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Reg. No. : ....

Code No. : 5372 **Sub. Code : ZMAM 24**

M.Sc.(CBCS) DEGREE EXAMINATION, APRIL 2023

Second Semester

Mathematics

DIFFERENTIAL GEOMETRY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The normal which is perpendicular to the osculating plane at a point is called———

(a) principal normal

(b) binormal

(c) tangent line

(d) curvature

2. The necessary and sufficient condition that a curve be a straight line is that ——— at all points.

(a) t = 0 (b) k = 1

(c) k = 0 (d) c = 0

3. The involute of a circular helix are plane curves, whose planes are ——— to the axis of the cylinder

(a) normal (b) point

(c) centre (d) length

4. If two curves have the same intrinsic equations, then they are ———

(a) equal (b) congruent

(c) parallel (d) constant

5. If the curvature and torsion are both constant, then the curve is———

(a) helix (b) circular helix

(c) sphere (d) constant curve

6. If (l,m) are direction coefficients of a direction on the surface ,then the numbers μ,λ which are ——— to (l,m) are called direction ratio

(a) parallel (b) orthogonal

(c) perpendicular (d) proportional

7. If the parametric curves are ——— then the curve $u = c$ will be geodesic iff $G_1 = 0$.

(a) constant (b) orthogonal

(c) parallel (d) equal

8. If $EE_2 + FE_1 - 2EF_1 = 0$ is satisfied for all values of u and v the parametric curves ——— are all geodesics.

(a) u=constant (b) v=constant

(c) u+v =constant (d) u-v =constant

9. If the orthogonal trajectories of the curve v=constant are geodesics, then______ is independent of u

(a) H/E2 (b) H^2/E

(c) $H/\sqrt{E}$ (d) $\sqrt{H}/E^2$

10. The curvature of a geodesic relative to itself is———

(a) constant (b) parallel

(c) zero (d) equal

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the curvature and torsion are equal for the curve $x = a(3u - u^3), y = 3au^2, z = a(3u + u^3)$

Or

(b) Derive Serret Frenet formula.

12. (a) Find the expression for the curvature and torsion if the curve is drawn on a parabolic cylinder so as to cut all the generators at the same angle.

Or

(b) Find curvature and torsion of an evolute.

13. (a) Find the direction coefficient making an angle $\pi/2$ with the direction coefficient (l,m).

Or

(b) Prove that the metric is invariant.

[P.T.O.]

14. (a) Prove that on the general surface ,a necessary and sufficient condition that the curve $v = c$ be a geodesics is $EE_2 + FE_1 - 2EF_1 = 0$ when v = c for all values of u.

Or

(b) Show that the curves $u + v =$ constant are geodesics on a surface with metric $(1+u^2)du^2 - 2uv\,du\,dv + (1+v^2)dv^2$.

15. (a) Prove that the necessary and sufficient condition for a curve on a surface to be a line of curvature is k dr + dN = 0 at each point of the line of curvature.

Or

(b) Discuss the nature geodesic on a plane.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Discuss the curvature and torsion of curve in the three cases.

Or

(b) Calculate the curvature and torsion of the cubic curve given by $r = (u, u^2, u^3)$.

17. (a) State and prove Existence theorem for space curves.

Or

(b) Derive the equation of an involute of a curve.

18. (a) Prove that the curves of family $v^3 / u^2 = c$ are geodesics on a surface with metric $v^2du^2 - 2uv\,du\,dv + 2u^2dv^2$.

Or

(b) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoids.

19. (a) Prove that metric is a positive definite quadratic form in du, dv Also discuss the invariance property.

Or

(b) Prove that every helix on a cylinder is a geodesic.

20. (a) State and prove Liovillie's formula.

Or

(b) Prove that the geodesic curvature vector of any curve is orthogonal to the curves.