

HKIE Geotechnical Division: 6 November 2007

# The magic of sands

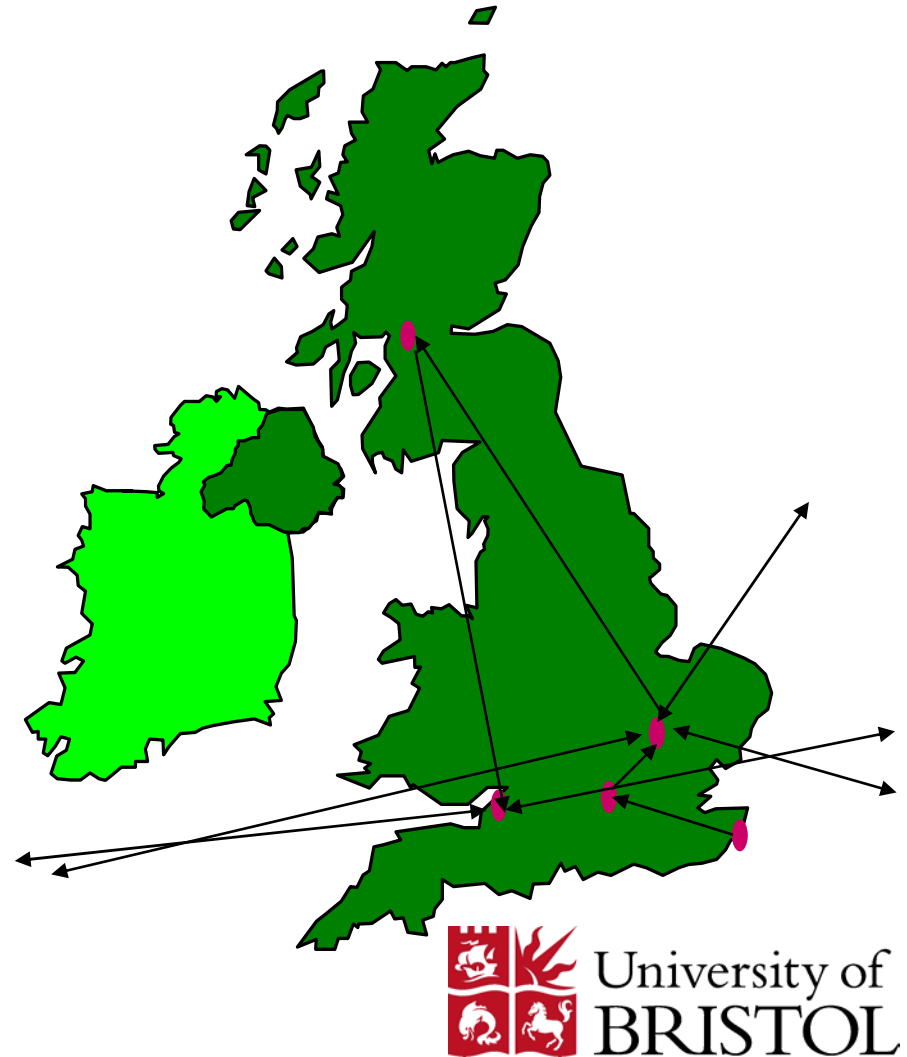
David Muir Wood  
University of Bristol, England

(20<sup>th</sup> Bjerrum Lecture: Oslo: November 2005)



# David Muir Wood

1949-1950 Folkestone  
1950-1970 Beaconsfield  
1967-1987 Cambridge  
1975 Oslo  
1978 Hong Kong  
1986 Boulder  
1987-1996 Glasgow  
1995- Bristol  
2000 Minneapolis  
2003 Tokyo



# The magic of sands

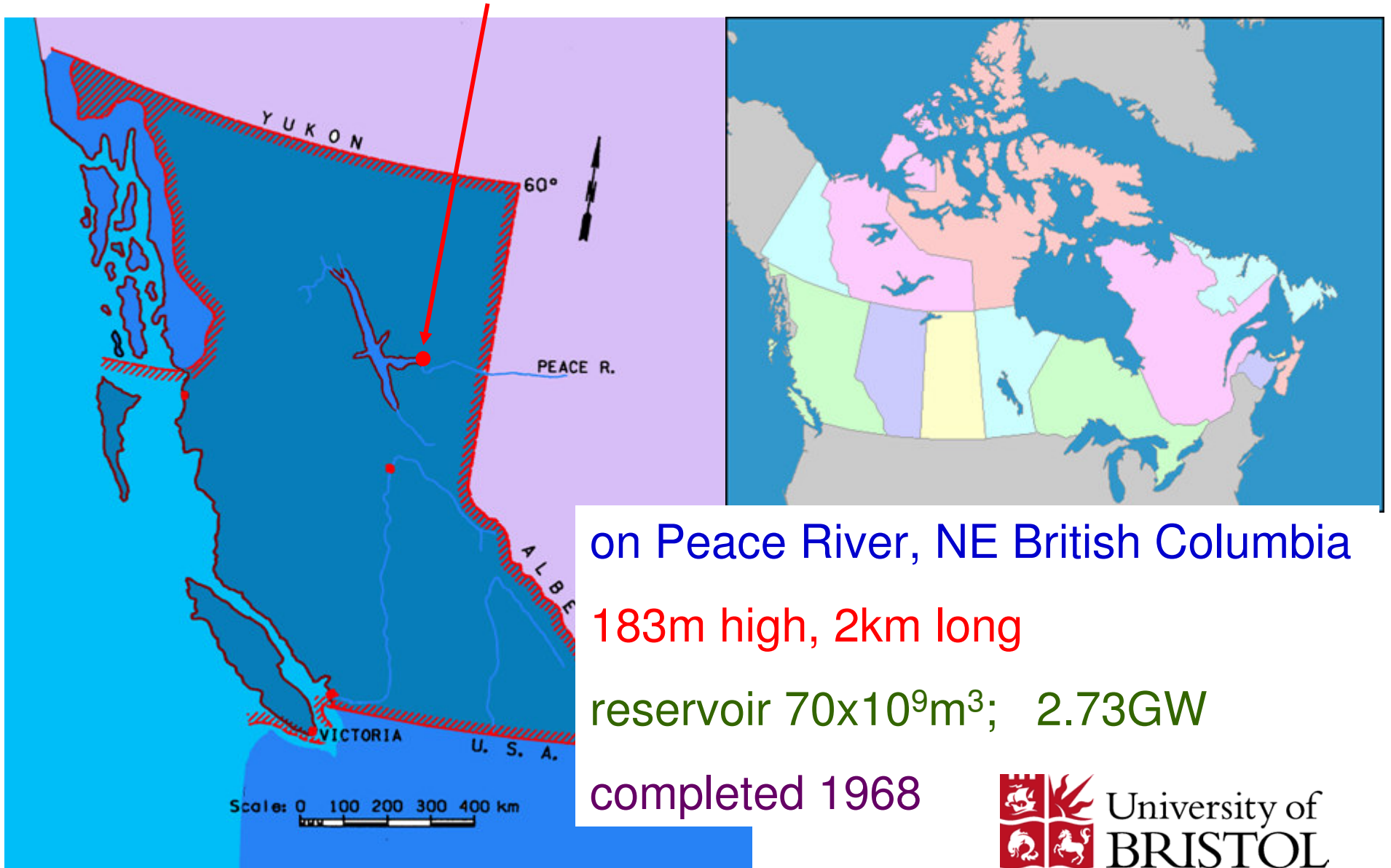
- Introduction: context: WAC Bennett Dam
- Particle-continuum duality
- Mohr-Coulomb model
- Elastic properties
- *Critical states*
- Severn-Trent sand
- Grading state index
- Conclusion

# The magic of sands

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# WAC Bennett Dam



# WAC Bennett Dam



no incidents until 1996

1

sinkholes

2

# sinkhole 1: June 1996

**BChydro** 



2.4m diameter at crest

6.7m deep hole

76m deep extremely  
loose zone

114m variable zone

## WAC Bennett Dam: sinkhole incident 1996



spillway flow  $3000\text{m}^3/\text{s}$  (> Canadian Niagara Falls)

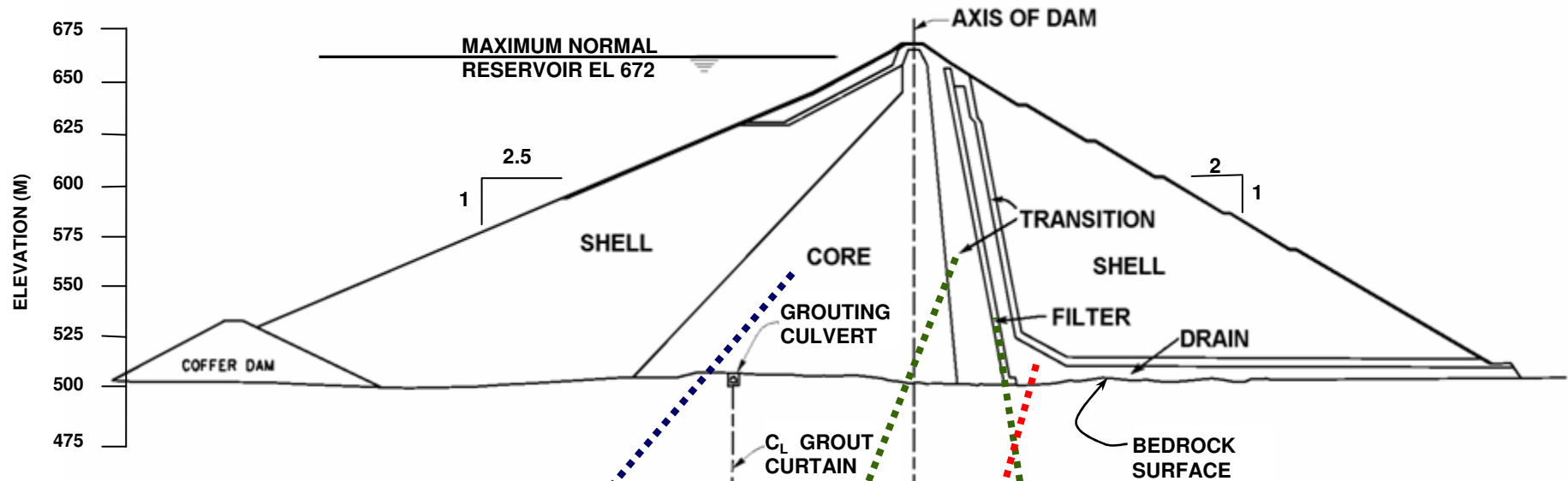
fall in reservoir level: 2m in 7 weeks

**BC hydro** 

 University of  
BRISTOL



# WAC Bennett Dam: cross section



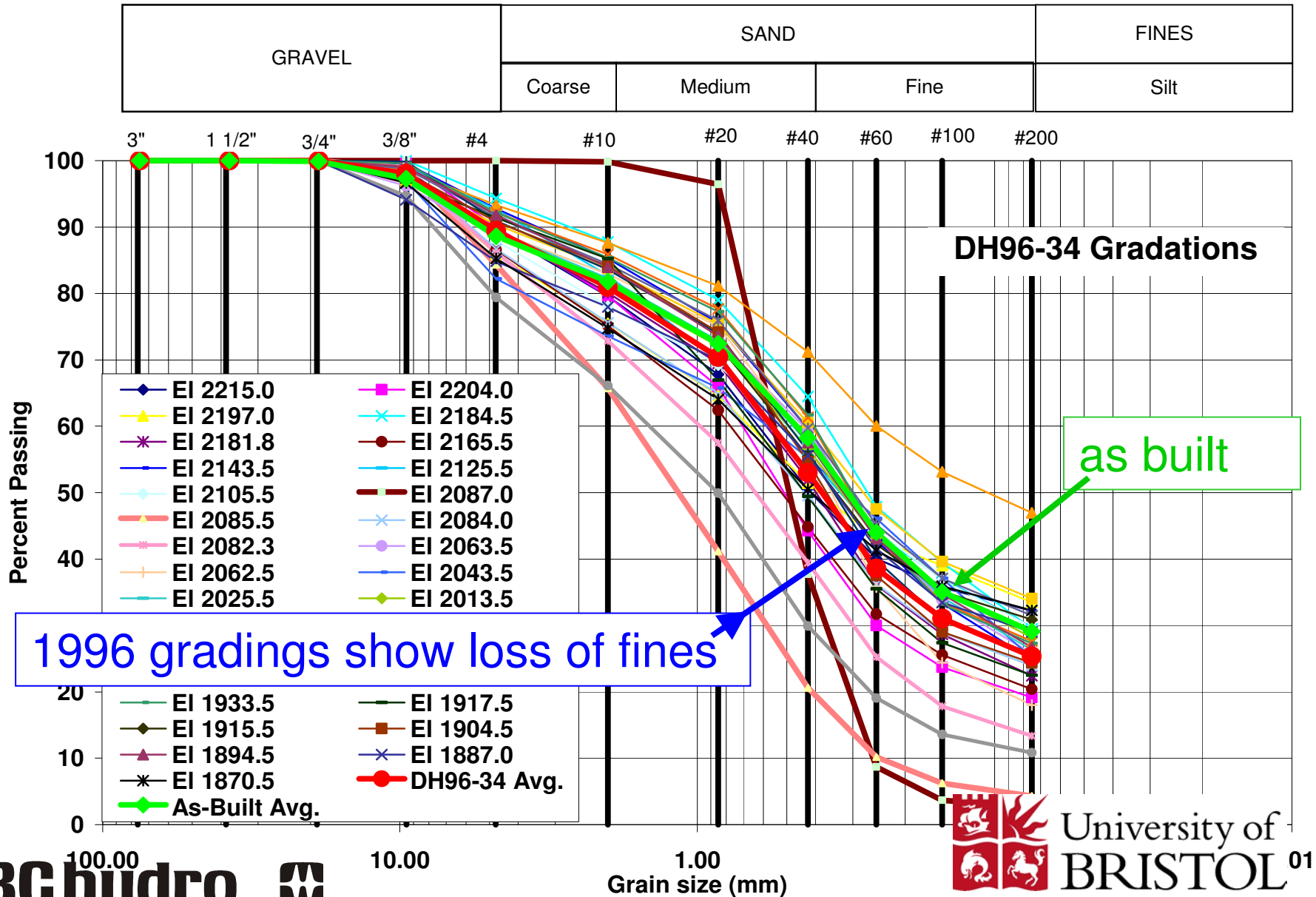
zoned earthfill dam

blended 'till-like' core

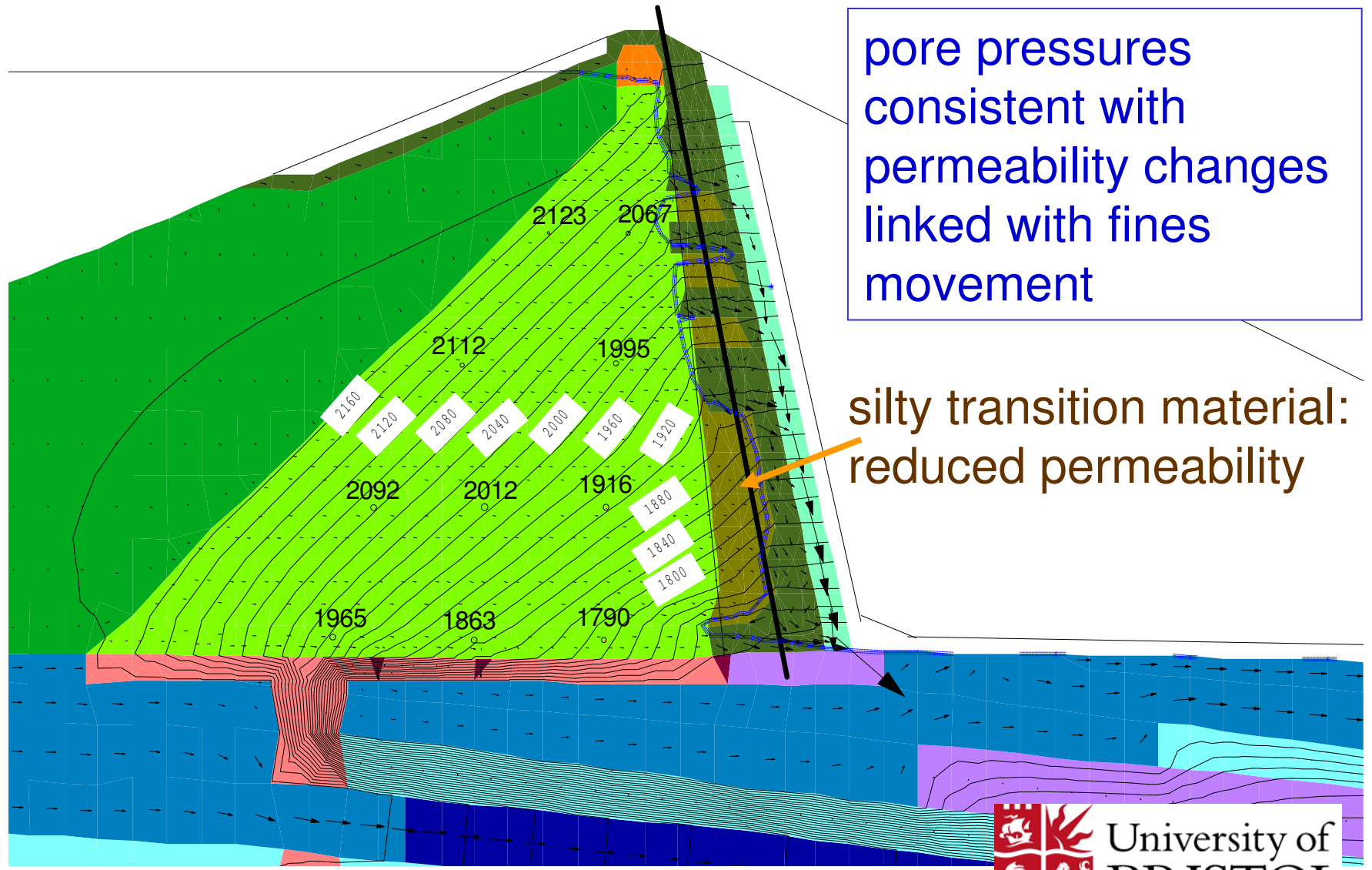
well-graded transition and filter

high capacity drain

# Core material (Sinkhole 1)



# Canyon section: 2003 seepage model



# WAC Bennett Dam: statement of problem

non-plastic material with changing granulometry and density

- what are consequences for mechanical response of dam?
- potential for future deformations?

need model of soil behaviour which can incorporate changes in density and grading of the soil

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# Particle-continuum duality

paradox of sand:

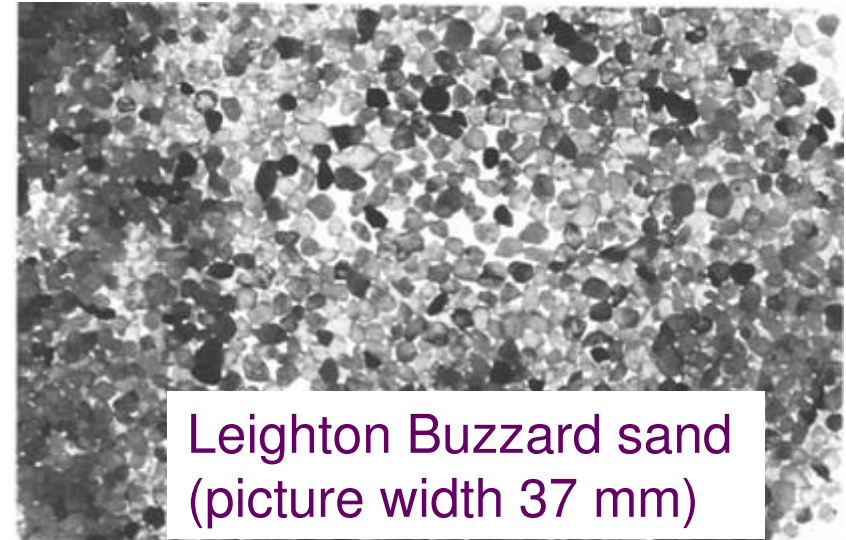
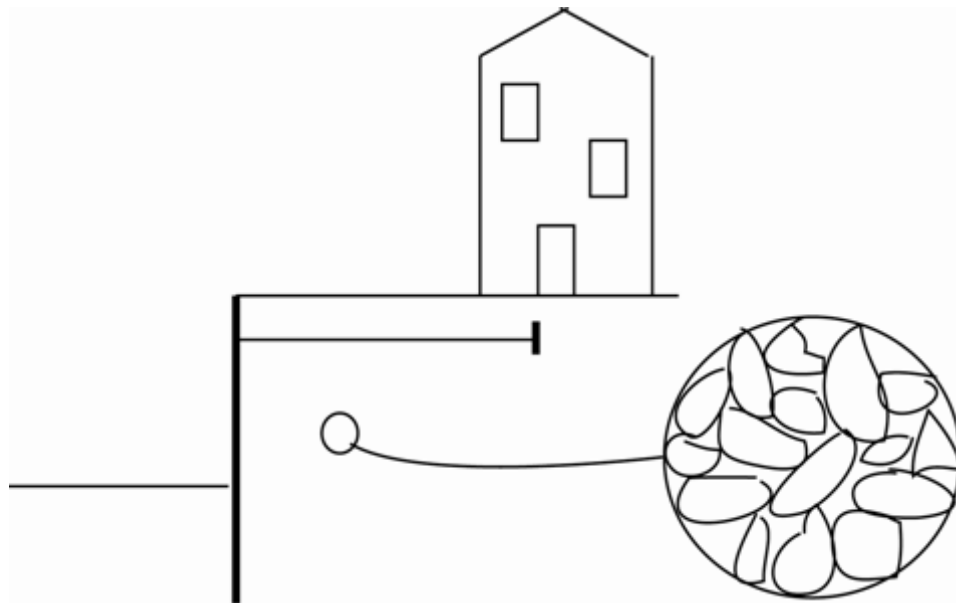
blows in the wind

flows like a liquid

but supports loads like a solid



# Particle-continuum duality



for analysis we need to treat soil as a continuum (stress, strain)

... but its properties emerge from its particulate nature

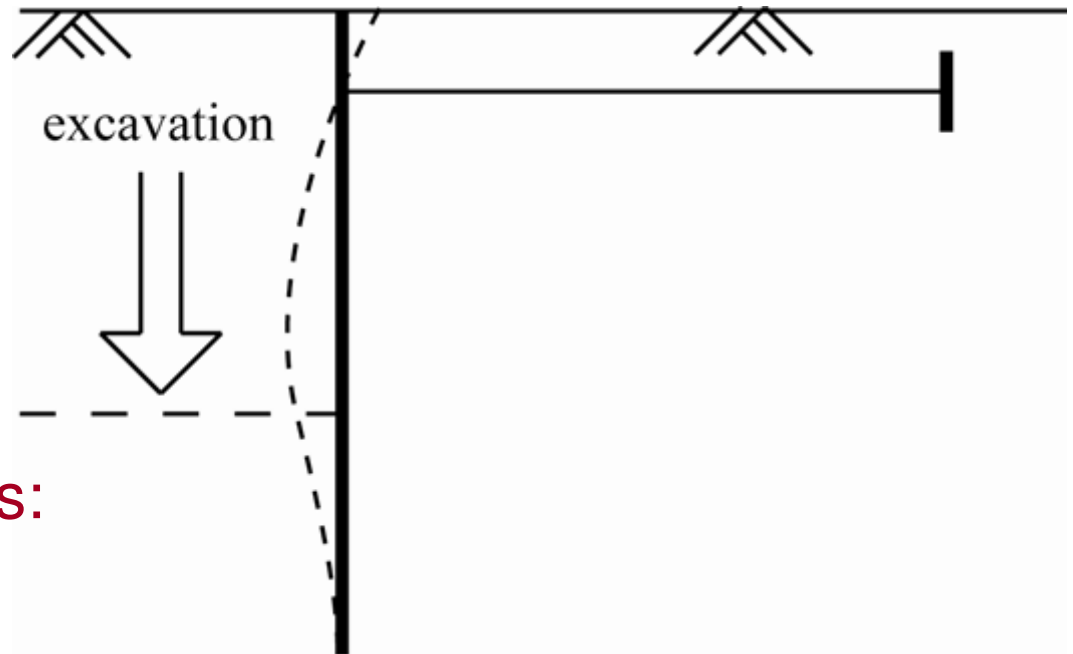
# The magic of sands

- Introduction: context: WAC Bennett Dam
- Particle continuum duality
- **Mohr-Coulomb model**
- Elastic properties
- *Critical states*
- **Severn-Trent sand**
- Grading state index
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# Mohr-Coulomb model

*model* = appropriate simplification of reality



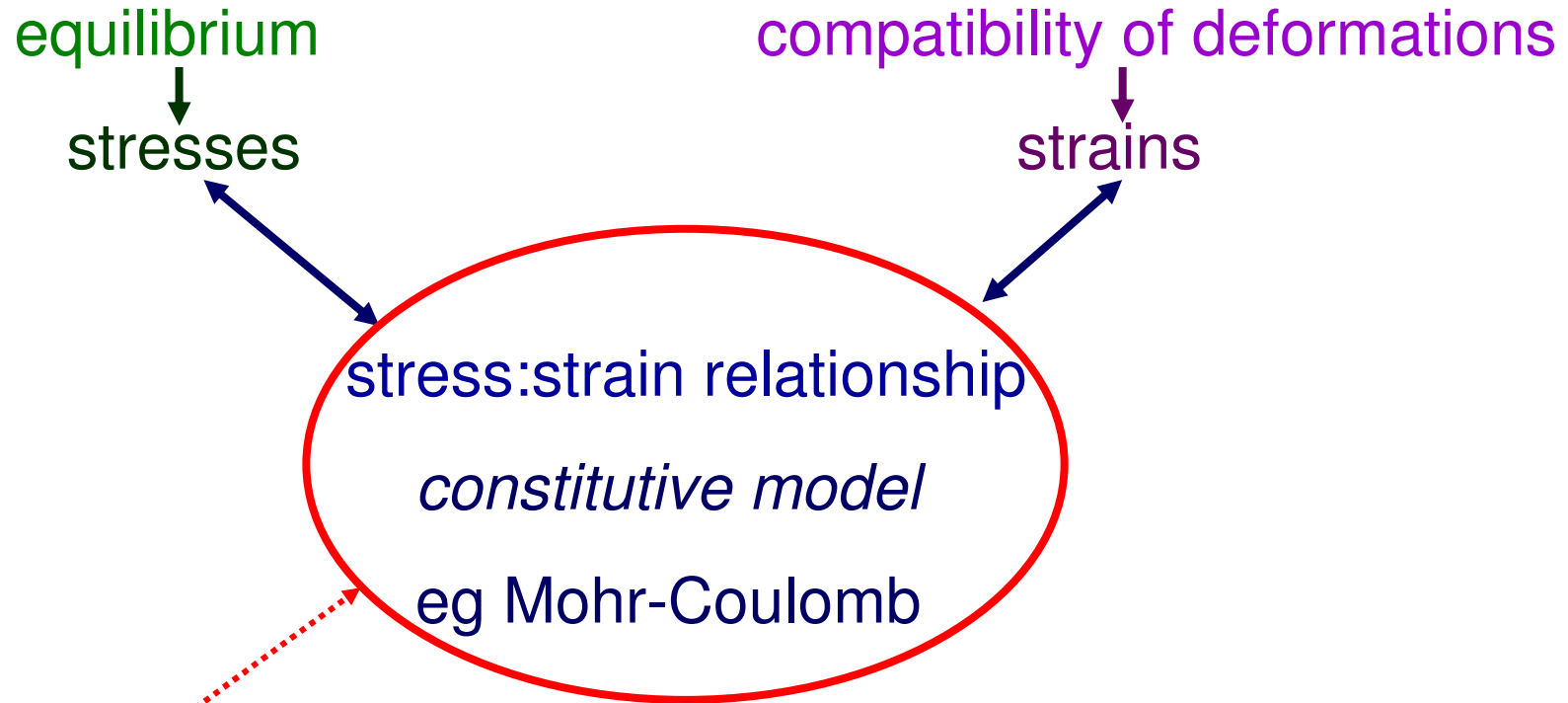
modelling for working loads:

- serviceability limit states
- deformations of geotechnical systems

usually requires numerical analysis:

- for example, finite element, finite difference

# Mohr-Coulomb model



*constitutive model*: central element of numerical modelling

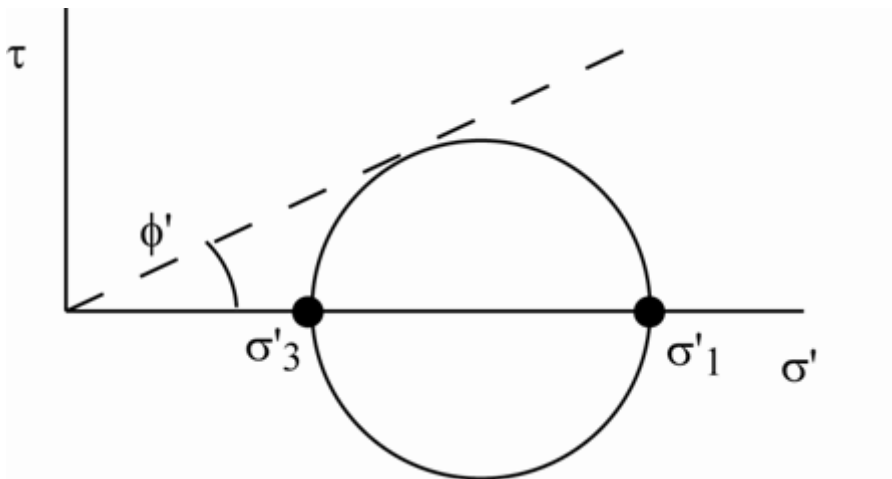
linking *change* in stress with *change* in strain

generalised evolving incremental stiffness

# Mohr-Coulomb model: failure

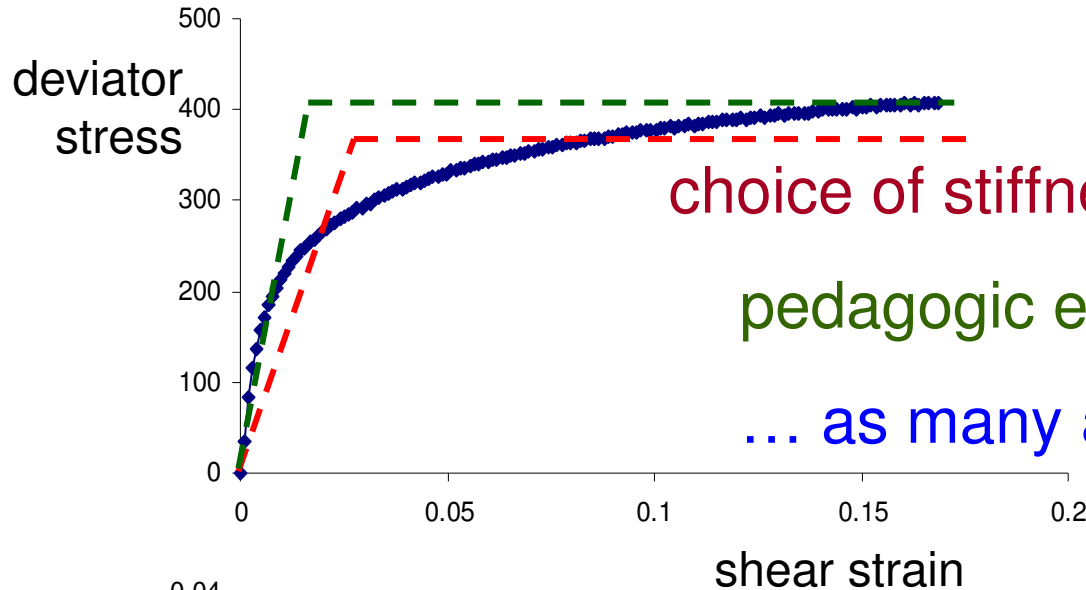
Mohr-Coulomb familiar as strength/failure model

limiting stress ratio characterised by frictional strength  $\phi'$   
(ignore cohesion)



drives laboratory and *in situ* investigation of soil properties

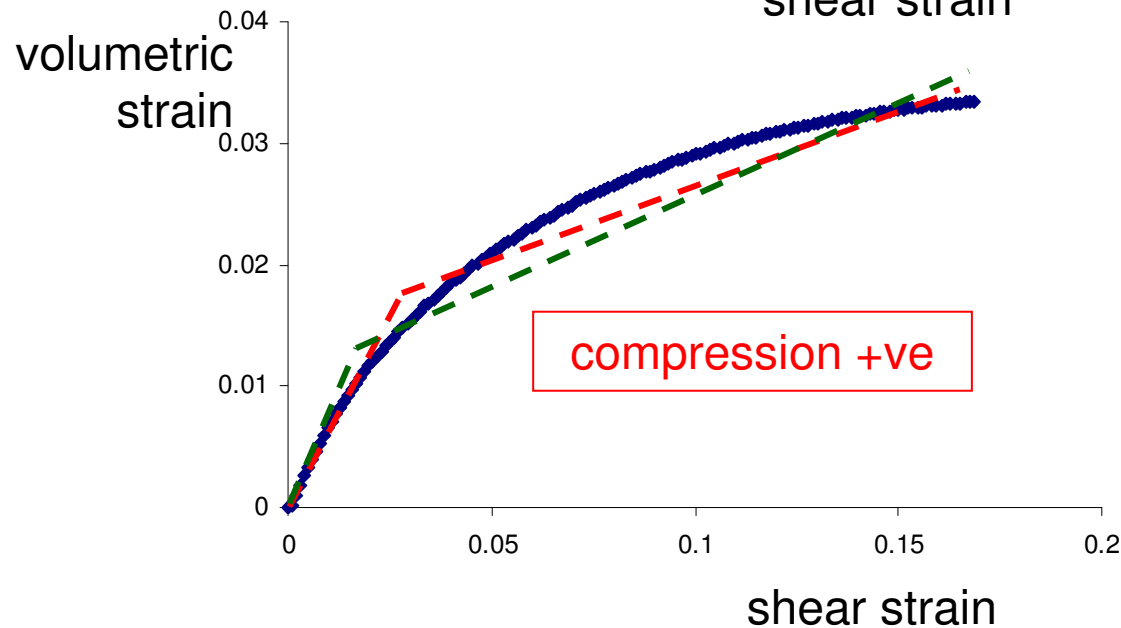
# Mohr-Coulomb model: parameter selection



choice of stiffness, strength, dilatancy

pedagogic exercise

... as many answers as people!



compression +ve

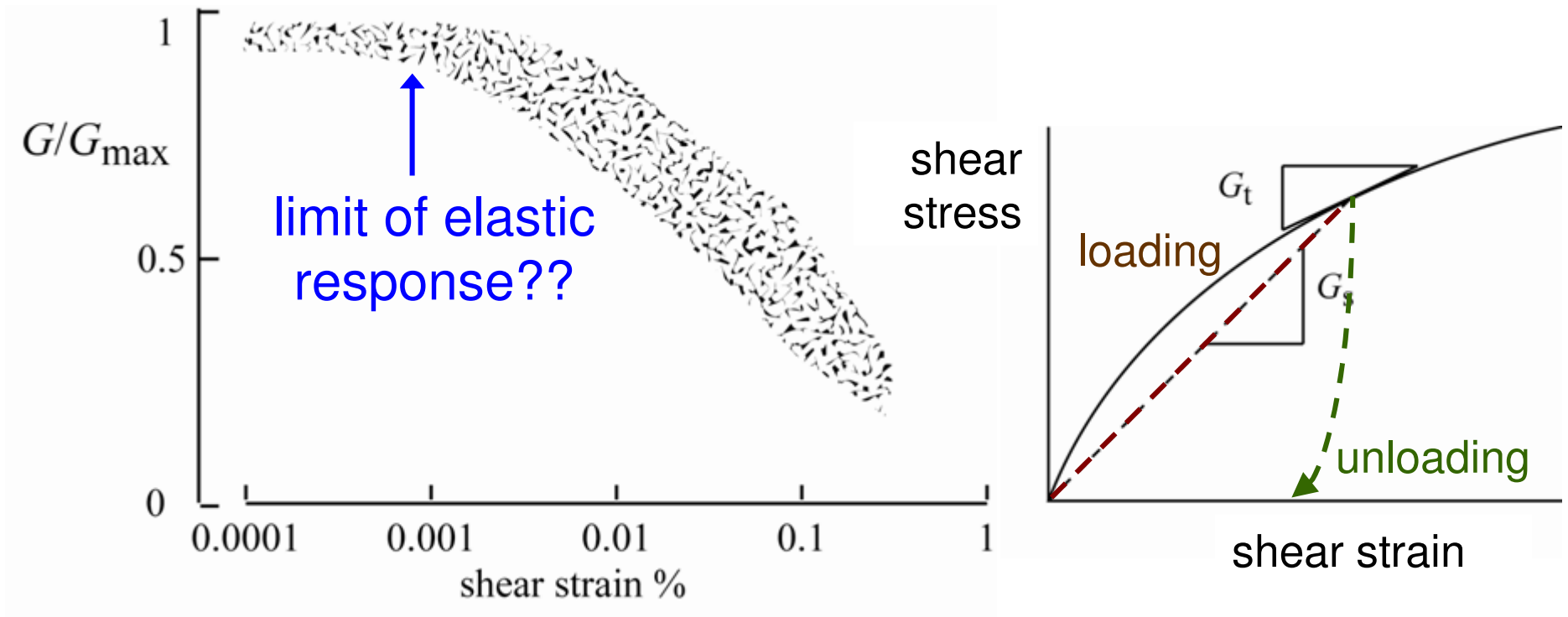
loose Hostun sand

Benahmed, 2001

# The magic of sands

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# Elastic properties



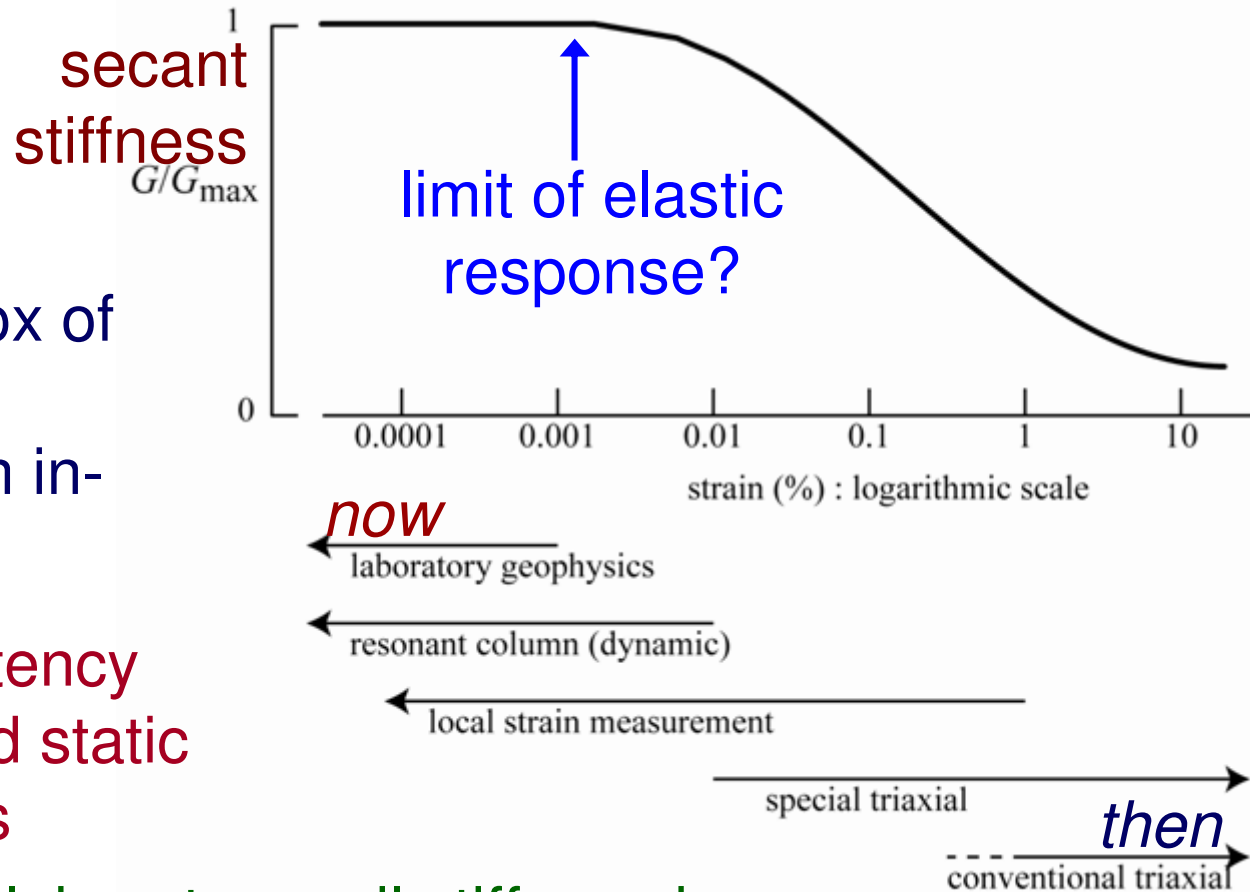
*stiffness* does not imply *elasticity* (recoverable deformations)

fall of stiffness  $\Rightarrow$  *plasticity* (permanent changes on unloading)

... hence need for constitutive models

Quiou sand (LoPresti *et al.*, 1997)

# Elastic properties



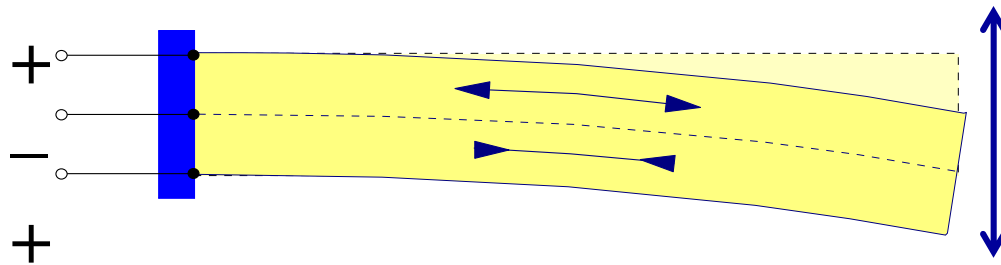
*then* ... paradox of low laboratory stiffness – high in-situ stiffness

*now* ... consistency of dynamic and static measurements

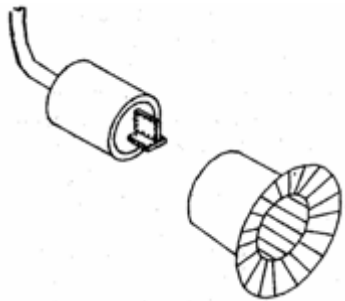
perceptions of laboratory soil stiffness have changed as instrumentation for measurement of deformations has improved

# laboratory geophysics: piezoceramic elements

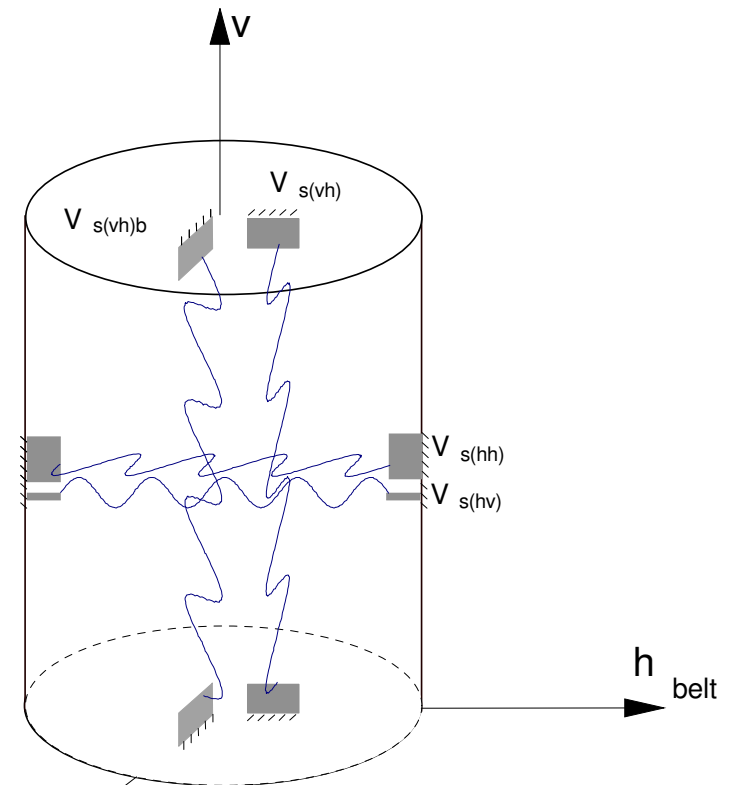
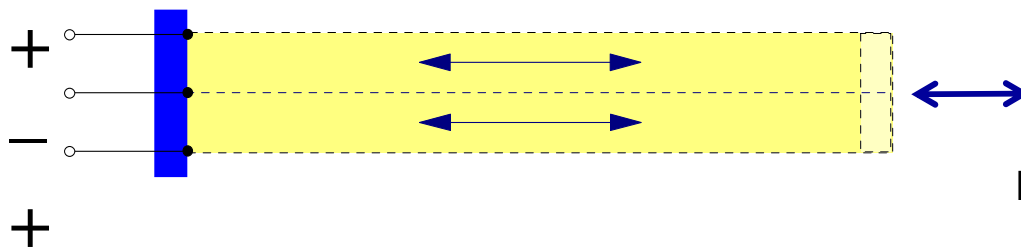
bender element (s-waves)



bender elements  
on triaxial sample



extender element (p-waves)



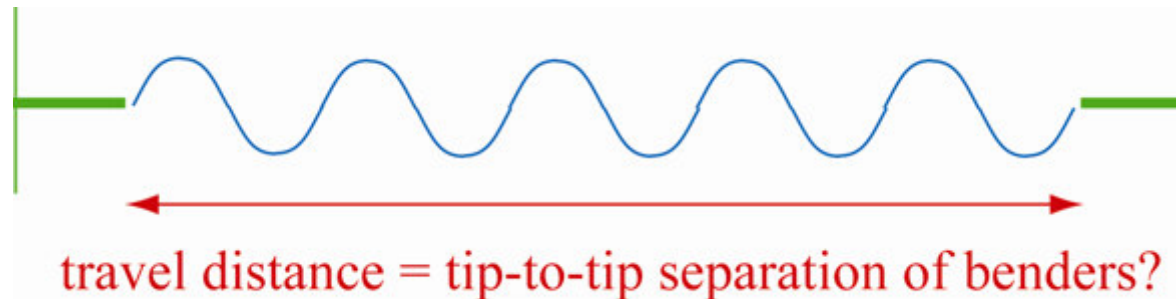


# deduction of elastic stiffness

elastic stiffness deduced from shear (or compression)

wave velocity  $G = \rho v_s^2$

velocity = distance/time

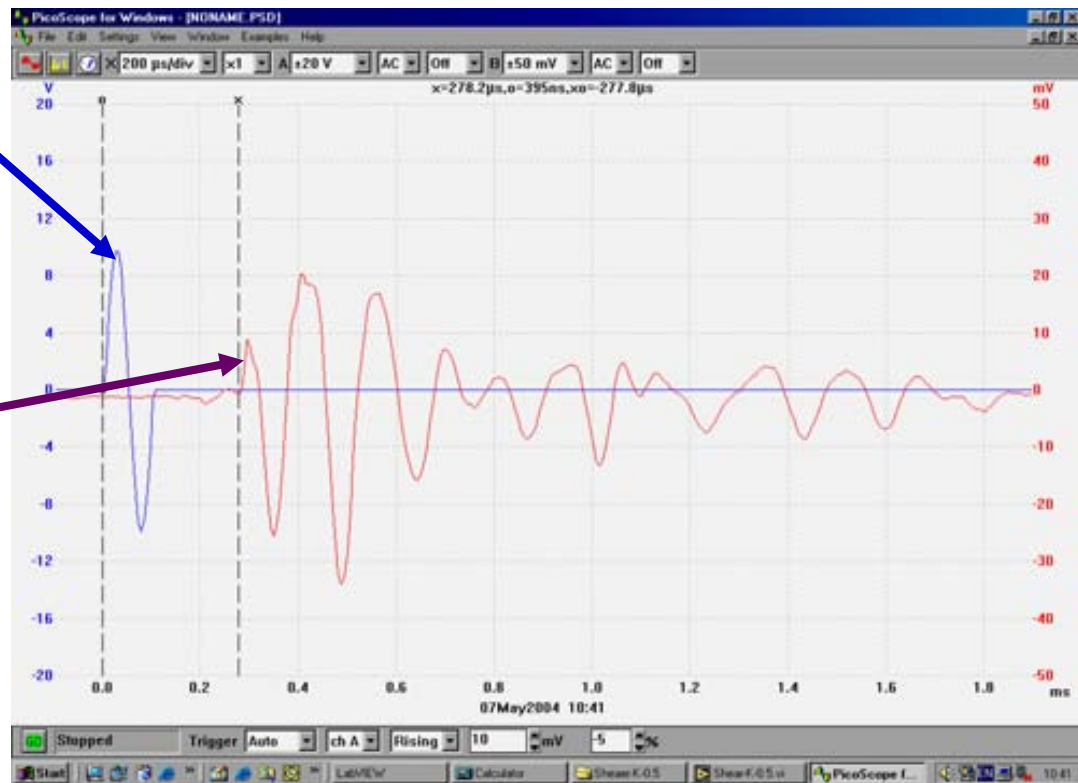


arrival time? complex (attenuated) received signals – reflections, dispersion

# the problem:

input signal: single sine pulse!!

output signal: complex

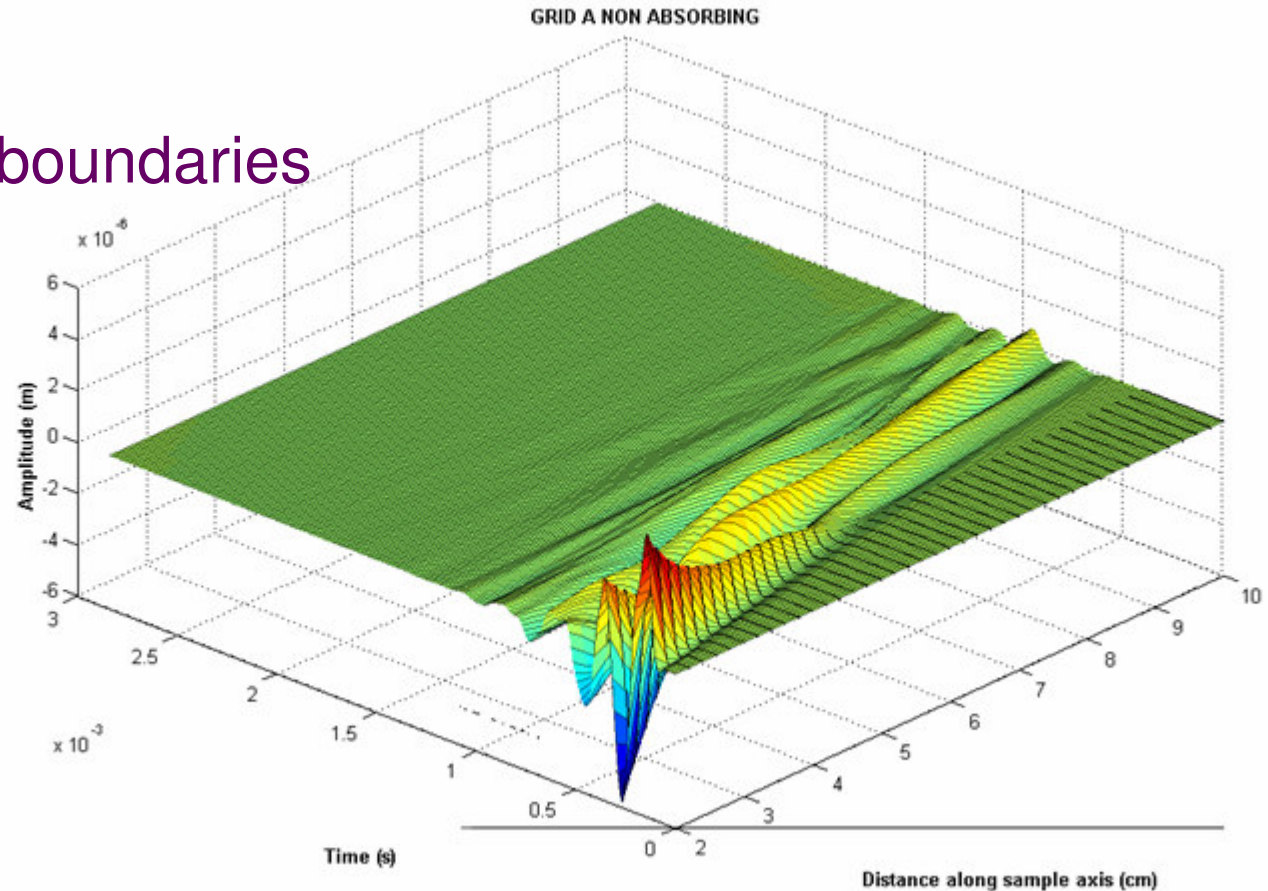
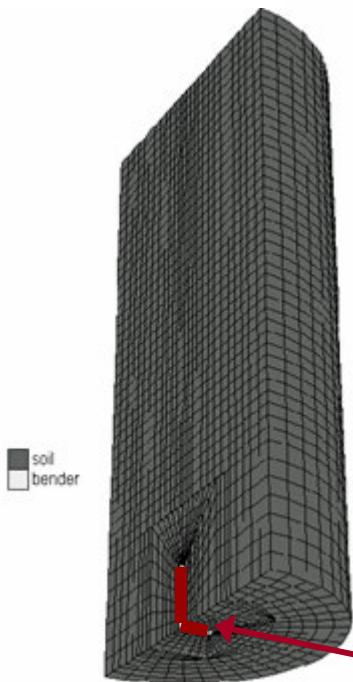


arrival time?

# elastic properties: numerical simulation

non-absorbing boundaries

FLAC-3D grid

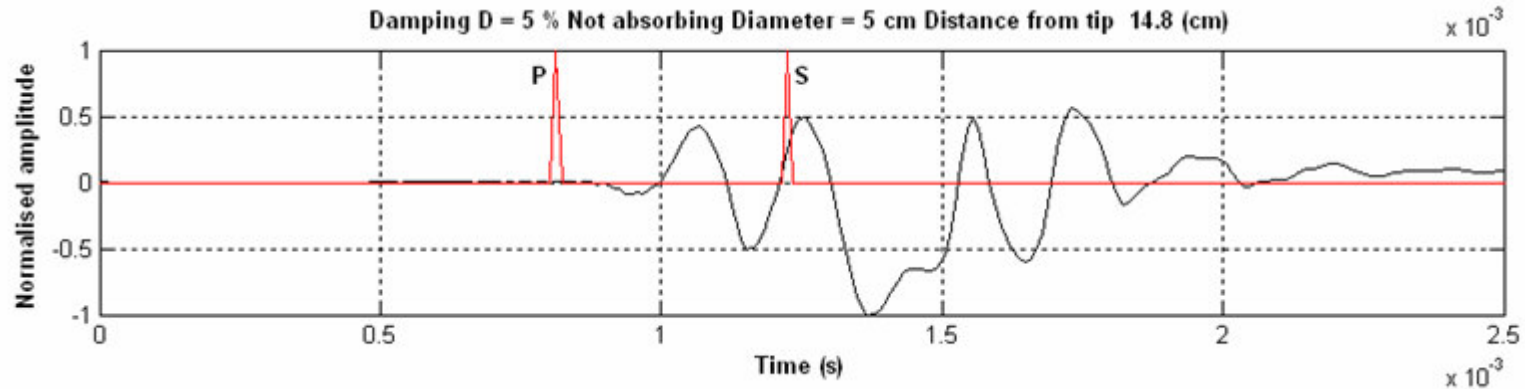


reflection at boundaries

conversion of *s*-wave to *p*-wave energy

bender element

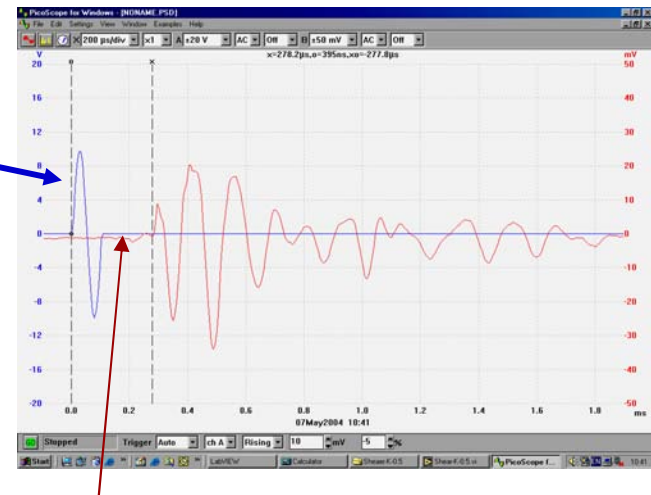
# interpretation of numerical simulations



numerical simulation: objective deduction of arrival time??

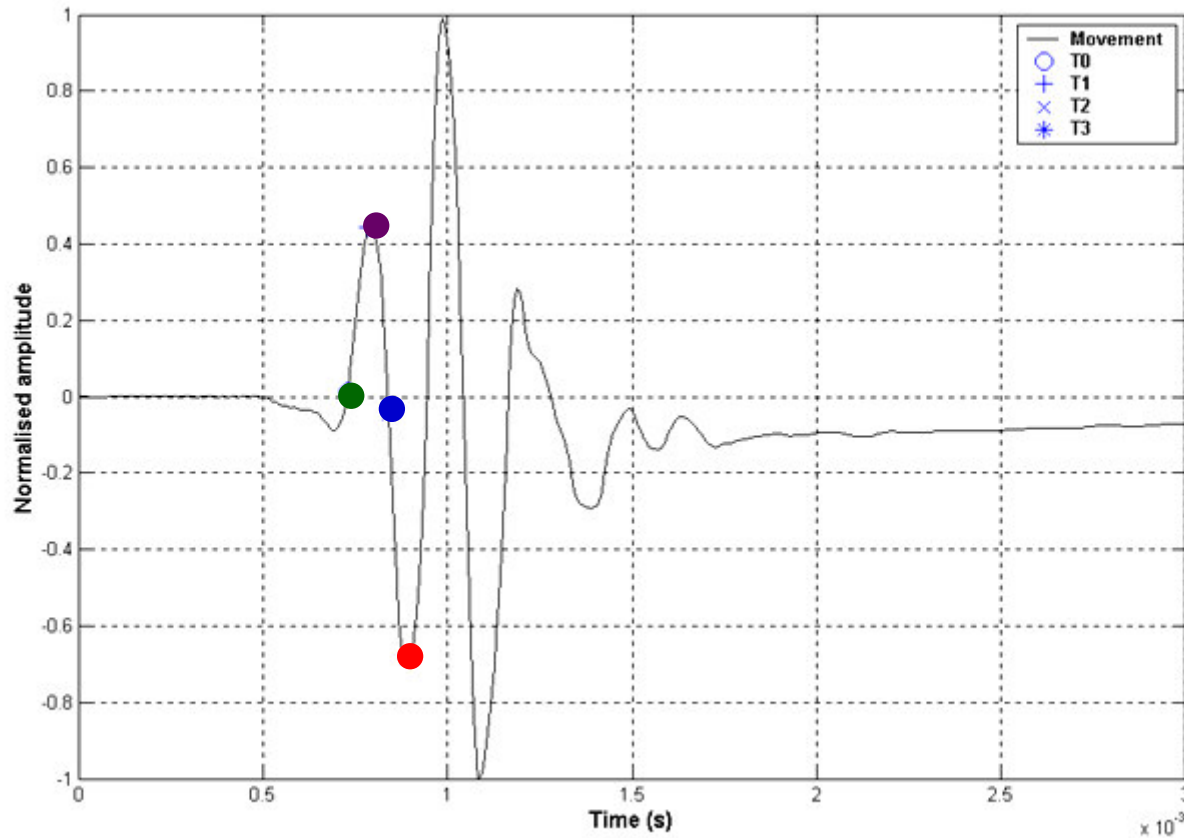
input signal: single sine pulse!!

compare theoretical  $p$ -wave and  $s$ -wave arrival times



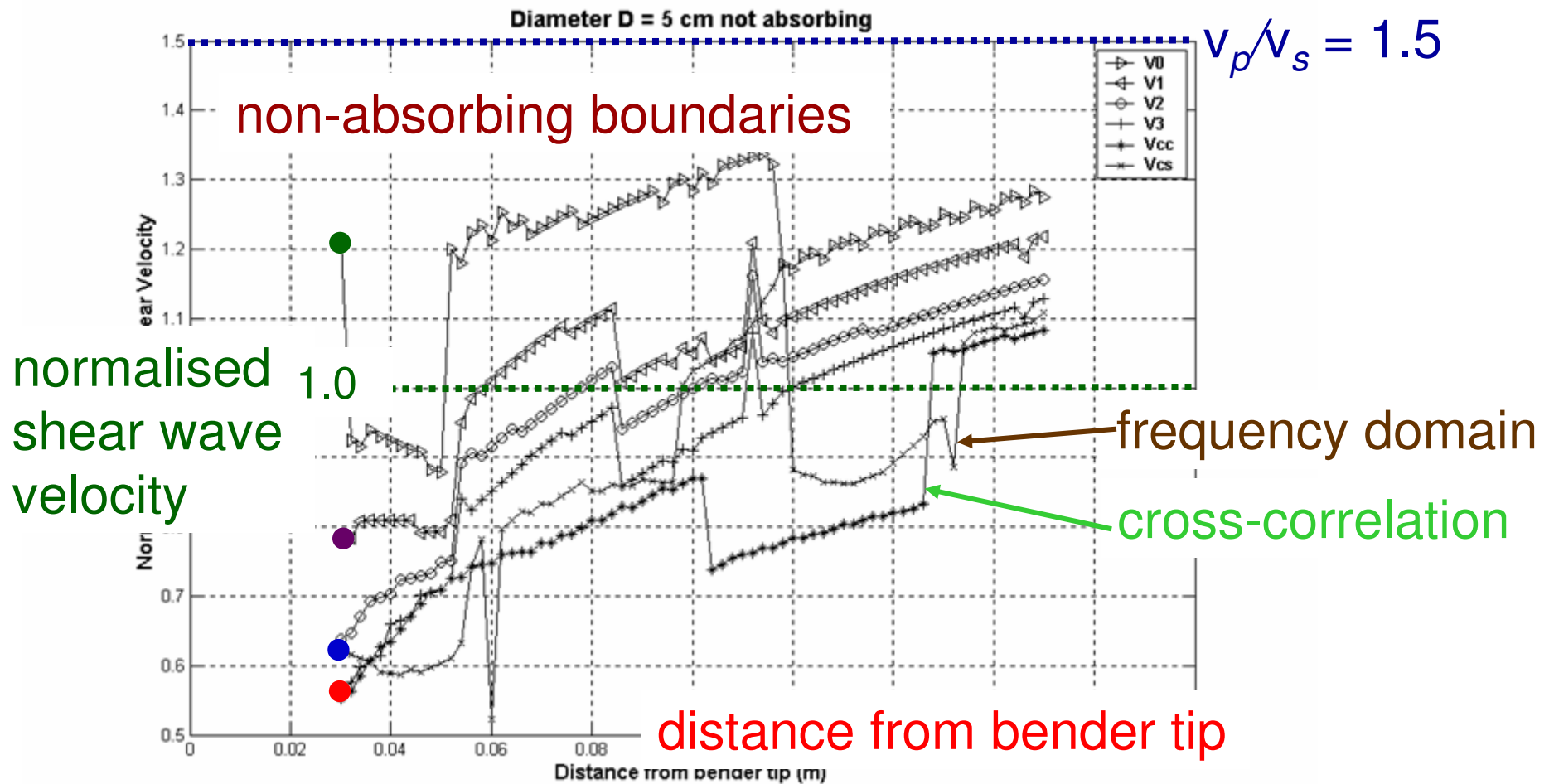
experimental observation: note  $p$ -wave arrival in  $s$ -wave trace

# interpretation of numerical simulations



typical choices for arrival time from inspection of received signal

# interpretation of numerical simulations



no clear trends?

$p$ -wave dominating with distance

# Severn-Trent sand

Severn – river adjacent to Bristol

Trent – Alessandro Gajo, Italy

- Particle continuum duality

• Mohr Coulomb model

Trento ... properties

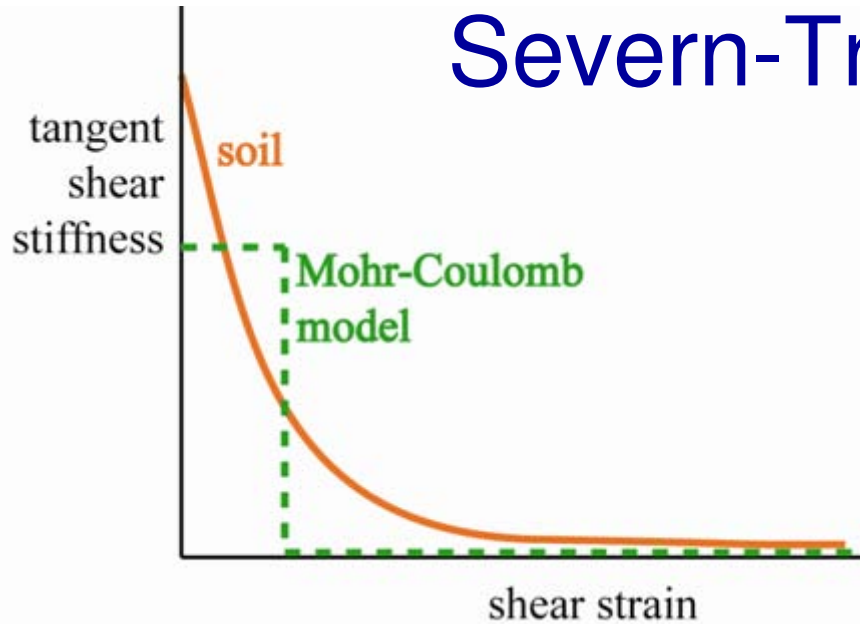
- C
- S
- G
- C



and

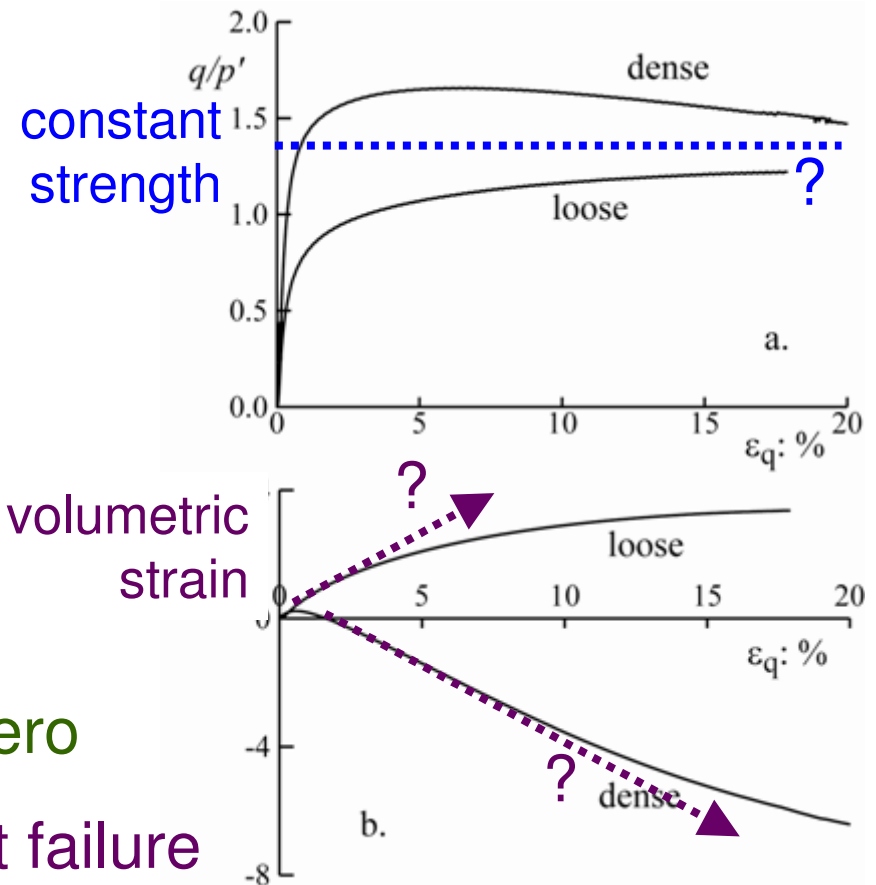


# Severn-Trent sand



## Mohr-Coulomb model: defects?

- incremental stiffness elastic or zero
- indefinite dilation/compression at failure
- strength chosen as soil constant



Hostun sand (Benahmed)



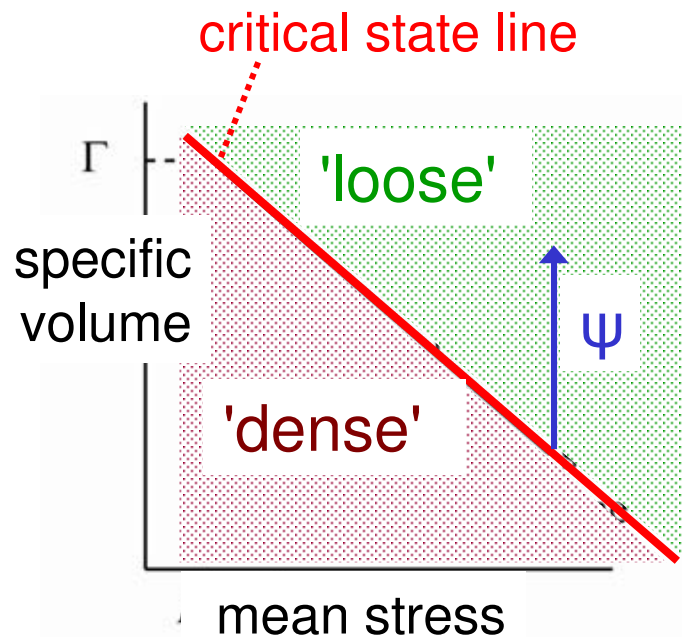
# Severn-Trent sand: strength

what is peak strength?

- property of the soil which changes with stress level, density

- dependent quantity*

# Severn-Trent sand: strength



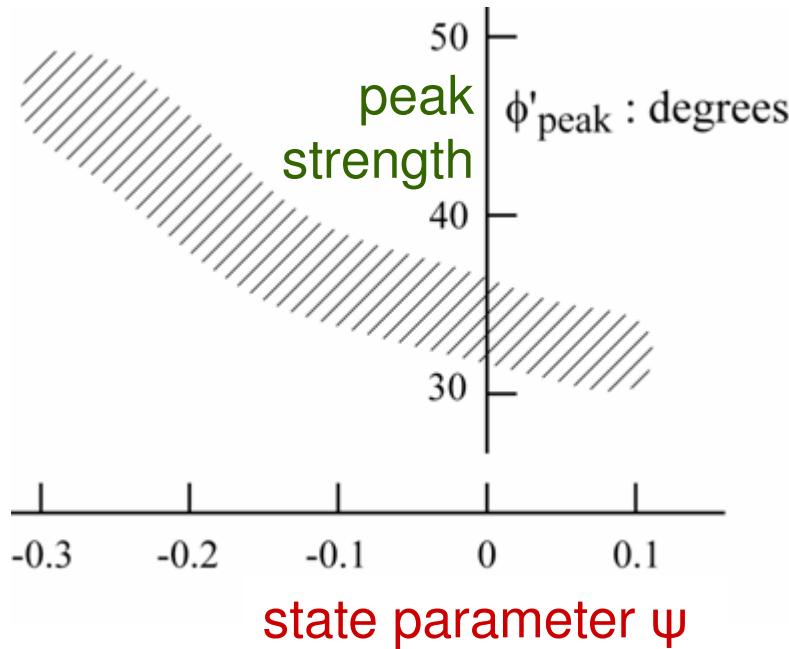
state parameter  $\psi$  = volume distance from critical state line

function of density *and* stress level

more useful than void ratio alone – indicating effect of density *and* stress

Been & Jefferies

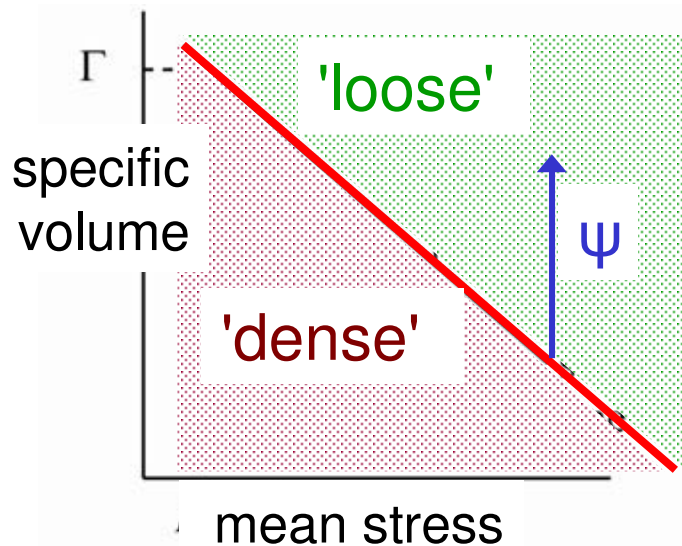
# Severn-Trent sand: strength



what is peak strength?

- property of the soil which changes with stress level, density

data confirm link between strength and state parameter  $\psi$



**Mohr-Coulomb model with *current* strength dependent on *current* stress level and density**

Been & Jefferies

# Severn-Trent sand: dilatancy

dilatancy: volume change during shearing

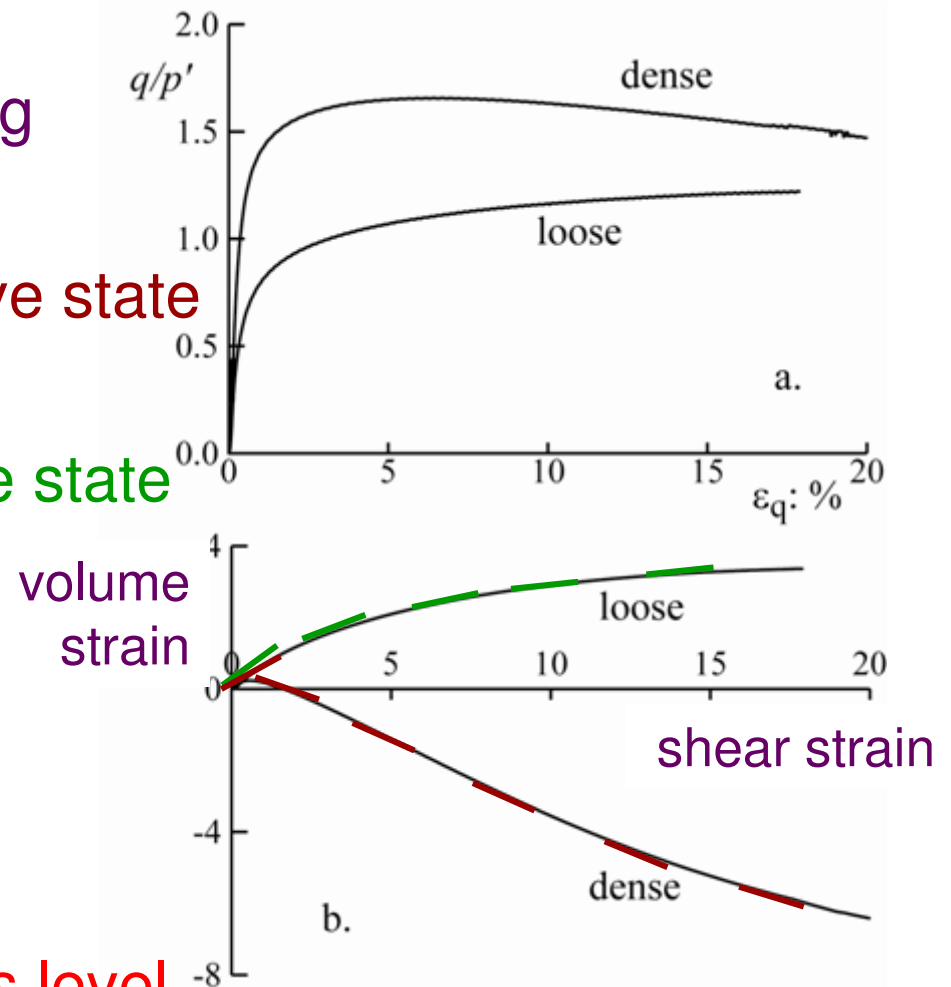
'dense' sand expands – negative state parameter

'loose' sand contracts – positive state parameter

dilatancy depends on density

dilatancy varies during test

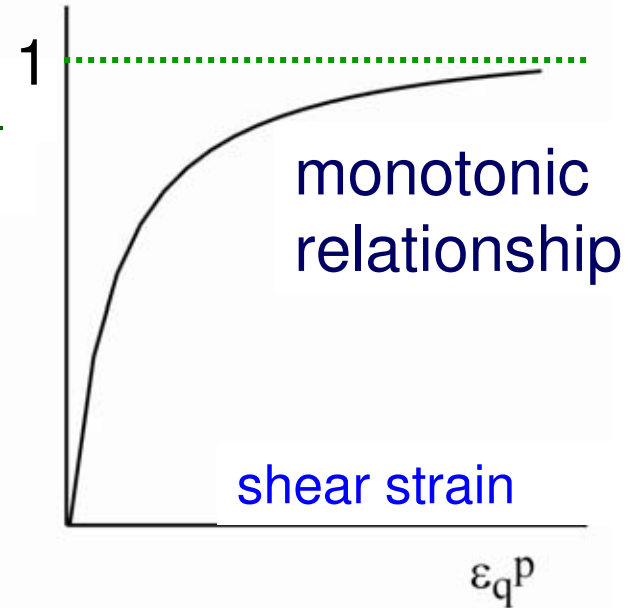
link with state parameter (stress level and density)



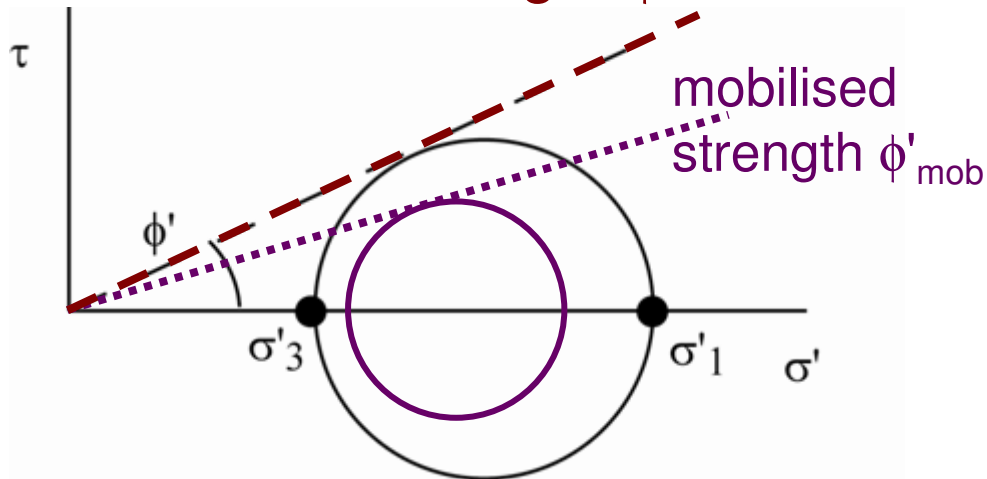
Benahmed

# Severn-Trent sand: stiffness evolution

ratio  $\frac{\text{currently mobilised strength}}{\text{currently available strength}}$



available strength  $\phi'$ : varies with  $\psi$



# Severn-Trent sand

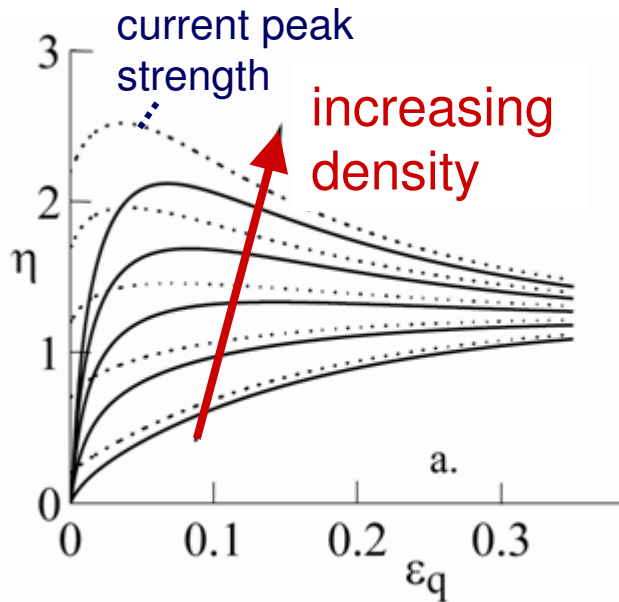
model complete

- monotonic travel towards *current* strength
- *current* strength depends on *current* density (state parameter)
- shearing leads to *change* in density (dilatancy): soil seeking critical state
- *change* in density leads to *change* in strength

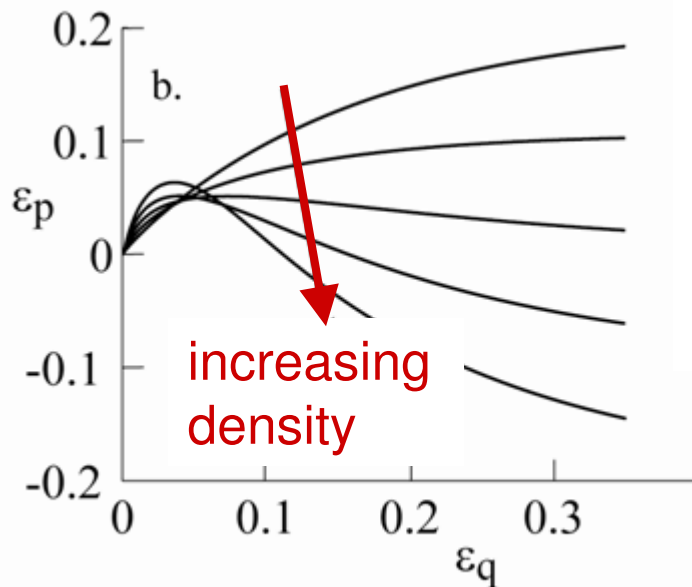
etc

predicted softening for dense sand as emergent property

# Severn-Trent sand: simulations



drained triaxial compression tests  
different initial density (state  
parameter)

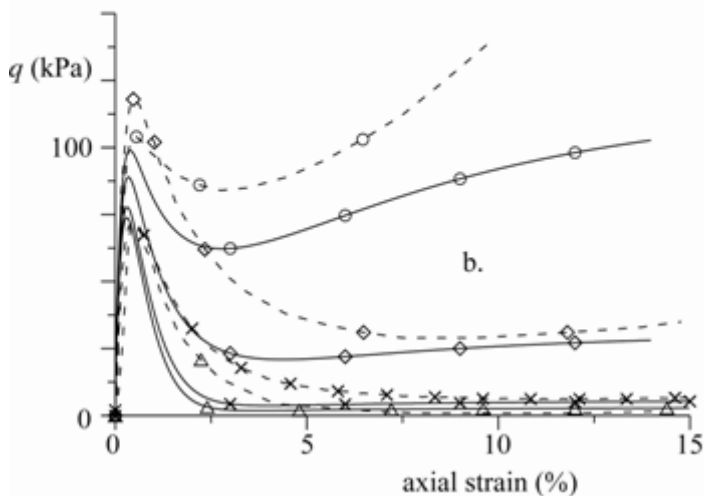
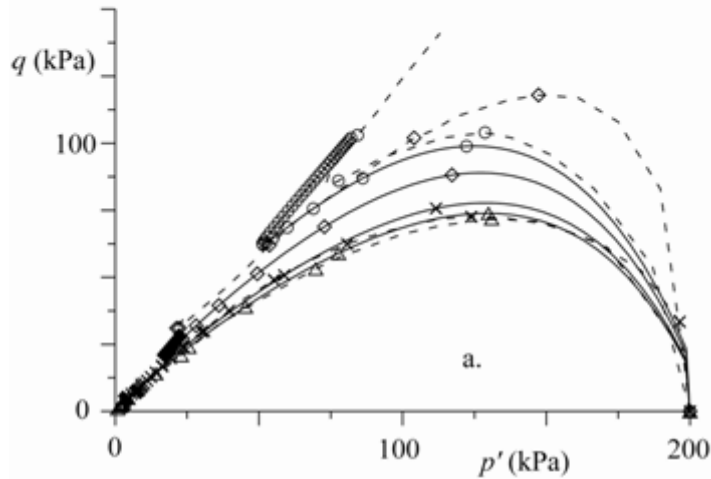


model automatically homes in on  
critical state

peak strength is moving target  
reached at infinite distortional strain –  
critical state

# Severn-Trent sand: calibration

- $e=0.83$  - batr06 (model)
- ◇  $e=0.87$  - batr05 (model)
- ×  $e=0.92$  - alert51 (model)
- △  $e=0.94$  - batr02 (model)



calibrated against triaxial test data  
for Hostun sand

undrained triaxial compression

effect of different initial density  
automatically described

Gajo & Muir Wood, 1999



# Severn-Trent sand

Mohr-Coulomb model with extra features

building on familiar foundations

- central role of state parameter

- feedback through dilatancy

- rich patterns of response simulated

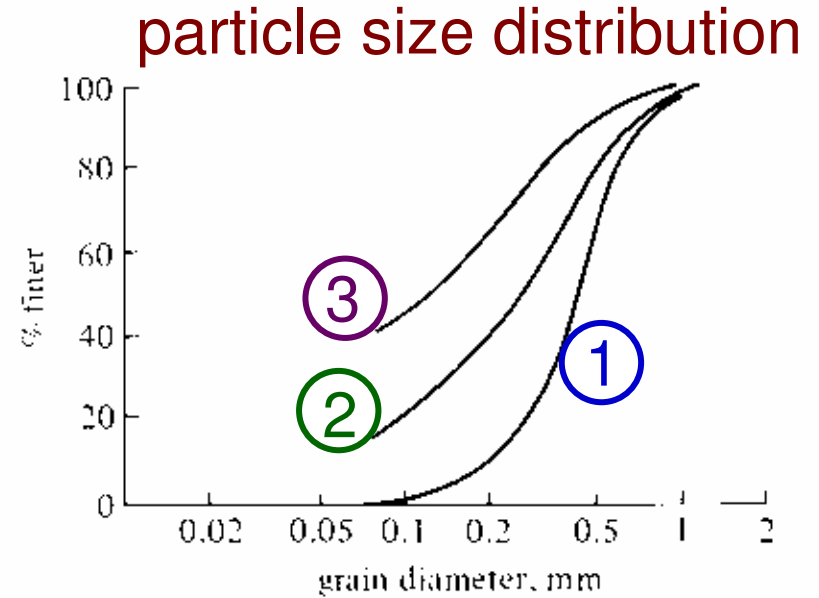
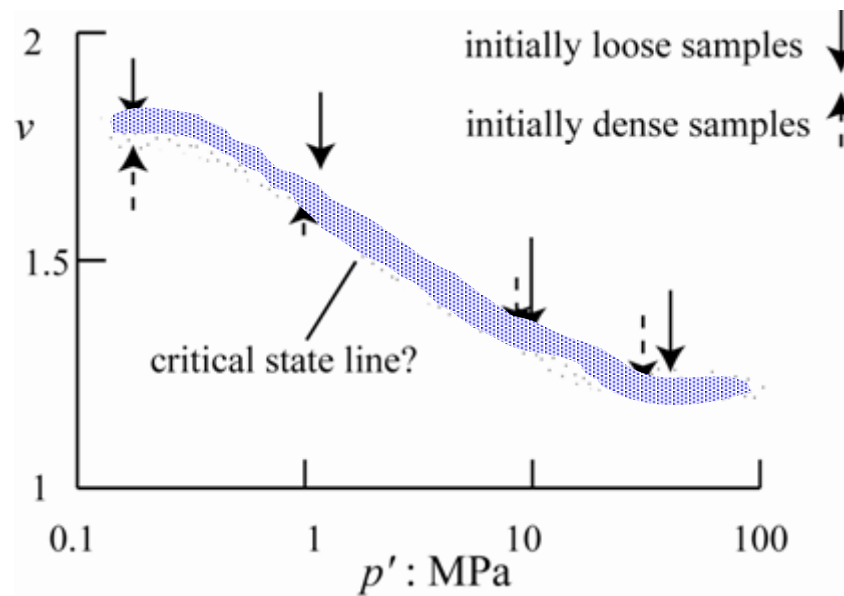
mathematically elegant

economical in demand for soil parameters

# The magic of sands

- Introduction: context: WAC Bennett Dam
- Particle continuum duality
- Mohr-Coulomb model
- Elastic properties
- *Critical states*
- Severn-Trent sand
- Grading state index
- Conclusion

# Grading state index



occurrence of crushing

change in grading

irreversible

1: before testing

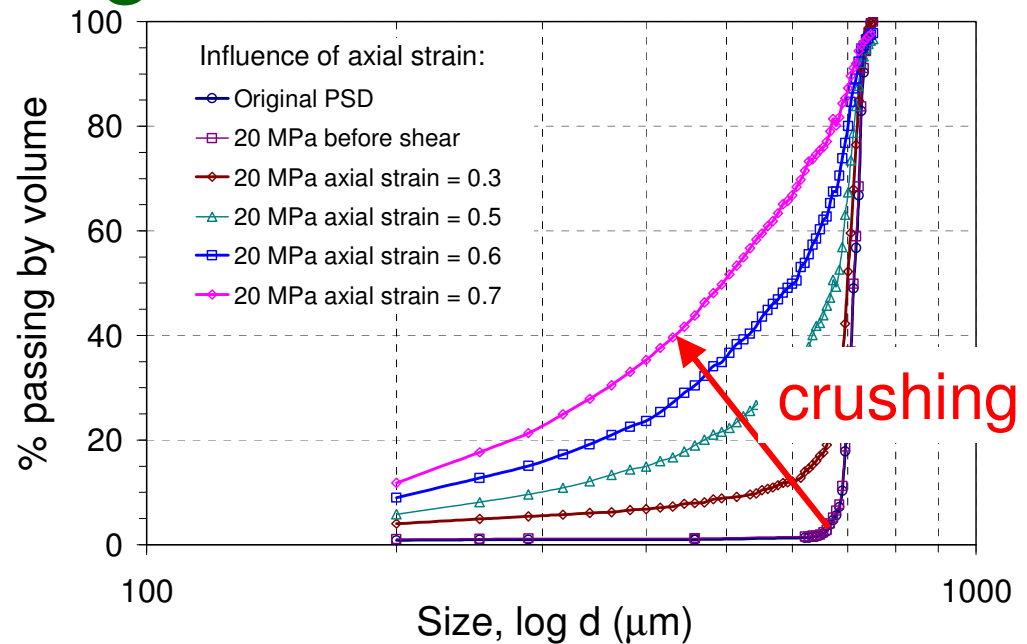
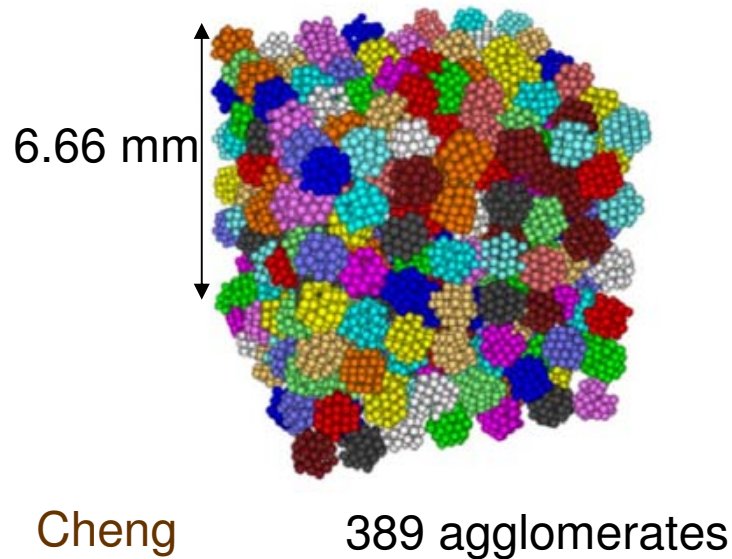
2: after compression to 6.21 MPa

3: after triaxial compression

Chattahoochee River sand

Vesic & Clough, 1968

# Grading state index

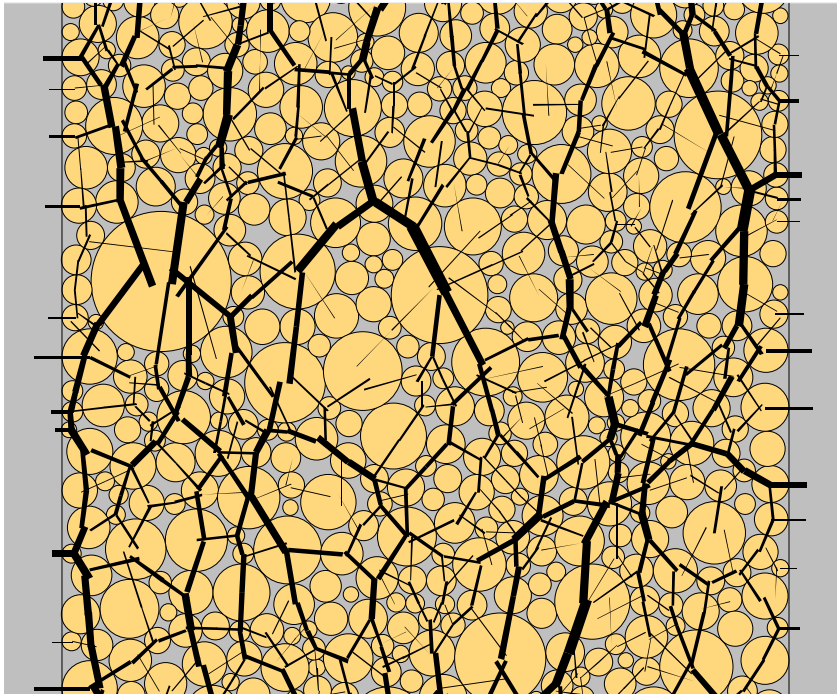


numerical simulations – compression and shearing of assembly of agglomerates

gradings tend to *self similar* 'fractal' grading

continuous 'fractal' grading: every void space filled with progressively smaller particles

# Grading state index: crushing

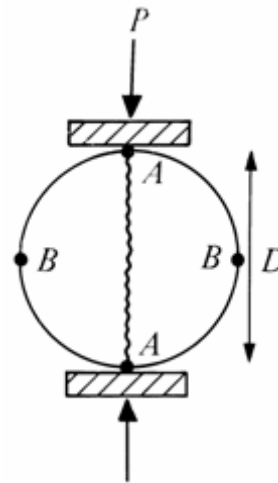


Maeda (2005)

crushing?

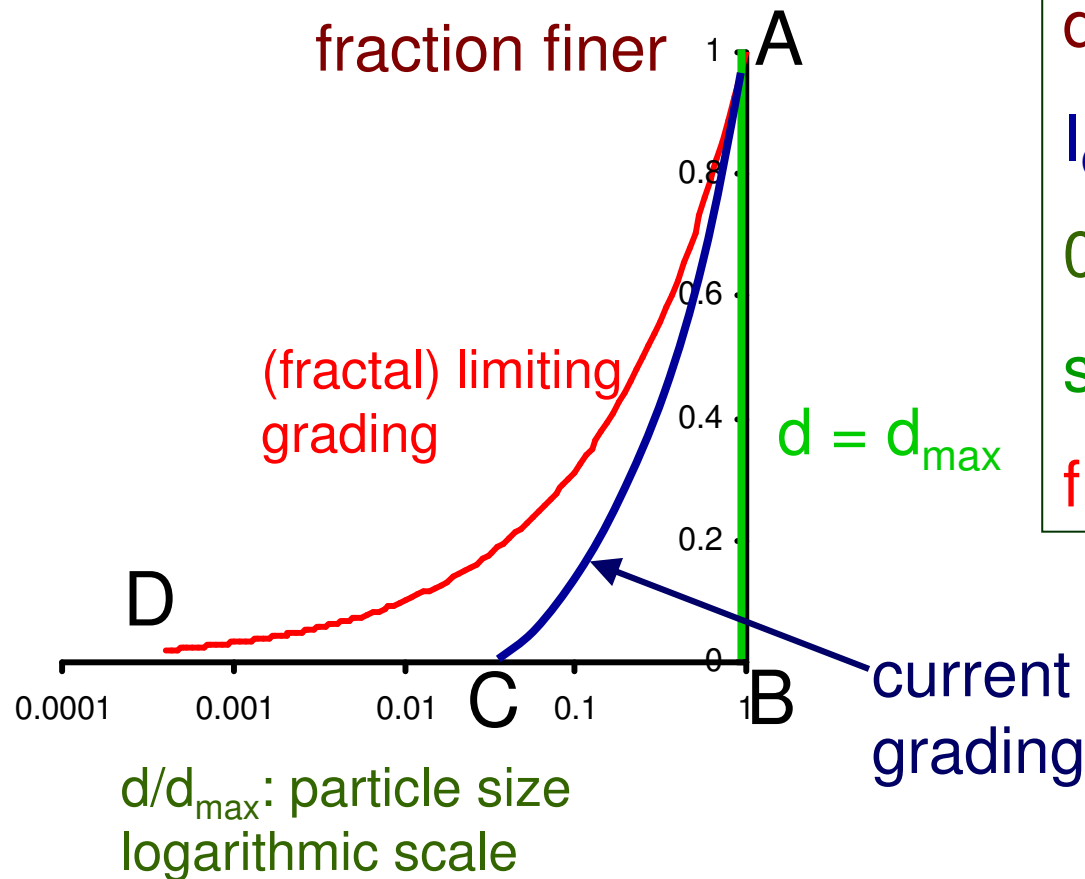
coordination number (number of contacts) larger for larger particles

*smaller* particles tend to crush



Brazil cylinder test:  
*tensile* strength of  
concrete

# Grading state index $I_G$ : definition



definition of  $I_G$ ?

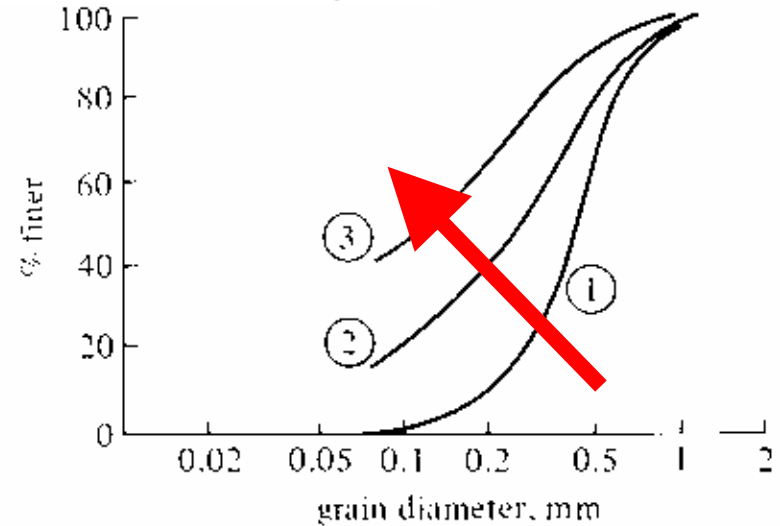
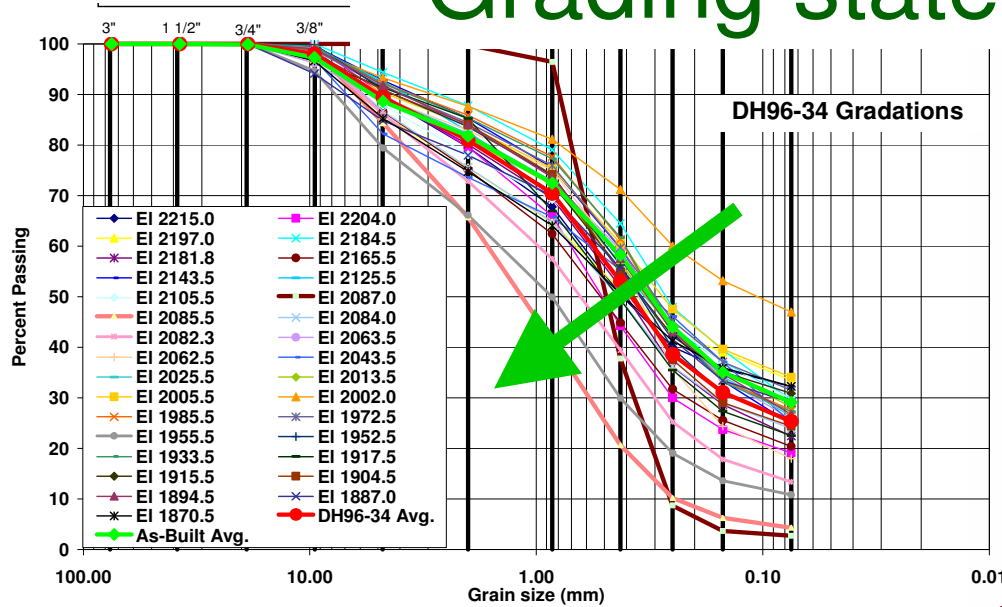
$$I_G = \text{area ABC} / \text{area ABD}$$

$$0 < I_G < 1$$

single size AB:  $I_G = 0$

fractal limit AD:  $I_G = 1$

# Grading state index $I_G$

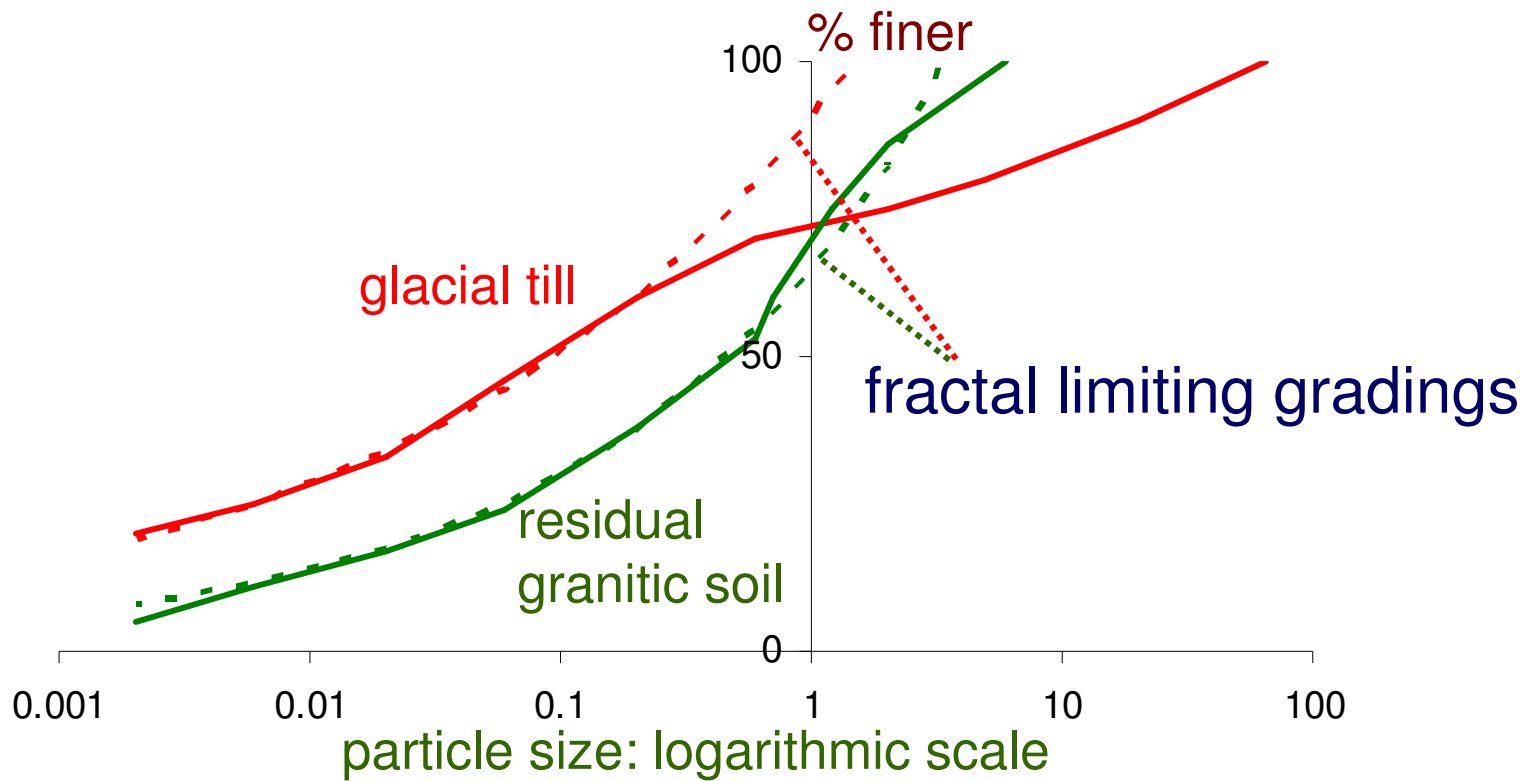


WAC Bennett Dam core  
fines removal:  $I_G$  falling

Chattahoochee River sand  
grain crushing:  $I_G$  increasing

- soil grading change: erosion/transport or crushing
- material changing (irreversibly) while being studied
- effect on mechanical behaviour?

# Grading state index



natural soils:

discovering fractal limiting gradings?



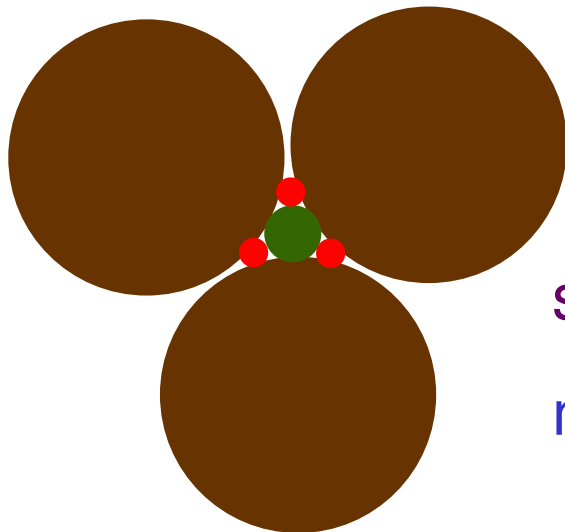
# Modelling grading change

- material changing (irreversibly) while being studied
- modelling requirements:
  - ❖ *characterisation* of evolving grading – additional grading state index
  - ❖ *evolution law* for grading state index (mass conservation, transport, crushing criteria, etc)
  - ❖ *influence* of grading state index on constitutive properties (for example, critical states)
- *research in progress: add  $I_G$  influence to existing model*

# Grading state index $I_G$ : influence

influence of grading state index on constitutive properties

- elastic properties – unchanged (first order)?
- friction/strength – unchanged (first order)?

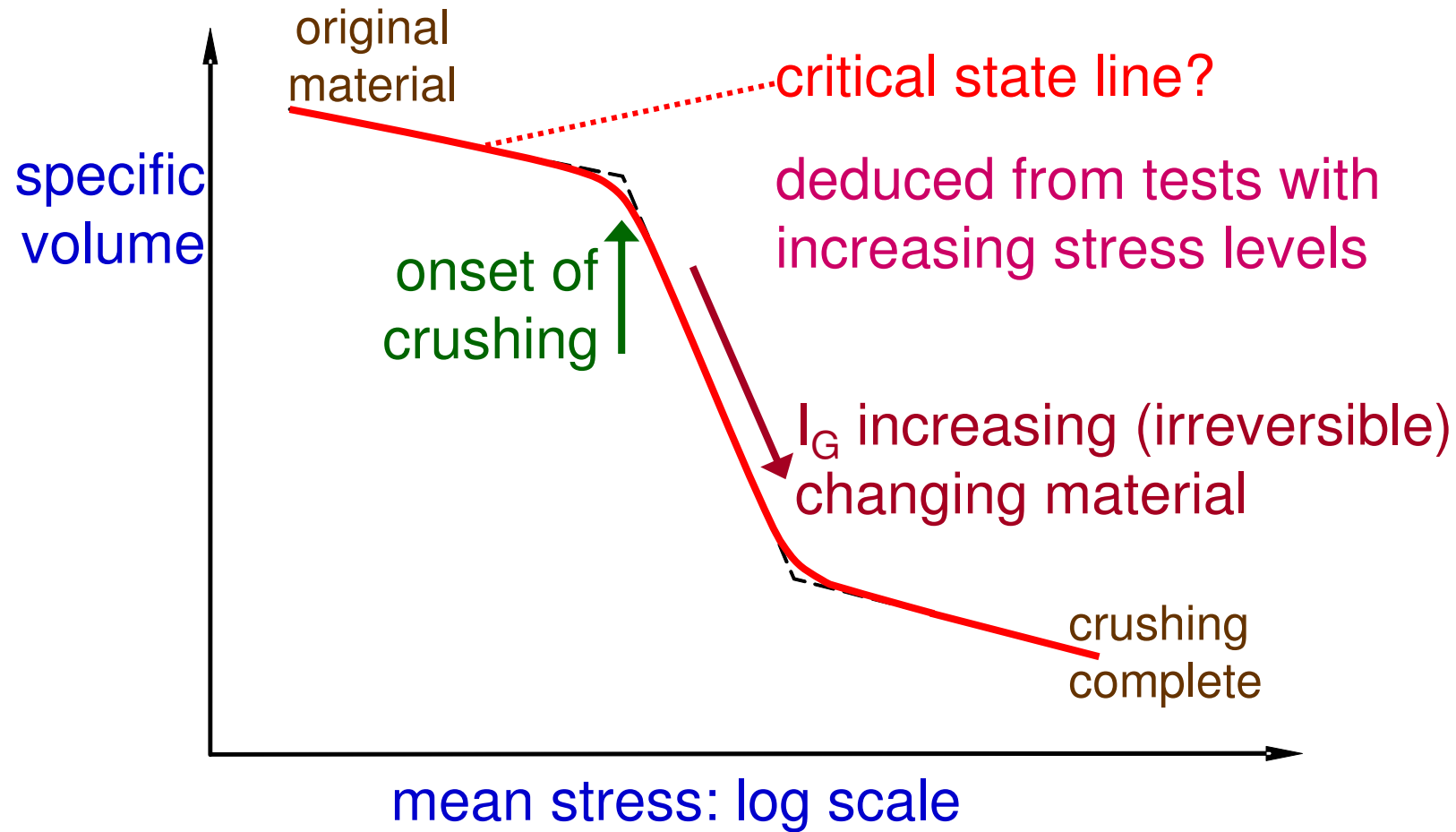


smaller particles tending to fill gaps

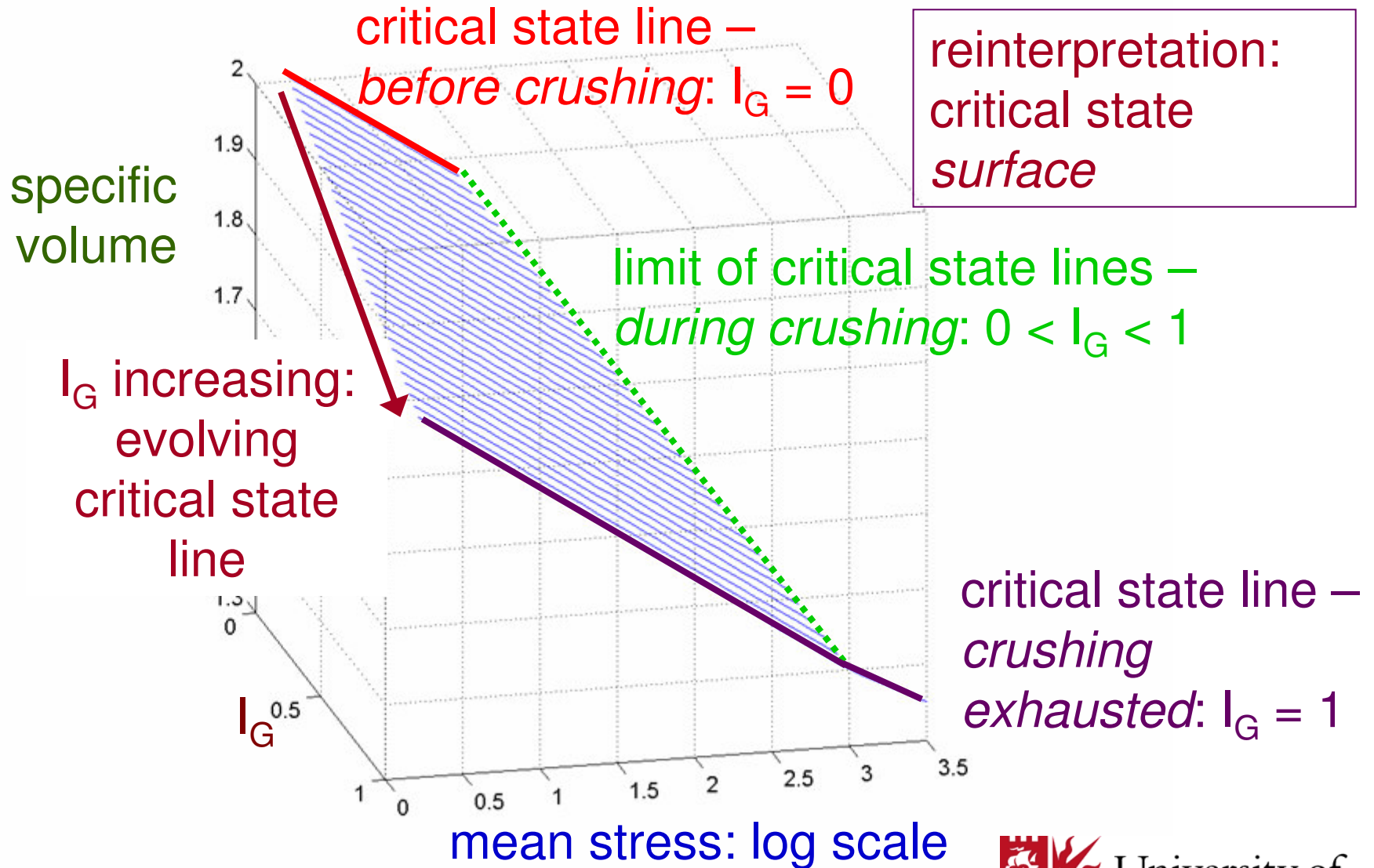
maximum and minimum void ratios  $\downarrow$  as  $I_G \uparrow$

- critical state line – expected to change!
- evidence?

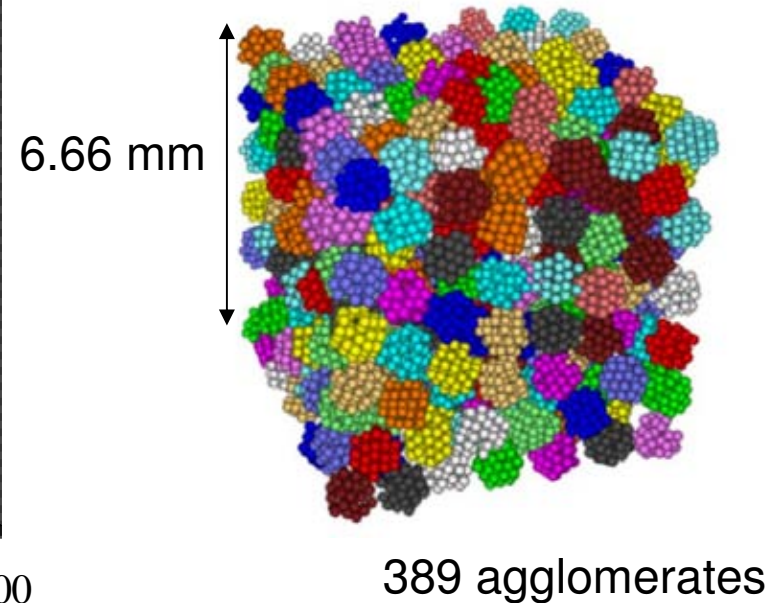
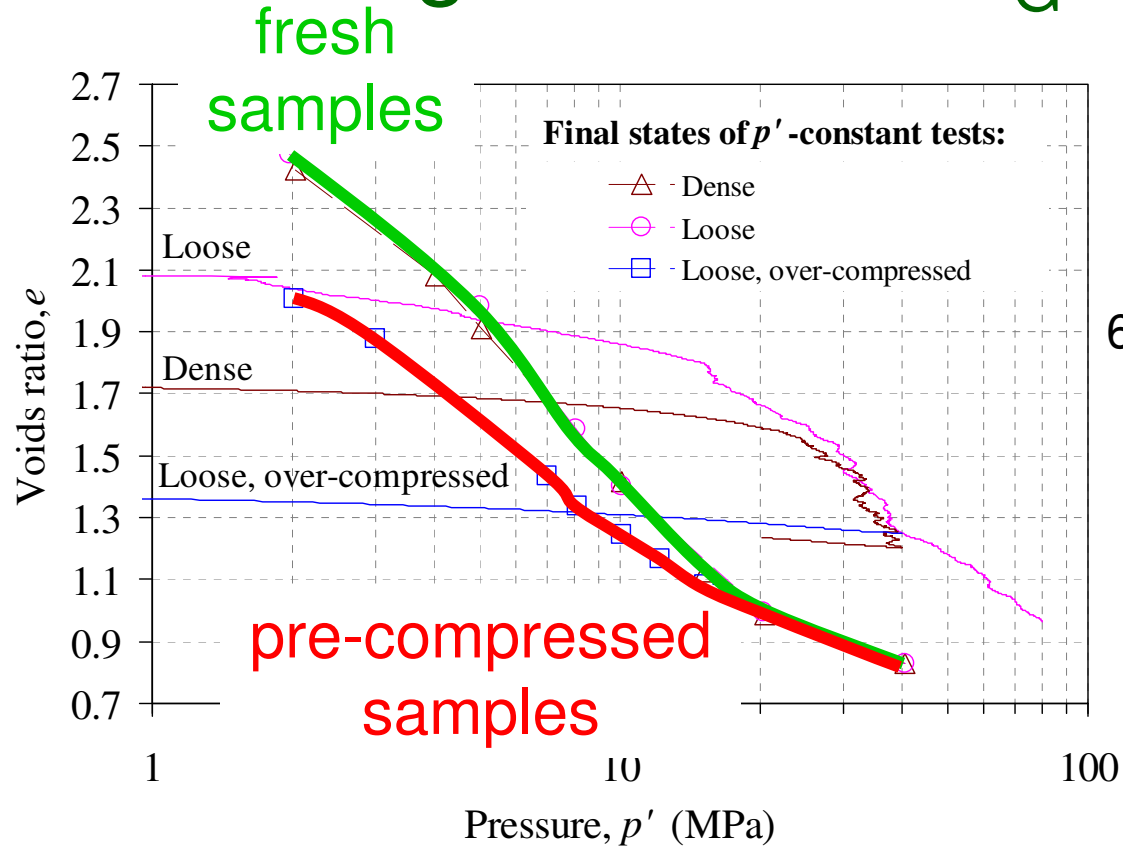
# Grading state index $I_G$ : critical states



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# Grading state index $I_G$ : critical states



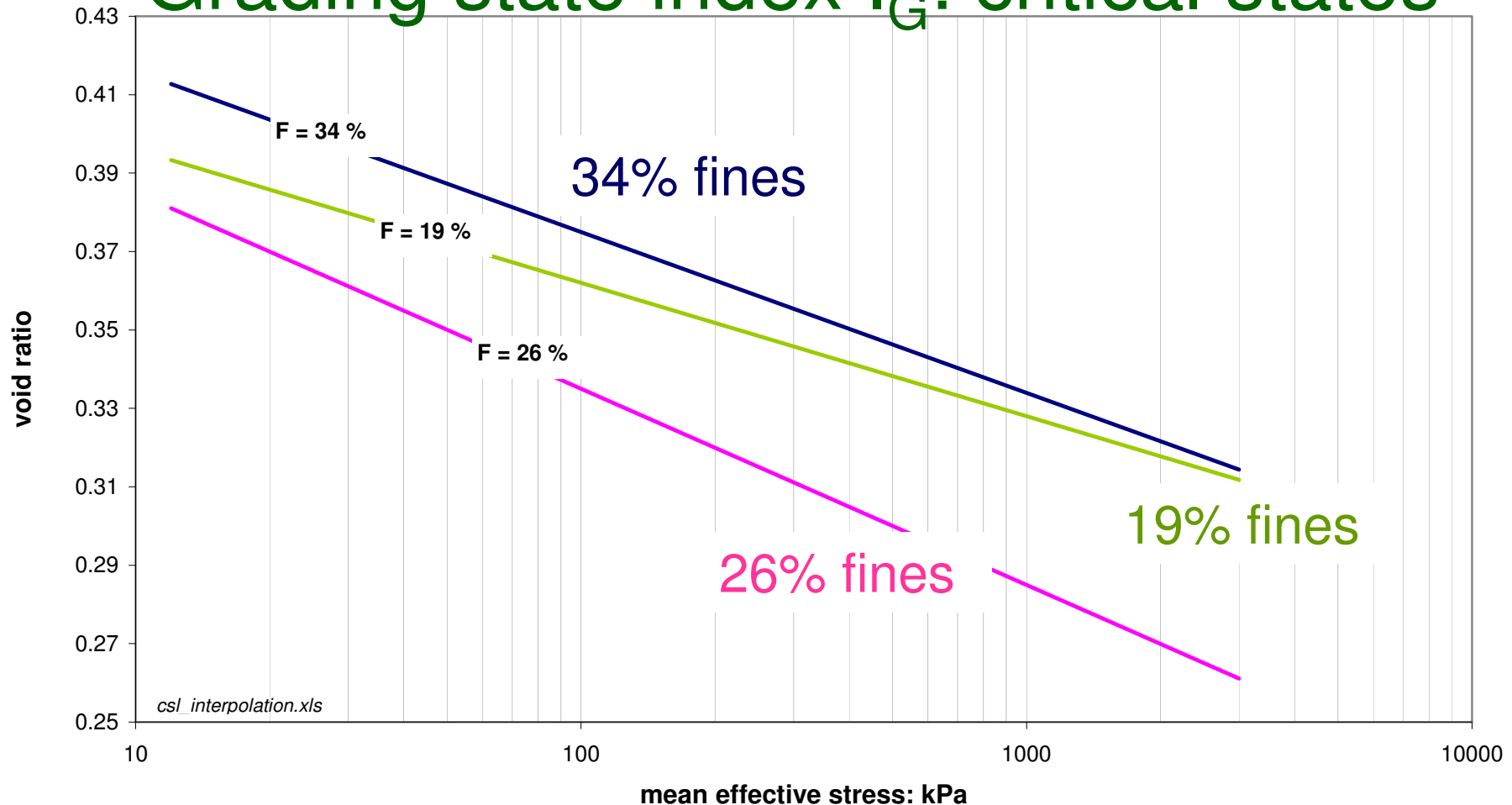
simulations for assemblies of agglomerates

critical state line *changes* with crushing

fresh samples — pre-compressed samples

Cheng, 2005

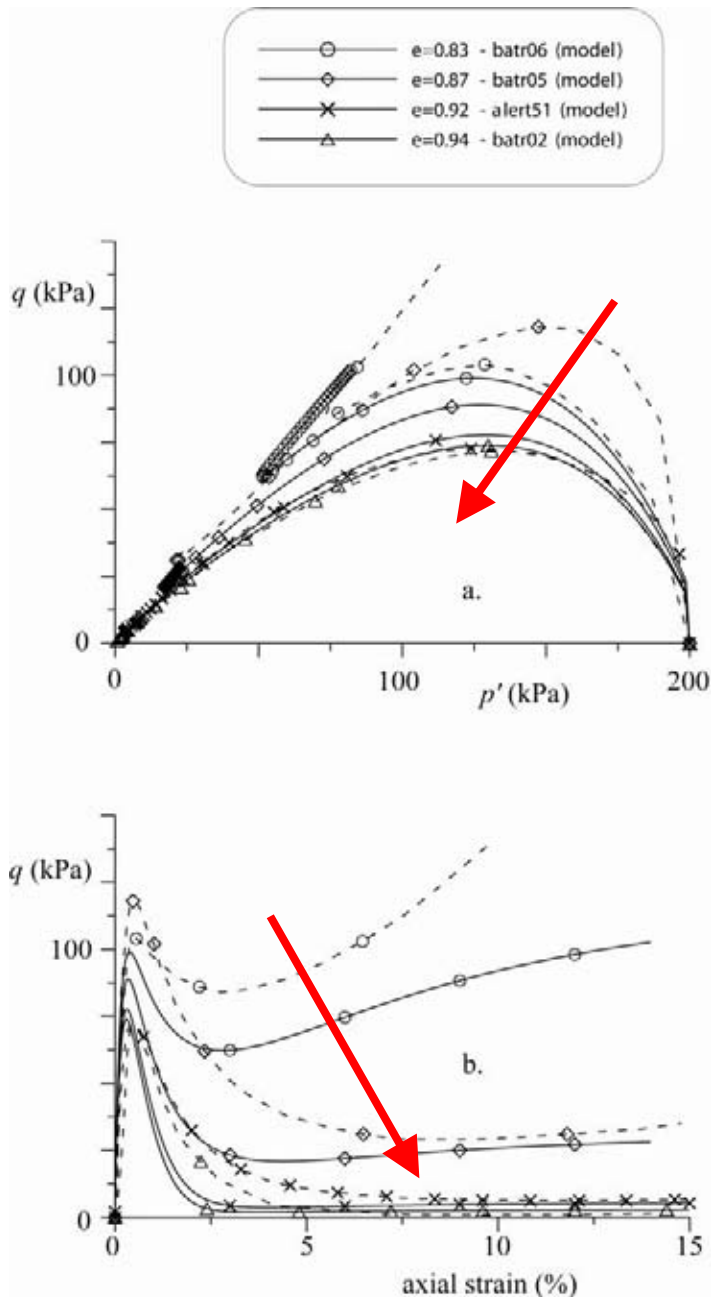
# Grading state index $I_G$ : critical states



WAC Bennett Dam: interpretation of effect of fines content on location of critical state line (triaxial tests, artificial mixtures)

non-monotonic...!?

# Grading state index



WAC Bennett Dam

Severn-Trent sand

transport of fines from core

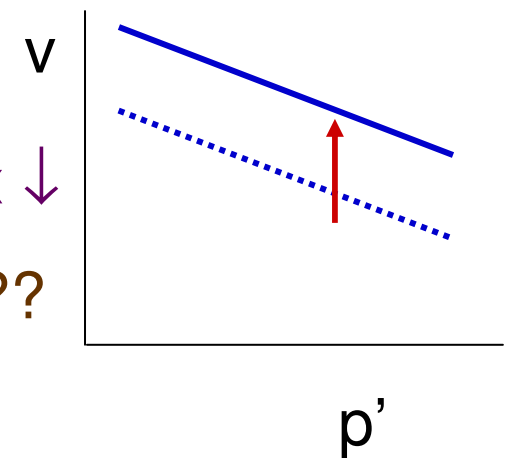
void ratio  $\uparrow$

grading state index  $\downarrow$

critical state line  $\downarrow??$

state parameter  $\uparrow$

soil feels looser 😞



# Grading state index

WAC Bennett Dam

Severn-Trent sand

transport of fines from core

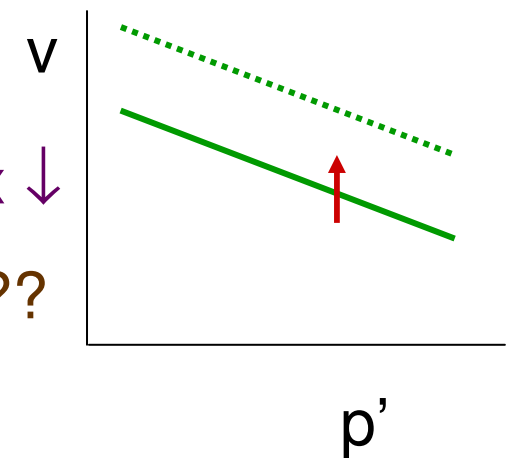
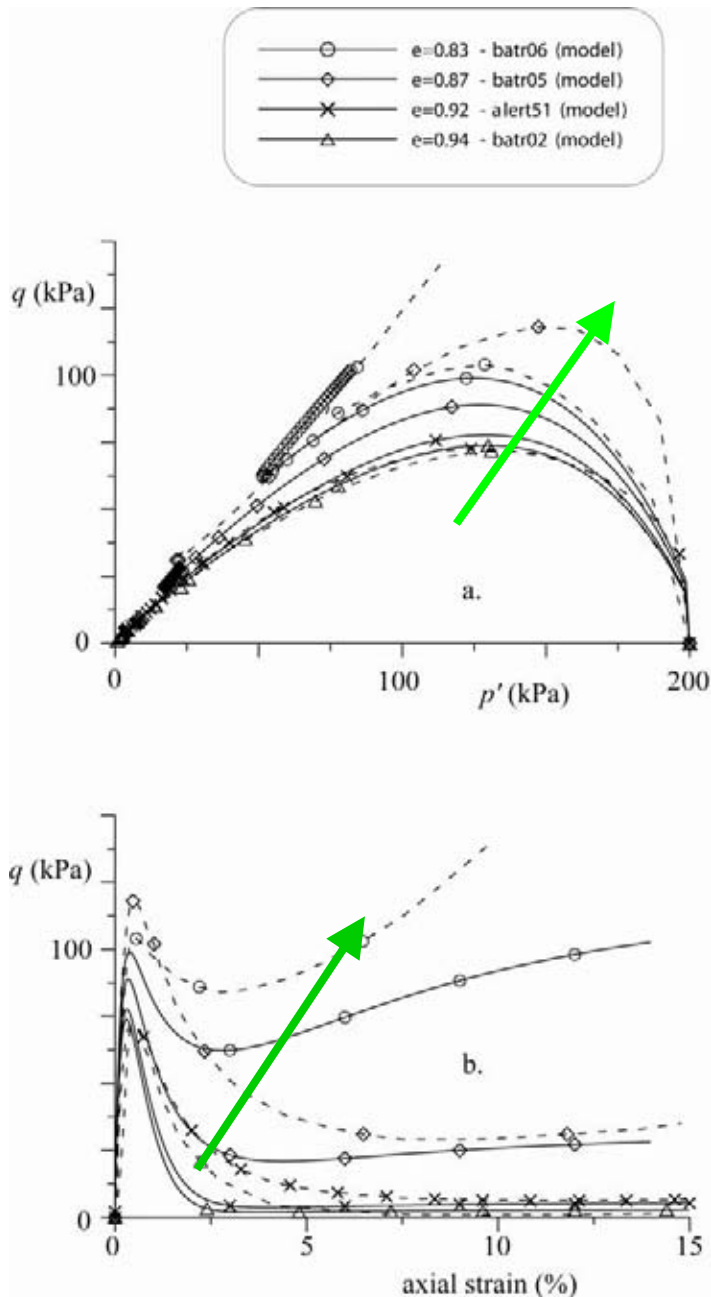
void ratio  $\uparrow$

grading state index  $\downarrow$

critical state line  $\uparrow??$

state parameter  $\downarrow$

soil feels denser 😊





# Grading state index

WAC Bennett Dam??

benefit of simple model that systematically incorporates changes in stress level *and* density *and* grading (making up *state* of soil)

model has to be honed – subtle data requirements for calibration

most testing has used artificially prepared mixtures

# The magic of sands

- Introduction: context: WAC Bennett Dam
- Particle continuum duality
- Mohr-Coulomb model
- Elastic properties
- *Critical states*
- Severn-Trent sand
- Grading state index
- Conclusion

# Conclusion

*WAC Bennett Dam:* Need for modelling able to include effects of density, stress *and* evolving grading.

*Mohr-Coulomb model:* Use as basis for development of more comprehensive models.

*Severn-Trent sand:* Mohr-Coulomb model with strength dependent on state parameter – softening as emergent property.

*Grading state index:* How does the changing grading of a soil affect its mechanical behaviour?



Nous nous sommes nourris de la magie des sables.

We are sustained by the magic of sands.

Antoine de St Exupéry: *Terre des hommes* (1939)



Thank you for your attention!