A Categorical Approach to Sexual Utility: The Case of Orgasm

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May 28, 2023

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1 Introduction

The analysis of human behavior and satisfaction, much like the study of the cosmos or the investigation into the depths of the human psyche, presents a vast array of complex and interconnected phenomena that often elude precise quantification. And yet, the pursuit of understanding urges us to construct frameworks and models that might enable us to discern the underlying patterns and principles at play. In the present inquiry, we turn our gaze to an arena of human experience that is as old as humanity itself, and yet continues to confound and fascinate: the realm of sexual satisfaction and, in particular, the phenomenon of orgasm.

In the broader field of economics and decision theory, scholars have long made use of the concept of utility to provide a measure of preference or satisfaction. This theoretical entity, often quantified in arbitrary units known as 'utils', serves to map the nebulous realm of human desires and satisfactions onto the more tractable landscape of numerical values. Yet, the application of this model to the domain of sexual satisfaction has remained notably underexplored.

The concept of orgasm, in particular, stands as a remarkable case. Unquestionably a significant facet of sexual satisfaction, orgasms have been variously described as the pinnacle of sexual pleasure, and yet their utility remains notoriously difficult to quantify. It is this challenge that forms the central preoccupation of the present paper.

In an endeavor to build upon the existing notions of utility, and to better articulate the contribution of orgasms to overall sexual satisfaction, we turn to the mathematical machinery of category theory. With its ability to capture the relational structure between different entities, category theory presents an apt framework for our task. In our proposed model, we define three functors - P, V, and O - to represent the penis, vagina, and orgasm respectively. These functors map from the category C (representing various states and actions in sexual activity) to the real numbers, R (representing the 'utils' or utility generated). In the following sections, we shall elaborate on the mathematical definitions and properties of these functors, and build a categorical framework that illuminates the utility of orgasm.

As we embark on this mathematical journey through the landscape of sexual satisfaction, it is worth emphasizing the exploratory and speculative nature of our endeavor. Much like the pursuit of the fundamental constituents of matter or the exploration of the unconscious mind, our quest is not merely

to validate established truths, but to expand the boundaries of understanding. We thus invite the reader to join us in this voyage of discovery, equipped with a spirit of curiosity and openness to new perspectives.

2 Preliminaries

2.1 The Basics of Category Theory

Category theory is an abstract branch of mathematics that has broad implications and applications in a number of other disciplines such as computer science, physics, and various aspects of mathematics itself. It offers a structured approach to describe mathematical entities, their relationships, and transformations.

A category, denoted by $\mathcal{C}$, comprises:

1. **Objects:** These could be diverse entities—sets, numbers, vectors, or in our discussion, states and actions. The characteristics of these objects hinge on the category in question.

2. **Morphisms:** These represent the links or arrows between objects. In essence, they are functions, transformations, or operations that map one object onto another. Each morphism f possesses a source object a and a target object b. This relationship is typically expressed as $f : a \to b$.

For each object a, there exists a unique **identity morphism**, represented as $id_a : a \to a$.

Morphisms adhere to two fundamental properties:

a. **Associativity:** For any three morphisms $f : a \to b$, $g : b \to c$, and $h : c \to d$, it holds true that $h \circ (g \circ f) = (h \circ g) \circ f$.

b. **Identity:** For any morphism $f : a \to b$, the property $f \circ id_a = f = id_b \circ f$ is satisfied.

3. **Functors:** A functor is a transformation between two categories that conserves the structural properties of the category. In effect, it maps objects to objects and morphisms to morphisms in a way that respects the identity and composition of morphisms. If $\mathcal{C}$ and $\mathcal{D}$ are categories, a functor $F : \mathcal{C} \to \mathcal{D}$ maps each object a in $\mathcal{C}$ to an object $F(a)$ in $\mathcal{D}$, and each morphism $f : a \to b$ in $\mathcal{C}$ to a morphism $F(f) : F(a) \to F(b)$ in $\mathcal{D}$ such that for any morphisms $f : a \to b$ and $g : b \to c$ in $\mathcal{C}$, it follows that $F(g \circ f) = F(g) \circ F(f)$, and for any object a in $\mathcal{C}$, $F(id_a) = id_{F(a)}$.

These foundational definitions from category theory will play a pivotal

role in building our model of utilitarian principles in the context of sexual activity. Subsequent sections will delve into how we can leverage these constructs to gain insights into the structure of sexual interactions, thereby deriving measures of satisfaction and utility.

2.2 Understanding Utility and Decision Theory

In the spheres of economics and similar disciplines, decision theory offers a structured approach to addressing situations where decisions are required under conditions of uncertainty. Within this paradigm, the concept of utility is of great significance. It represents the satisfaction or preference related to the potential outcomes of a decision. The underlying idea is that every individual or system has preferences, which can be represented by a utility function.

A utility function U(x) maps from a collection of possible outcomes (or states) to real numbers with the intent of quantifying the happiness, satisfaction, or preference associated with each outcome. A higher number signifies a more favored outcome. The selection of a utility function is subjective, determined by the individual or system's preferences. It's typical to measure utility in an arbitrary unit known as "utils". For instance, a utility of 10 utils merely suggests that one outcome is preferred over another.

Classical economics frequently assumes that individuals or systems make decisions to maximize their expected utility. The principle of utility maximization can be perceived as a mathematical expression of the notion that people (or systems) make decisions in their best interest.

Utility functions can be either ordinal or cardinal. With the ordinal approach, we are solely interested in the order of preferences, whereas the cardinal approach considers the magnitudes of the differences in utility. The latter enables interpersonal comparison of utility and constitutes the foundation of social choice theory, a key component of welfare economics.

In the context of this paper, we will employ a cardinal utility function to represent the 'utils' or satisfaction derived from various states of sexual activity. We will use category theory to offer a structured model for these states and actions.

In forthcoming sections, we will present the category symbolizing these states and actions, as well as the functors symbolizing the penis, vagina, and orgasm. These functors map these states to their corresponding utilities.

3 The Category of States and Actions

As we aim to scrutinize sexual satisfaction using category theory tools, we introduce the category $\mathcal{C}$, which incorporates various states and actions associated with sexual activity.

The **objects** in $\mathcal{C}$ signify different possible states during sexual activity. These could comprise states like foreplay, intercourse, or orgasm, among others. Each state embodies a specific configuration of a sexual encounter, encompassing substantial information - the physical status of the participants, the emotional context, the progression of the encounter, and so forth.

The **morphisms** in $\mathcal{C}$ signify actions that transition between these states. An action could range from a change in position, to a shift in the pace or style of activity, to communication between partners. Each action represents a transformation from one state to another.

Considering two objects (states) S_1 and S_2 in $\mathcal{C}$, a morphism (action) a would be denoted as $a : S_1 \to S_2$, indicating that the action a moves us from state S_1 to state S_2.

Notably, in line with category theory principles, we have an identity morphism for each state, representing the 'do nothing' action that maintains the sexual encounter in the same state. Furthermore, the composition of morphisms signifies the chaining of actions.

It's crucial to underline that the definitions of these states and actions, encapsulated within the category $\mathcal{C}$, abstractly depict the complex dynamics of sexual activity. However, it's through this abstraction that we can start applying the powerful mathematical apparatus of category theory, as will be seen in the subsequent sections.

4 Introducing the Functors P, V, and O

4.1 Penis Functor (P)

Let's now consider the functor P, or as we call it, the "Penis Functor." This functor, P: $\mathcal{C} \to \mathcal{R}$, maps objects (states) in the category $\mathcal{C}$ to the real numbers. These real numbers represent the utility (in 'utils') derived from these states.

The functor P also maps morphisms (actions) in $\mathcal{C}$ to functions between real numbers. If a is a morphism transitioning from state S_1 to state S_2, then

P(a) is a function transitioning from $\mathrm{P}(S_1)$ to $\mathrm{P}(S_2)$. This function quantifies the change in utility for the penis arising from the action a.

The exact definition of P depends on the specific characteristics of the states and actions in $\mathcal{C}$. Generally, we anticipate that P assigns higher utility values to states and actions that are more pleasurable or satisfying for the penis. For instance, P might assign a high utility value to the state of orgasm, and a positive change in utility to an action transitioning from foreplay to intercourse.

Notably, P is a homomorphism, meaning it conserves the structure of $\mathcal{C}$ in its mapping to $\mathcal{R}$. In essence, for any two actions $a : S_1 \to S_2$ and $b : S_2 \to S_3$, we have $\mathrm{P}(b \circ a) = \mathrm{P(b)} \circ \mathrm{P(a)}$. Additionally, for any state S, $\mathrm{P}(\mathrm{id}_S) = \mathrm{id}_{P(S)}$.

In the following sections, we will discuss the Vagina Functor (V) and the Orgasm Functor (O), and delve into how these three functors interact to model the overall utility derived from a sexual encounter.

4.2 Vagina Functor (V)

The "Vagina Functor" V, analogous to the Penis Functor, maps states and actions from our category $\mathcal{C}$ to real numbers, symbolizing the utility or satisfaction obtained from these states and actions for the vagina.

Formally, V: $\mathcal{C} \to \mathcal{R}$ maps objects (states) in the category $\mathcal{C}$ to real numbers, with $\mathrm{V}(S)$ denoting the utility of the state S for the vagina. For instance, V may assign a high utility value to a state involving clitoral stimulation, reflecting the physiological response to such a state.

V also maps morphisms (actions) in $\mathcal{C}$ to functions between real numbers. If a is a morphism transitioning from state S_1 to state S_2, then V(a) is a function transitioning from $\mathrm{V}(S_1)$ to $\mathrm{V}(S_2)$. This function encapsulates the change in utility arising from the action a.

Like the Penis Functor, the Vagina Functor is a homomorphism, meaning that it conserves the structure of $\mathcal{C}$. Specifically, for any two actions $a : S_1 \to S_2$ and $b : S_2 \to S_3$, we have $\mathrm{V}(b \circ a) = \mathrm{V(b)} \circ \mathrm{V(a)}$. Furthermore, for any state S, $\mathrm{V}(\mathrm{id}_S) = \mathrm{id}_{V(S)}$.

The introduction of this second functor enables a more comprehensive modeling of the dynamics of a sexual encounter. The interaction between P and V, which mirror the preferences and responses of

the two participants, is crucial to our comprehension of the overall utility of an encounter, as we will see when we introduce the Orgasm Functor (O).

4.3 Orgasm Functor (O)

Finally, we introduce the "Orgasm Functor" O, which signifies the pinnacle of sexual satisfaction in our model. The functor O: $\mathcal{C} \to \mathcal{R}$ maps states and actions in the category $\mathcal{C}$ to real numbers, with these real numbers denoting the utility or satisfaction derived from orgasm.

The object mapping of O assigns to each state S a utility value O(S). This value signifies the orgasmic utility of that state for both participants. For instance, the orgasm state would usually be assigned a high utility value.

The morphism mapping of O takes an action $a : S_1 \to S_2$ and maps it to a function O(a): O(S_1) $\to$ O(S_2), which represents the change in orgasmic utility arising from action a.

The functor O, like P and V, is also a homomorphism. For any two actions $a : S_1 \to S_2$ and $b : S_2 \to S_3$, we have O($b \circ a$) = O(b) $\circ$ O(a). Additionally, for any state S, O(id_S) = $\mathrm{id}_{O(S)}$.

The Orgasm Functor, in conjunction with the Penis and Vagina Functors, offers a powerful tool to analyze and understand the utility dynamics of sexual encounters. In the subsequent sections, we will explore how these three functors interact and combine to represent the overall utility of an encounter.

5 Combined Satisfaction Functor (H)

With the Penis, Vagina, and Orgasm Functors introduced, we now seek to consolidate them into a singular measure of overall satisfaction or utility derived from a sexual encounter. To achieve this, we define the Combined Satisfaction Functor H: $\mathcal{C} \to \mathcal{R}$.

The functor H integrates the utilities of P, V, and O into one utility measure. We stipulate H to map a state S to a real number H(S) equivalent to a weighted average of P(S), V(S), and O(S). These weights can be calibrated to reflect the relative significance of the penis, vagina, and orgasm utilities in the overall satisfaction of the encounter.

Formally, let ω_P, ω_V, and ω_O be nonnegative real numbers such that ω_P + ω_V + ω_O = 1. We then define H(S) = ω_P P(S) + ω_V V(S) + ω_O O(S) for each state S.

Likewise, for an action $a : S_1 \to S_2$, we define H(a): H(S_1) $\to$ H(S_2) to be a function representing the combined change in penis, vagina, and orgasm

utilities resulting from action a.

As with the previous functors, H is a homomorphism. For any two actions $a : S_1 \to S_2$ and $b : S_2 \to S_3$, we have $\mathrm{H}(b \circ a) = \mathrm{H(b)} \circ \mathrm{H(a)}$. Additionally, for any state S, $\mathrm{H}(\mathrm{id}_S) = \mathrm{id}_{H(S)}$.

The Combined Satisfaction Functor permits us to encapsulate the entire utility dynamics of a sexual encounter within a singular mathematical entity. In the upcoming sections, we will probe the implications of this functor and employ it to scrutinize various scenarios and inquiries.

6 Optimization Problem and Extreme Value Theorem

With the Combined Satisfaction Functor H defined, we can articulate an optimization problem to identify the sequence of actions maximizing utility.

The optimization problem is framed as follows: find a morphism a in $\mathcal{C}$ such that H(a) is maximized. Put simply, we're looking for an action resulting in the highest increase in combined utility.

The application of the Extreme Value Theorem in our category-theoretical context requires caution, as its straightforward applicability to numerical sets doesn't directly translate to categories. However, by imposing specific conditions, we can guarantee the existence of an optimal action.

Specifically, we require that the category $\mathcal{C}$ be a "small" category and the functor H be a continuous functor. In this scenario, a "small" category is one where the objects and morphisms form sets (instead of proper classes), and a continuous functor is one that preserves limits.

With these conditions met, we can apply a categorical version of the Extreme Value Theorem to argue for the existence of an optimal action a^* that maximizes H(a).

The categorical Extreme Value Theorem can be formulated as follows: if $\mathcal{C}$ is a small category and H: $\mathcal{C} \to \mathcal{R}$ is a continuous functor, then there exists a morphism a^* in $\mathcal{C}$ such that $\mathrm{H}(a^*)$ ¿= H(a) for all morphisms a in $\mathcal{C}$.

In essence, under the correct

conditions, there will always exist an action that maximizes the overall utility, as gauged by the Combined Satisfaction Functor. This optimality provides a framework to explore the pinnacle of satisfaction attainable through the actions available within the states of a sexual encounter.

7 Natural Transformations and Policy Changes

Natural transformations, another concept from category theory, provide us a method to model shifts in societal norms, values, or satisfaction measurement. These transformations act on functors, essentially creating a "map of maps".

In our context, a natural transformation η: H1 -¿ H2 between two Combined Satisfaction Functors H1 and H2 can represent a change in policy, societal norms or even in the way satisfaction is evaluated. Each component η_S of the natural transformation is a function η_S: H1(S) -¿ H2(S) that maps the old utility of a state to its new utility.

This transformation would be applied across all states and actions in the category $\mathcal{C}$. For instance, if societal norms change such that the relative importance of the orgasm utility (O) increases compared to the penis (P) and vagina (V) utilities, the weights ω_O, ω_P, and ω_V in the definition of H would change accordingly, resulting in a new Combined Satisfaction Functor H2.

Formally, a natural transformation η: H1 -¿ H2 is a collection of morphisms η_S: H1(S) -¿ H2(S) in $\mathcal{R}$ for each state S in $\mathcal{C}$, such that for every action a: S -¿ T in $\mathcal{C}$, the following diagram commutes:

$$\begin{array}{ccc} H1(S) & \xrightarrow{\eta_S} & H2(S) \\ {\scriptstyle H1(a)}\downarrow & & \downarrow{\scriptstyle H2(a)} \\ H1(T) & \xrightarrow{\eta_T} & H2(T) \end{array}$$

Through these natural transformations, our category-theoretic model of utilitarianism can capture complex shifts in societal norms and values, making it a versatile tool for analyzing various societal scenarios in the context of sexual encounters.

8 Conclusion

In this paper, we have presented a category-theoretical framework for modeling the utility of sexual encounters. We defined functors to represent the utilities of the penis, vagina, and orgasm, and consolidated these into a singular Combined Satisfaction Functor. We then set up an optimization problem to identify the action that maximizes this combined utility, and showed that under certain conditions, such an optimal action exists.

This work showcases how abstract mathematical concepts from category theory can be applied to model complex phenomena in the realm of human sexuality. The utilization of categories, functors, and natural transformations allows us to capture the interplay of various factors and the impact of societal changes on individual satisfaction.

Although the model presented here is a simplification, it paves the way for future research. The relative weights in the Combined Satisfaction Functor could be adjusted to reflect different societal norms or individual preferences. More detailed models could also incorporate additional factors such as emotional connection, intimacy, or other aspects of sexual and romantic relationships.

Additionally, this framework could be extended to other areas of human behavior and decision making, beyond just sexual encounters. The overarching approach of using category theory to model utilitarian principles could have broad applications in economics, decision theory, and social sciences.

Overall, this work is a step towards a more rigorous and nuanced understanding of human satisfaction and utility, leveraging the power of abstract mathematics to shed light on complex and deeply human experiences.