(8 pages) Reg. No. : ....

Code No. : 5370 **Sub. Code : ZMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics – Core

ANALYSIS – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If P^{*} is a retirement of P than
   (a) $L(P,f,\alpha) \geq L(P^{*},f,\alpha)$
   (b) $U(P^{*},f,\alpha) \geq U(P,f,\alpha)$
   (c) $L(p^{*},f,\alpha) \leq U(P^{*},f,\alpha)$
   (d) $U(P,f,\alpha) - L(P,f,\alpha) < E$

2. If $a < s < b$, f is bounded on $[a,b]$, f is continuous at s, and $\alpha(x) = I(x-s)$, then $\int_a^b f\,d\alpha$ is
   (a) $f(s)$ (b) $f(0)$
   (c) $\alpha(s)$ (d) $\begin{cases} 0 & if\ x \leq 0 \\ 1 & if\ x > 0 \end{cases}$

3. For $m = 1, 2, \ldots$, $n = 1, 2, 3, \ldots$, let $s_{m,n} = \frac{m}{m+n}$. Then $\lim_{m \to \infty} \lim_{n \to \infty} s_{m,n}$ is
   (a) 1 (b) 0
   (c) ∞ (d) $\frac{m}{m+n}$

4. If $f \in \zeta(x)$, its supremum noun is defined by
   (a) $\|f\| = \sup_{x \in X} |f(x)|$
   (b) $\|f\| = \inf_{x \in X} |f(x)|$
   (c) $\|f\| = \lim_{x \to \infty} |f(x)|$
   (d) $\|f\| = f(x_0)$ where x_0 is any point of x

5. $\{f_n\}$ is uniformly bounded on E is
   (a) there exists a finite valued function ϕ on E such that $|f_n(x)| < \phi(x)$ $(x \in E, n = 1, 2, \ldots)$
   (b) there exists a number M such that $|f_n(x)| < M$ $(x \in E, n = 1, 2, \ldots)$
   (c) there exists a sequence of number m_n st $|fn(x)| < m_n (x \in E)$ $n = 1, 2, 3 \ldots$
   (d) there exists a function f such that $f_n(x) \to f(x)$ as $n \to \infty$

6. Which one of the following is true
   (a) Every convergent sequence contains a uniformly convergent subsequence
   (b) Every member of an equicontinuous family is uniformly continuous
   (c) The uniform convergence of $\{f_n\}$ impairs the uniform convergence of $\{f_n'\}$
   (d) If $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set E, then there exist a subsequence which converges pointwise on E

7. The uniform closure of set of polynomials on $[a,b]$ is
   (a) the set of polynomials on $[a,b]$
   (b) empty
   (c) the set of functions on $[a,b]$
   (d) the set of continuous functions $[a,b]$

8. Suppose $\sum_1^{\infty} C_n = A$. Let $f(x) = \sum_0^{\infty} c_n x^n (-1 < x < 1)$ then $\lim_{x \to 1} f(x)$ is
   (a) $c_1 + c_2 + c_n$ (b) 0
   (c) 1 (d) A

9. The Fourier coefficient c_m of f is
   (a) $\frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-imx} dx$ (b) $\frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{imx} dx$
   (c) $\frac{1}{2\pi i} \int_{-\pi}^{\pi} f(x) e^{-imx} dx$ (d) $\frac{1}{2\pi i} \int_{-\pi}^{\pi} f(x) e^{imx} dx$

10. The value of $\Gamma(1/2)$ is
   (a) π (b) $\sqrt{\pi}$
   (c) 1/2 (d) 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11\. (a) Prove that $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha$.

Or

(b) State and prove the fundamental theorem.

12\. (a) Give an example to show that a convergent series of continuous functions may have a discontinuous sum.

Or

(b) When do we say the sequence $\{f_n\}$ converges uniformly on E to a function f? State and prove a very convenient test for uniform convergence due to weierstress.

13\. (a) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$ for $n = 1,2,\ldots$ and suppose $f_n \to f$ uniformly on $[a,c]$. Prove that $f \in R(\alpha)$..

Or

(b) If K is a compact metric space, if $f_n \in \zeta(K)$ for $n = 1,2,3,\ldots$ and if $\{f_n\}$ converges uniforms on K, prove that $\{f_n\}$ is equicontinuous on K.

14\. (a) Define the uniform closure B of an algebra A of bounded functions. Prove that B is a uniformly closed algebra.

Or

(b) Find the limit $\lim_{x \to 0} \frac{b^x - 1}{x} (b < 0)$.

15\. (a) Prove that every nonconstant polynomial with complex coefficients has a complex root.

Or

(b) If $x > 0$ and $y > 0$, prove that $\int_0^1 f^{x-1}(1-f)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16\. (a) When do you say that f is integrable w.r.t α, in the Riemann sense? Prove that $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(p,f,\alpha) - L(p,f,\alpha) < \Sigma$.

Or

(b) Suppose $f \in R(\alpha)$ on $[a,b]$, $m \leq f \leq M$, ϕ is continuous on $[m,M]$ and $h(x) = \phi(f(x))$ on $[a,b]$. Prove that $h \in R(\alpha)$ on $[a,b]$.

17\. (a) If γ' is continuous on $[a,b]$, prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |r'(t)| dt$.

Or

(b) Suppose $f_n \to f$ uniformly on E, a metric space. Prove that $\lim_{t \to x} \lim_{h \to \alpha} f_n(t) = \lim_{n \to \alpha} \lim_{t \to x} f_n(t)$ where x is a limit point of E.

18\. (a) Prove that there exists a real continuous function on the red line which is nowhere differentiable.

Or

(b) If $\{f_n\}$ is a pointwise founded sequence of complex functions on a countable set E, prove that $\{f_n\}$ has a subsequence $\{fn_k\}$ such that $\{fn_k^{(x)}\}$. Converges for every $x \in E$.

19\. (a) State and prove that Weierstrass theorem.

Or

(b) State and prove the Afel's theorem for powerserviece.

20\. (a) State and prove parseval's theorem.

Or

(b) If f is a positive function on $(0, \propto)$ such that $f(x+1) = xf(x)$, $f(1) = 1$ and $\log f$ is convex, then prove that $f(x) = \Gamma(x)$.
