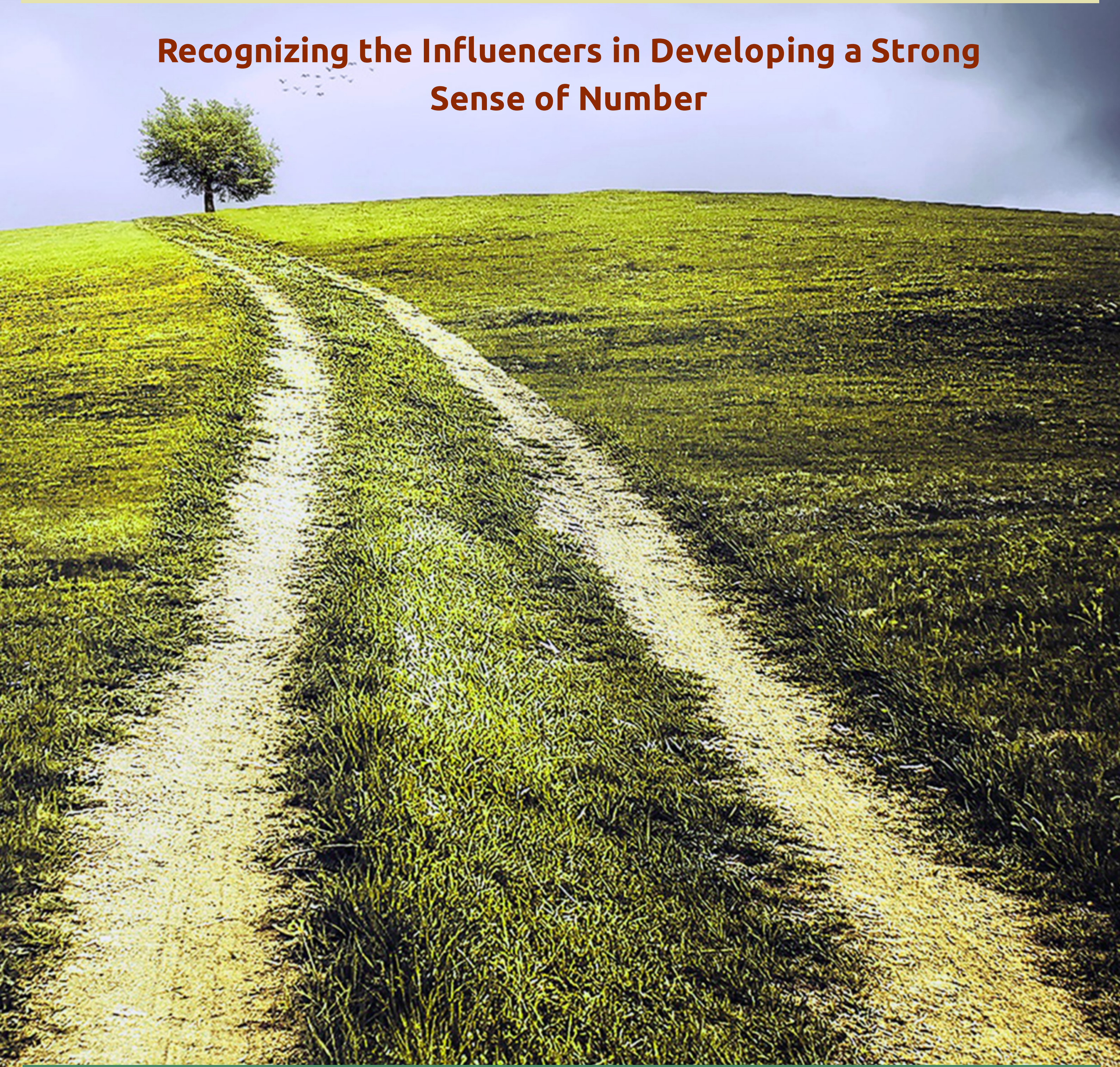
The Developmental Road to Learning Mathematics

Recognizing the Influencers in Developing a Strong Sense of Number



Lisa Ann de Garcia, MEd

If you are reading this book, you probably have a child or a student who struggles during math. It is not uncommon for a teacher to complain of having a student who "cannot do anything" in math. However, in my experience, outside the rare exception, that is never true. I will ask those teachers if the child can at least count, to which they always reply affirmatively. It just shows that what teachers really mean is that the child cannot do anything in the level that their class is functioning, but it doesn't mean that they do not have any ability to do math.

Every child can do something, it is just our job to figure what that something is so we know how to move them forward.

Number Sense is defined by Hilde Howden as "having good intuition about numbers and their relationships."

If our child or student is exhibiting poor number sense, we need to first, before employing any interventions, have a solid global perspective of what is happening so you can develop a precise plan of action.

We first have to recognize what are the specific developmental levels through which each child must pass in order to build mathematical understanding.

So, just like motor milestones, there are developmental milestones in mathematics. Children do not simply start off multiplying. They have to pass through specific phases first, and if they get stuck in a phase and their peers are working in one or two phases ahead of them, they cannot simply be doing what everyone else is doing and be expected to catch up. They have to go through all of the phases.

A good sense of number and relationships are established when the child has enough rich exploratory mathematical experiences in each of these developmental phases, and has developed specific concepts that can be used to build additional concepts in subsequent phases. Some children pass through each phase quicker than others, but children need plenty of mathematical experiences in each phase for them to move on.

The Developmental Math Phases of Number and Operations

The outline of the elementary age mathematical phases of number and operation are listed below. The grade listed is when that phase is typically addressed. However, it doesn't mean that all students are working in the same phase even though they are in the same grade.

Emergent (pre-school): The things that the child needs to learn before they get to kindergarten, such as that numbers are different from letters and that they represent quantity.

Matching (kindergarten): A lot of 1-1 counting. When they solve problems, they mostly have to act them out, either with themselves or tools. And they are doing very small part-part- whole-type problems.

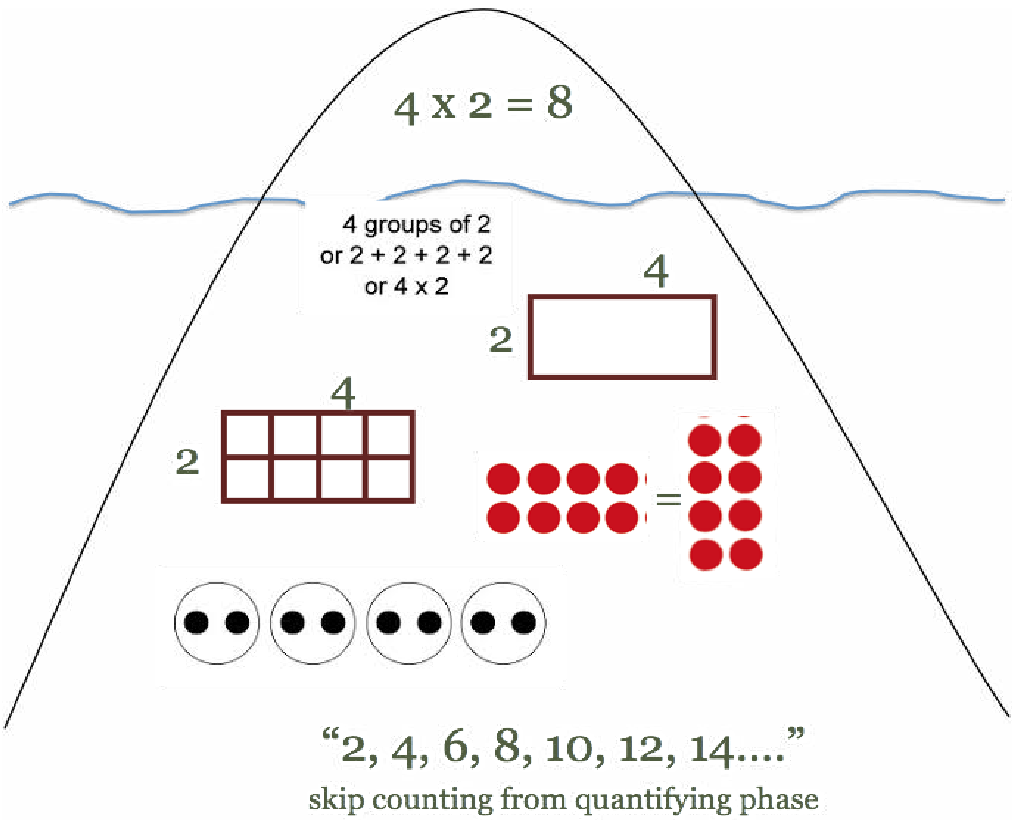
Quantifying (1st grade): Children are learning to skip count, add and subtract, as well as starting to explore tens and ones.

Partitioning (2nd grade): This phase is all about tens and ones, and addition and subtraction with regrouping with 2 and 3 digit numbers.

Factoring (3rd & 4th grade): Children have to add and subtract with larger numbers and begin multiplication, division, and fractions.

Operating (5th and 6th grades): Students have to operate with fractions and decimals by adding, subtracting, multiplying, and dividing.

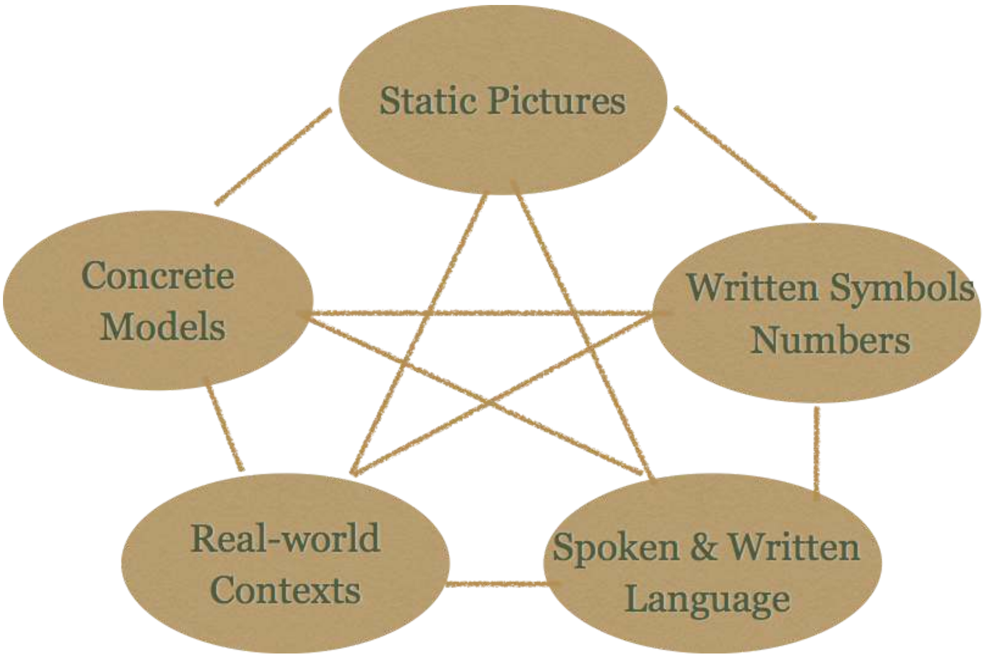
If we think of learning in the form of an iceberg, we can consider that the part that is sticking up out of the water is what the child can demonstrate or what might be shown on a test, such as 4 x 2 = 8, which would be in the factoring phase. However, underneath that peak are a lot of skills and concepts that have to first be developed in order for the child to know that 4 x 2 = 8.



If the child is only able to spit out the answer by memory without having developed these underlying concepts, they don't have the foundation and will not be able to figure it out if they forget or apply it to something larger.

There are some strategic ways we can ensure that children spend quality time within a phase in order to "develop a good intuition about numbers and their relationships," and grow a healthy base of the iceberg related to that phase. So, for argument sake, we can consider each iceberg as it's own developmental phase. At the tip of the iceberg are what the children should be able to show or do in that phase.

1. Use Multiple Representations



First, students build mathematical understanding through the repeated use of 5 representations, which are: real-life contexts, building a model, drawing a picture, writing a number sentence, and communicating the idea either orally or in writing.

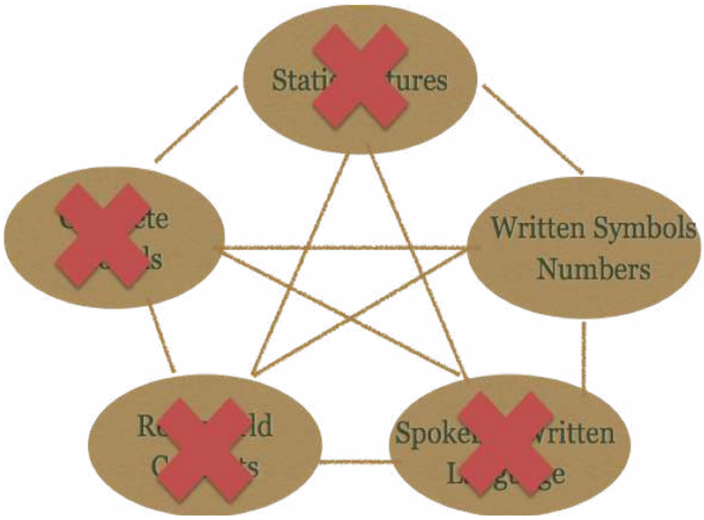
On the above image, you can see the different representations. The lines show the connections that should be made between them. The more students engage in as many of these representations as possible, the stronger the knitting will be, resulting in a stronger concept and mathematical understanding.

2. Work from the Concrete to the Abstract

Secondly, children need to have concrete experiences before being able to make sense of abstract ideas. If they are solely presented concepts abstractly, or through numerical representations, they are limited, at best, in their understanding and how those concepts connect in different contexts. However, the greater risk is that children make unreasonable errors because they cannot recognize that their answers do not make sense.

For example, when I was a senior in high school. I was in AP calculus and even passed my test. However, everything in mathematics up to that point from elementary school was taught to me in a very abstract way. For most of my mathematical school career, it was good enough for me to get by and move from class to class. I was even considered the top of many of my classes. However, I found that calculus was my breaking point as I did not know what was going on. But because I didn’t have the experiences growing up of connecting things like multiplication to a model or picture, I didn’t understand that what I was missing in calculus was the connection to all the different representations.

So, if we go back to the image of multiple representations, looking at symbols and numbers, we see that it is only one-fifth of all the representations. Numbers and symbols cannot be the only way that children are exposed to mathematics, or they run the risk of really struggling.

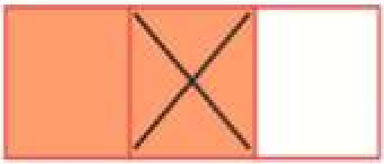


Introducing a new concept needs to be done concretely. If it is problematic enough, children will generally need to build it or act it out. They then might move towards pictures and draw what they build. They use these pictures as a way to solve the problem, leaving behind the concrete tools. They then start making number sentences, or equations, of what they are drawing or building to connect those numbers to the picture.

Students in ALL phases should be working concretely and making connections to build abstract understanding. So, if they are building ideas around addition and subtraction they would start by building it. If they are in 5th grade and developing understanding around addition and subtraction of fractions, they STILL have to build it. Even though they have an understanding of addition and subtraction, the new number type makes the problem more complex and building gives the child access and allows him to make connections.

It is necessary to teach children how to make connections from their pictures to the numbers. I vividly recall going into one 5th grade class where the students were learning how to multiply fractions. The teacher told them to solve a problem with a picture, which they did, then had them solve the same problem with numbers, which they also did. However, no one noticed that the picture they all drew had nothing to do with the way they solved it with the numbers. Therefore, we have to make sure that even though they can solve it with a picture or build it, their numbers are connected to it.

$\frac{1}{2} \times \frac{2}{3}$



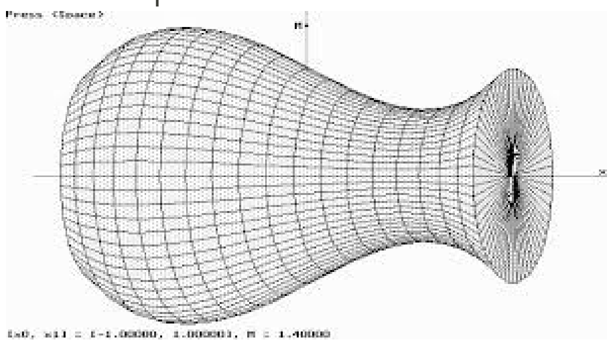
Picture:
The student thinks, "half of two-thirds" and draws a picture to match.

Numbers: $\frac{2}{3} \times \frac{1}{2} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

The above illustration shows how a child might draw a picture of 1/2 x 2/3 when understanding that it literally means "half of 2/3". The numerical representation below is how it is typically solved through a procedure. The two, although correct, are not necessarily directly related.

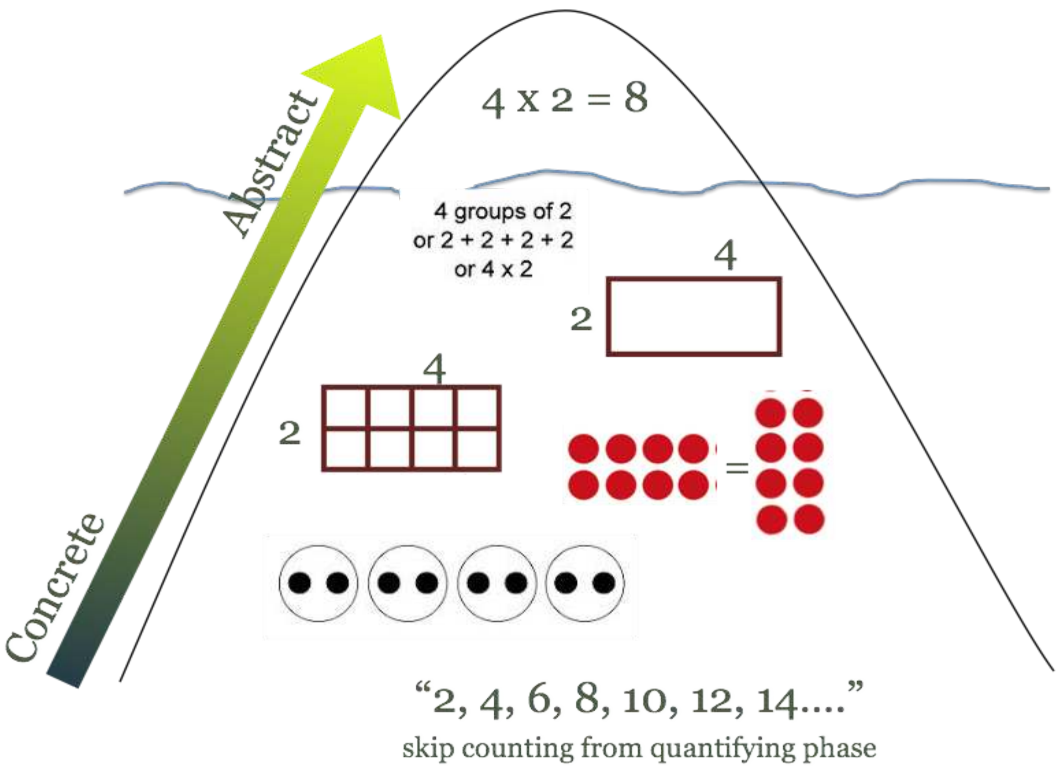
The primary grades (k-2) typically need to spend a larger percentage of the time working concretely than older students, who might spent more time in the abstract. This does not mean that the older students should only be in the abstract.

Once I had the opportunity to take a hands-on calculus course for teachers. I was very excited because I wanted to finally understand the calculus that I took back in high school. Unfortunately, the course was dropped due to low interest and I was quite disappointed. The point being is that every level of mathematics can be taught via pictures and real-world contexts. In fact, I want my students to understand that math is the real-world application of science and that the numbers can be found in the picture or model. So a simple technique is when the student draws a picture, ask him where the different numbers are in his representation.



If the child has not had opportunities to explore skip counting, repeated addition, and multiple groups and is just made to memorize that 4 x 2 = 8, then their understanding is quite fragile and the risk of forgetting is quite great. However, if the child has had the proper experiences and forgets the answer, the likelihood that s/he is able to figure out the answer is high.

We will see in the illustration below that as we move up the iceberg, the child moves from concrete to abstract representations by making explicit connections between the two.



When functioning at different parts of the iceberg, children may approach the problem differently from one another. A child who is struggling with a mathematical concept may be directly modeling or acting out the problem with manipulatives. Once they have their mind wrapped around the context and has developed some strategies for solving the problem, they may begin to draw pictures rather than build. However, initially they may have to draw what they build in order for their pictures to have meaning for them. As they move up, the need for pictures is reduced and children will usually wean away from them on their own and stick to solving the problems just with numbers.

3. Reduce the Number Size

Thirdly, when introducing a concept, you may find it helpful to reduce the number size. When the numbers are too large, it can get in the way of making sense of the relationships and what is happening between them in a given context. Once they develop and internalize some sense of relationships, increasing the number size can be appropriate.

Within one phase, there is a number range. If you increase the number too much or you go from whole numbers to integers or fractions, then you are moving across phases.

When you are in one phase, you need to stick to the number size. However, let's say you are in the operating phase and the students have to learn to multiply fractions. Take them all the way back down to an earlier phase just so they can deal with the problem. They may have to briefly re-explore what multiplication really means with whole numbers before applying it to fractions.

Number size influences the way a child solves a problem. If you increase the numbers too much, they may revert back to direct modeling or building out the problem because they cannot handle those numbers. So, depending on your purpose, this may or may not be what you are trying to achieve.

For example, maybe you want your child to explore a story problem such as:

> After school, Tim went with his friends to go fishing. Tim caught some fish. His friends caught 5 fish among them. There were 9 fish caught altogether. How many fish did Tim catch?

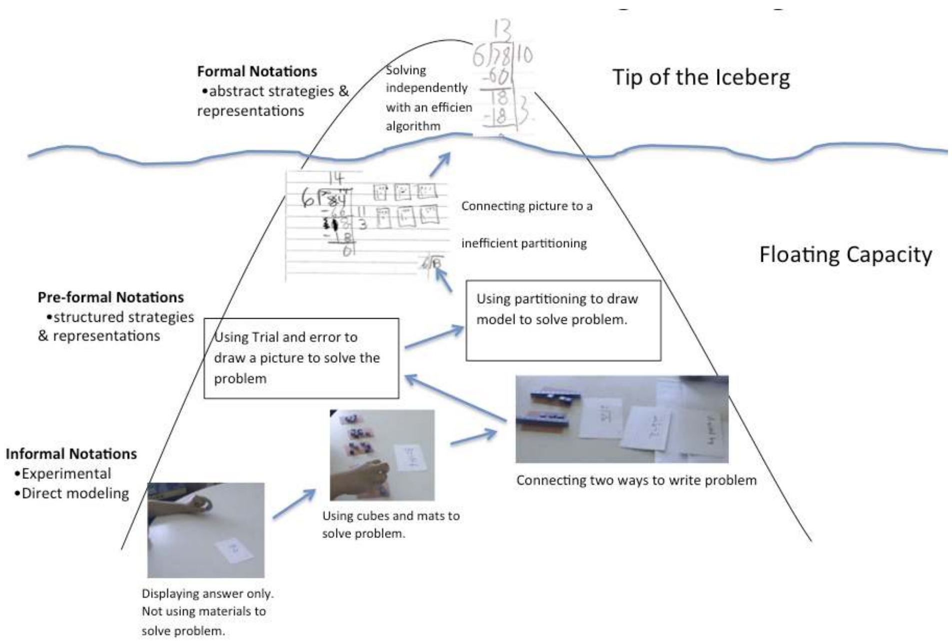
Keeping the numbers nice and small will give students access to the structure of the context. Once they realize that they might add up or subtract to solve this problem, the numbers can be raised to the level in which they are working. Some may need to stay within single digits, while others may be able to add two digits with regrouping. A problem like the one on the previous page could be used in 6th grade, but using fractions, decimals, or negative numbers as well as adjusting the context, since catching a 1/2 of a fish would be pretty gross.

Note that the same context can be used for students working in different phases, but the number size and type determines in which phase they are performing.

What About Learning Disabilities?

These principles apply even if the child has a learning disability, although the process of moving up the iceberg, or to abstract representations where the child can solve problems using numbers and symbols will be in smaller steps and will take longer. In this situation it is even more critical to be able to identify the phase in which s/he is functioning and then slowly build that understanding through the use of as many of the 5 representations as s/he is able depending on the disability. Then using lots of hands-on and visual experiences, and adjusting the number size so the child is able to appropriately reason about numbers and the problem.

I know because I have a 17 year old son with autism and a severe language processing disorder. When working with him in elementary school, I was trying to figure out how I was going to teach him multiplication and division. Due to his lack of communication skills, I had to essentially treat him like a second language learner when approaching these concepts and skills. Nonetheless, I took him through this exact the same processes that I describe here and he was able to learn how to both multiply and divide.



If you want to know how to identify and assess your child or student's mathematical developmental phase so you can know how to best support him/her towards developing a strong sense of numbers, then I invite you to learn more about my book and accompanying tutorial, "A Qualitative and Quantitative Developmental Math Assessment and Intervention Protocol." The online tutorial comes with a digital copy of the book and fillable recording forms. Click the link below.

LEARN MORE!

Hard copies of my book can be found **here**.

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