (d) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

2. A rational function $R(z)$ of order p has ———— zeros and ———— poles

   (a) $p, p-1$ (b) $p-1, p$

   (c) p, p (d) $p, p+1$

3. If $w = s(z) = \frac{az+b}{cz+d}$, $ad-bc \neq 0$, then $s^{-1}(w)$ is given by

   (a) $\frac{b-dw}{-cw+a}$ (b) $\frac{dw-b}{-cw+a}$

   (c) $\frac{dw-b}{a-cw}$ (d) $\frac{cz+d}{az+b}$

4. (z_1, z_2, z_3, z_4) is the image of z_1 under the linear transformation which carries z_2, z_3, z_4 into

   (a) 0, 1, ∞ (b) 1, ∞, 0

   (c) 1, 0, ∞ (d) 1, 1, 1

5. If $\int_\gamma f(z)dz = 5+i$, then $-\int_\gamma f(z)dz$ is

   (a) 0 (b) $5+i$

   (c) $5-i$ (d) $\sqrt{26}$

6. When C is a circle about G, then $\int_C \frac{dz}{z-a}$ is

   (a) 0 (b) 2π

   (c) $2\pi i$ (d) $2\pi a i$

7. If $n(\gamma, a) = 5$ then $n(-\gamma, a) - n(\gamma, -a)$ is

   (a) -10 (b) 5

   (c) 10 (d) 0

8. The value of $\int_{|z|=1} \frac{e^z}{z} dz$ is

   (a) 2π (b) $2\pi i$

   (c) 0 (d) ∞

9. The residue of $\frac{e^z}{(z-a)^2}$ at $z=a$ is

   (a) e^a (b) ∞

   (c) $\frac{e^a}{z-a}$ (d) 1

10. If f has a pole or order h, then $f\frac{1}{f}$ has the radius

    (a) h (b) $-h$

    (c) 0 (d) ∞

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Derive Cauchy-Riemann differential equation for any analytic function.

    Or

    (b) Prove that $\sum a_n z^n$ and $\sum n a_n z^{n-1}$ have the same radius of convergence.

12. (a) Explain a conformal mapping.

    Or

    (b) Prove that the reflection $z \to \bar{z}$ is not a linear transformation.

13. (a) Prove that the time integral $\int_\gamma Pdx + qdy$ defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x,y)$ in Ω such that $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

    Or

    (b) Compute $\int_{|z|=r} x\,dz$ for the positive sense of the circle.

[P.T.O.]

14. (a) State and prove Morera's theorem.

Or

(b) State and prove the fundamental theorem of algebra.

15. (a) State and prove the residue theorem.

Or

(b) Compute $\int_0^\pi \frac{d\theta}{a+\cos\theta}; a>1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If all zeros of a polynomial $p(z)$ lies in a half plane, prove that all zeros of the derivative $p'(z)$ lie in the same half plane.

Or

(b) Find the radius of convergence of the power series

(i) $\sum n^p z^n$

(ii) $\sum n! z^n$

17. (a) If $T_1 z = \frac{z+2}{z+3}$, $T_2 z = \frac{z}{z+1}$, find $T_1 T_2 z$, $T_2 T_1 z$ and $T_1^{-1} T_2 z$.

Or

(b) Investigate the geometric significance of symmetry when

(i) C is a straight line

(ii) C is a circle of center a and radius R.

18. (a) If the function $f(z)$ is analytic on a reactance R, prove that $\int_{\partial R} f(z)dz = 0$.

Or

(b) If $f(z)$ is analytic in an open disc Δ, prove that $\int_\gamma f(z)dz = 0$ for every closed curve γ in Δ.

19. (a) With usual notation prove that $F_n'(z) = nF_{n+1}(z)$ if $F_n(z) = \int_\gamma \frac{\phi(\xi)\, d\xi}{(\xi - z)n}$.

Or

(b) State and prove Weierstrass theorem for an essential singularity.

20. (a) State and prove Roucher's theorem.

Or

(b) Evaluate $\int_{-\infty}^{+\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

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