(6 pages) **Reg. No. :** ....

Code No. : 5378 **Sub. Code : ZMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let G be simple. $\varepsilon = \binom{\gamma}{2}$ if and only if G is

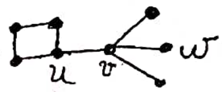
   (a) A cycle

   (b) A path

   (c) Complete

   (d) A complete bipartite graph

2. Consider the graph G :



   Then $d(u)+d(v)-d(w)$ is

   (a) 6 (b) 5

   (c) 7 (d) 4

3. Number of spanning trees of the cycle C_4 is

   (a) 1 (b) 2

   (c) 4 (d) 8

4. The relation connecting K and K' is

   (a) $K' \le K$ (b) $K \le K'$

   (c) $K + K' = \gamma$ (d) $K + K' = \varepsilon$

5. A connected graph has an Euler trail if and only if it has ———— vertices of odd degree

   (a) At most two (b) At least two

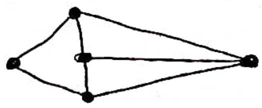
   (c) Exactly two (d) No

6. In $C_{1,5}$ the number of edges is

   (a) 4 (b) 5

   (c) 6 (d) 7

7. The edge chromatic number of the graph



   is

   (a) 1 (b) 4

   (c) 3 (d) 2

8. The value of $r(3, 3)$ is

   (a) 6 (b) 9

   (c) 3 (d) 0

9. If G is 7-critical then

   (a) $\delta = 6$ (b) $\delta \ge 6$

   (c) $\delta \ge 7$ (d) $\delta \le 6$

10. If G is a tree, with 4 vertices then $\prod_k(G)$ is

   (a) $k(k-1)^3$

   (b) k^4

   (c) $(k-1)^4$

   (d) $k(k-1)(k-2)(k-3)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Draw any graph with 4 vertices and 6 edges. For your graph, write down the incidence matrix.

    Or

    (b) Define the degree of a vertex and Show that in any graph, the number of vertices of odd degree is even.

12. (a) In a tree, prove that any two vertices are connected by a unique path.

    Or

    (b) If e is a link of G, prove that

    $\tau(G) = \tau(G-e) + \tau(G.e)$

13. (a) For $1 \le m < n/2$, define the graph $C_{m,n}$ with an example and short that $C_{m,n}$ is non Hamiltonian.

    Or

    (b) Prove that every 3-regular graph without cut edges has a perfect matching.

14. (a) Let $\mathscr{E} = (E_1, E_2, .., E_k)$ be an optimal k-edge colouring of G. If there is a vertex u in G and colours i and j such that i is not represented at u and j is represented at least twice at u, prove that the component of G $[E_i \cup E_j]$ that contains u is an odd cycle.

Or

(b) Define the numbers α and β and show that $\alpha + \beta = \gamma$.

15. (a) In a critical graph, prove that no vertex cut is a clique.

Or

(b) If G is simple, prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ for any edge e of G.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) Show that, in any group of two or more people, there are always two with exactly the same number of friends inside the group

(ii) Show that if G is simple and $\delta > [r/2] - 1$, then G is connected.

Or

(b) Prove that a graph is bipartite if and only if it contains no odd cycle.

17. (a) Show that any spanning tree $T^x = G[\{e_1, e_2, e_{r-1}\}]$ constructed by Kruskal's algorithm is an optimal tree.

Or

(b) With usual notations, prove that $K \le K' \le \delta$.

18. (a) If G is a simple graph with $\gamma \ge 3$ and $\delta \ge r/2$, prove that G is Hamiltonian.

Or

(b) Prove that G has a perfect matching if and only if $O(G-S) \le (S)$ for all $S \subset V$.

19. (a) State and prove Vizing's theorem.

Or

(b) Prove that $r(k, k) \ge 2^{k/2}$.

20. (a) If G is 4-chromatic, Prove that G contains a subdivision of K4.

Or

(b) If G is a connected simple graph and is neither an odd cycle nor a complete graph, Prove that $\chi \le \Delta$.