CHAPTER 1 Electric Circuit Variables

IN THIS CHAPTER

1.1 *Introduction*

A circuit consists of electrical elements connected together. Engineers use electric circuits to solve problems that are important to modern society. In particular:

1. Electric circuits are used in the generation, transmission, and consumption of electric power and energy.
2. Electric circuits are used in the encoding, decoding, storage, retrieval, transmission, and processing of information.

In this chapter, we will do the following:

- Represent the current and voltage of an electric circuit element, paying particular attention to the reference direction of the current and to the reference direction or polarity of the voltage.
- Calculate the power and energy supplied or received by a circuit element.
- Use the passive convention to determine whether the product of the current and voltage of a circuit element is the power supplied by that element or the power received by the element.
- Use scientific notation to represent electrical quantities with a wide range of magnitudes.

1.2 *Electric Circuits and Current*

The outstanding characteristics of electricity when compared with other power sources are its mobility and flexibility. Electrical energy can be moved to any point along a couple of wires and, depending on the user's requirements, converted to light, heat, or motion.

An **electric circuit** or electric network is an interconnection of electrical elements linked together in a closed path so that an electric current may flow continuously.

Solution

Recall that the unit of charge is coulombs, C. Then the current, from Eq. 1.2-1, is

$$i = \frac{dq}{dt} = 12\ \text{A}$$

where the unit of current is amperes, A.

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EXAMPLE 1.2-2 Charge from Current

Find the charge that has entered the terminal of an element from $t = 0$ s to $t = 3$ s when the current entering the element is as shown in Figure 1.2-6.

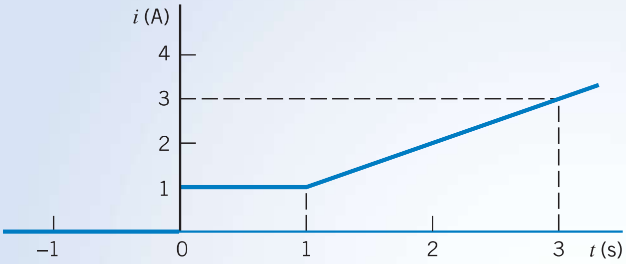


FIGURE 1.2-6 Current waveform for Example 1.2-2.

Solution

From Figure 1.2-6, we can describe $i(t)$ as

$$i(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t \le 1 \\ t & t > 1 \end{cases}$$

Using Eq. 1.2-2, we have

$$\begin{aligned} q(3) - q(0) &= \int_0^3 i(t)dt = \int_0^1 1\,dt + \int_1^3 t\,dt \\ &= t\Big|_0^1 + \frac{t^2}{2}\Big|_1^3 = 1 + \frac{1}{2}(9-1) = 5\,\text{C} \end{aligned}$$

Alternatively, we note that integration of $i(t)$ from $t = 0$ to $t = 3$ s simply requires the calculation of the area under the curve shown in Figure 1.2-6. Then, we have

$$q = 1 + 2 \times 2 = 5\,\text{C}$$

EXERCISE 1.2-1 Find the charge that has entered an element by time t when $i = 8t^2 - 4t$ A, $t \ge 0$. Assume $q(t) = 0$ for $t < 0$.

Answer: $q(t) = \frac{8}{3}t^3 - 2t^2$ C

EXERCISE 1.2-2 The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \ge 0$, and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

Answer: $i(t) = \frac{d}{dt} 4 \sin 3t = 12 \cos 3t$ A

1.3 Systems of Units

In representing a circuit and its elements, we must define a consistent system of units for the quantities occurring in the circuit. At the 1960 meeting of the General Conference of Weights and Measures, the representatives modernized the metric system and created the Système International d'Unites, commonly called SI units.

SI is *Système International d'Unités* or the International System of Units.

The fundamental, or base, units of SI are shown in Table 1.3-1. Symbols for units that represent proper (persons') names are capitalized; the others are not. Periods are not used after the symbols, and the symbols do not take on plural forms. The derived units for other physical quantities are obtained by combining the fundamental units. Table 1.3-2 shows the more common derived units along with their formulas in terms of the fundamental units or preceding derived units. Symbols are shown for the units that have them.

Table 1.3-1 SI Base Units

	SI UNIT	
QUANTITY	NAME	SYMBOL
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Table 1.3-2 Derived Units in SI

QUANTITY	UNIT NAME	FORMULA	SYMBOL
Acceleration — linear	meter per second per second	m/s^2	
Velocity — linear	meter per second	m/s	
Frequency	hertz	s^{-1}	Hz
Force	newton	$kg \cdot m/s^2$	N
Pressure or stress	pascal	N/m^2	Pa
Density	kilogram per cubic meter	kg/m^3	
Energy or work	joule	$N \cdot m$	J
Power	watt	J/s	W
Electric charge	coulomb	$A \cdot s$	C
Electric potential	volt	W/A	V
Electric resistance	ohm	V/A	Ω
Electric conductance	siemens	A/V	S
Electric capacitance	farad	C/V	F
Magnetic flux	weber	$V \cdot s$	Wb
Inductance	henry	Wb/A	H

Table 1.3-3 SI Prefixes

MULTIPLE	PREFIX	SYMBOL
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

The basic units such as length in meters (m), time in seconds (s), and current in amperes (A) can be used to obtain the derived units. Then, for example, we have the unit for charge (C) derived from the product of current and time (A · s). The fundamental unit for energy is the joule (J), which is force times distance or N · m.

The great advantage of the SI system is that it incorporates a decimal system for relating larger or smaller quantities to the basic unit. The powers of 10 are represented by standard prefixes given in Table 1.3-3. An example of the common use of a prefix is the centimeter (cm), which is 0.01 meter.

The decimal multiplier must always accompany the appropriate units and is never written by itself. Thus, we may write 2500 W as 2.5 kW. Similarly, we write 0.012 A as 12 mA.

EXAMPLE 1.3-1 SI Units

A mass of 150 grams experiences a force of 100 newtons. Find the energy or work expended if the mass moves 10 centimeters. Also, find the power if the mass completes its move in 1 millisecond.

Solution

The energy is found as

$$\text{energy} = \text{force} \times \text{distance} = 100 \times 0.1 = 10\ \text{J}$$

Note that we used the distance in units of meters. The power is found from

$$\text{power} = \frac{\text{energy}}{\text{time period}}$$

where the time period is 10^{-3} s. Thus,

$$\text{power} = \frac{10}{10^{-3}} = 10^4\ \text{W} = 10\ \text{kW}$$

EXERCISE 1.3-1 Which of the three currents, $i_1 = 45\ \mu\text{A}$, $i_2 = 0.03$ mA, and $i_3 = 25 \times 10^{-4}$ A, is largest?

Answer: i_3 is largest.

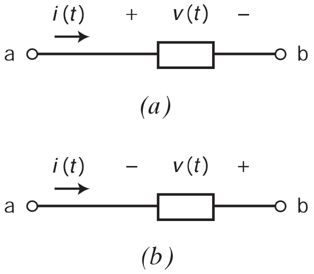


FIGURE 1.5-1 (*a*) The element voltage and current **adhere** to the passive convention. (*b*) The element voltage and current **do not adhere** to the passive convention.

where p is power in watts, w is energy in joules, and t is time in seconds. The power associated with the current through an element is

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \tag{1.5-2}$$

From Eq. 1.5-2, we see that the power is simply the product of the voltage across an element times the current through the element. The power has units of watts.

Two circuit variables are assigned to each element of a circuit: a voltage and a current. Figure 1.5-1 shows that there are two different ways to arrange the direction of the current and the polarity of the voltage. In Figure 1.5-1*a*, the current is directed from the + toward the − of the voltage polarity. In contrast, in Figure 1.5-1*b*, the current is directed from the − toward the + of the voltage polarity.

First, consider Figure 1.5-1*a*. When the current enters the circuit element at the + terminal of the voltage and exits at the − terminal, the voltage and current are said to "adhere to the passive convention." In the passive convention, the voltage pushes a positive charge in the direction indicated by the current. Accordingly, the power calculated by multiplying the element voltage by the element current

$$p = vi$$

is the power **received** by the element. (This power is sometimes called "the power absorbed by the element" or "the power dissipated by the element.") The power received by an element can be either positive or negative. This will depend on the values of the element voltage and current.

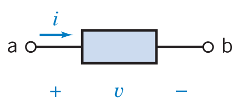
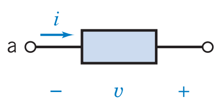
Next, consider Figure 1.5-1*b*. Here the passive convention has not been used. Instead, the current enters the circuit element at the − terminal of the voltage and exits at the + terminal. In this case, the voltage pushes a positive charge in the direction opposite to the direction indicated by the current. Accordingly, when the element voltage and current do not adhere to the passive convention, the power calculated by multiplying the element voltage by the element current is the power **supplied** by the element. The power supplied by an element can be either positive or negative, depending on the values of the element voltage and current.

The power received by an element and the power supplied by that same element are related by

$$\text{power received} = -\text{power supplied}$$

The rules for the passive convention are summarized in Table 1.5-1. When the element voltage and current adhere to the passive convention, the energy received by an element can be determined

Table 1.5-1 Power Received or Supplied by an Element

POWER RECEIVED BY AN ELEMENT	POWER SUPPLIED BY AN ELEMENT
	
Because the reference directions of v and i adhere to the passive convention, the power $p = vi$ is the power received by the element.	Because the reference directions of v and i do not adhere to the passive convention, the power $p = vi$ is the power supplied by the element.

CHAPTER 2 Circuit Elements

IN THIS CHAPTER

2.1 *Introduction*

Not surprisingly, the behavior of an electric circuit depends on the behaviors of the individual circuit elements that comprise the circuit. Of course, different types of circuit elements behave differently. The equations that describe the behaviors of the various types of circuit elements are called the constitutive equations. Frequently, the constitutive equations describe a relationship between the current and voltage of the element. Ohm's law is a well-known example of a constitutive equation.

In this chapter, we will investigate the behavior of several common types of circuit element:

- Resistors.
- Independent voltage and current sources.
- Open circuits and short circuits.
- Voltmeters and ammeters.
- Dependent sources.
- Transducers.
- Switches.

2.2 *Engineering and Linear Models*

The art of engineering is to take a bright idea and, using money, materials, knowledgeable people, and a regard for the environment, produce something the buyer wants at an affordable price.

Engineers use *models* to represent the elements of an electric circuit. A model is a description of those properties of a device that we think are important. Frequently, the model will consist of an equation relating the element voltage and current. Though the model is different from the electric device, the model can be used in pencil-and-paper calculations that will predict how a circuit composed of actual devices will operate. Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

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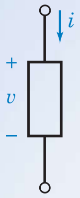
EXAMPLE 2.2-3 A Model of a Linear Device

A linear element has voltage v and current i as shown in Figure 2.2-2*a*. Values of the current i and corresponding voltage v have been tabulated as shown in Figure 2.2-2*b*. Represent the element by an equation that expresses v as a function of i. This equation is a model of the element. Use the model to predict the value of v corresponding to a current of $i = 100$ mA and the value of i corresponding to a voltage of $v = 18$ V.



(*a*)

v, V	i, mA
4.5	10
11.25	25
22.5	50

(*b*)

FIGURE 2.2-2 (*a*) A linear circuit element and (*b*) a tabulation of corresponding values of its voltage and current.

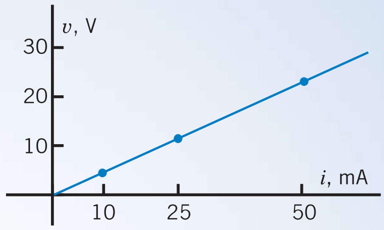


FIGURE 2.2-3 A plot of voltage versus current for the linear element from Figure 2.2-2.

Solution

Figure 2.2-3 is a plot of the voltage v versus the current i. The points marked by dots represent corresponding values of v and i from the rows of the table in Figure 2.2-2*b*. Because the circuit element is linear, we expect these points to lie on a straight line, and indeed they do. We can represent the straight line by the equation

$$v = mi + b$$

where m is the slope and b is the v-intercept. Noticing that the straight line passes through the origin, $v = 0$ when $i = 0$, we see that $b = 0$. We are left with

$$v = mi$$

The slope m can be calculated from the data in any two rows of the table in Figure 2.2-2*b*. For example:

$$\frac{11.25 - 4.5}{25 - 10} = 0.45\ \frac{\text{V}}{\text{mA}},\ \frac{22.5 - 11.25}{50 - 25} = 0.45\ \frac{\text{V}}{\text{mA}},\ \text{and}\ \frac{22.5 - 4.5}{50 - 10} = 0.45\ \frac{\text{V}}{\text{mA}}$$

Consequently,

$$m = 0.45\ \frac{\text{V}}{\text{mA}} = 450\ \frac{\text{V}}{\text{A}}$$

and

$$v = 450i$$

This equation is a model of the linear element. It predicts that the voltage $v = 450(0.1) = 45$ V corresponds to the current $i = 100\ \text{mA} = 0.1$ A and that the current $i = 18/450 = 0.04\ \text{A} = 40$ mA corresponds to the voltage $v = 18$ V.

2.3 *Active and Passive Circuit Elements*

We may classify circuit elements in two categories, *passive* and *active*, by determining whether they absorb energy or supply energy. An element is said to be passive if the total energy delivered to it from the rest of the circuit is always nonnegative (zero or positive). Then for a passive element, with the current flowing into the + terminal as shown in Figure 2.3-1*a*, this means that

$$w = \int_{-\infty}^{t} vi\,d\tau \geq 0 \tag{2.3-1}$$

for all values of t.

A **passive element** absorbs energy.

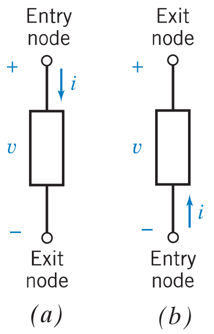


FIGURE 2.3-1 (*a*) The entry node of the current i is the positive node of the voltage v; (*b*) the entry node of the current i is the negative node of the voltage v. The current flows from the entry node to the exit node.

An element is said to be *active* if it is capable of delivering energy. Thus, an active element violates Eq. 2.3-1 when it is represented by Figure 2.3-1*a*. In other words, an active element is one that is capable of generating energy. Active elements are potential sources of energy, whereas passive elements are sinks or absorbers of energy. Examples of active elements include batteries and generators. Consider the element shown in Figure 2.3-1*b*. Note that the current flows into the negative terminal and out of the positive terminal. This element is said to be active if

$$w = \int_{-\infty}^{t} vi\,d\tau \geq 0 \tag{2.3-2}$$

for at least one value of t.

An **active element** is capable of supplying energy.

Example 2.3-1 An Active Circuit Element

A circuit has an element represented by Figure 2.3-1*b* where the current is a constant 5 A and the voltage is a constant 6 V. Find the energy supplied over the time interval 0 to T.

Solution

Because the current enters the negative terminal, the energy *supplied* by the element is given by

$$w = \int_{0}^{T} (6)(5)d\tau = 30T \text{ J}$$

Thus, the device is a generator or an active element, in this case a dc battery.

2.4 Resistors

The ability of a material to resist the flow of charge is called its *resistivity*, ρ. Materials that are good electrical insulators have a high value of resistivity. Materials that are good conductors of electric current have low values of resistivity. Resistivity values for selected materials are given in Table 2.4-1. Copper is commonly used for wires because it permits current to flow relatively unimpeded. Silicon is commonly used to provide resistance in semiconductor electric circuits. Polystyrene is used as an insulator.

Table 2.4-1 Resistivities of Selected Materials

MATERIAL	RESISTIVITY ρ (OHM.CM)
Polystyrene	1×10^{18}
Silicon	2.3×10^{5}
Carbon	4×10^{-3}
Aluminum	2.7×10^{-6}
Copper	1.7×10^{-6}

> **Resistance** is the physical property of an element or device that impedes the flow of current; it is represented by the symbol R.

Georg Simon Ohm was able to show that the current in a circuit composed of a battery and a conducting wire of uniform cross-section could be expressed as

$$i = \frac{Av}{\rho L} \tag{2.4-1}$$

where A is the cross-sectional area, ρ the resistivity, L the length, and v the voltage across the wire element. Ohm, who is shown in Figure 2.4-1, defined the constant resistance R as

$$R = \frac{\rho L}{A} \tag{2.4-2}$$

Ohm's law, which related the voltage and current, was published in 1827 as

$$v = Ri \tag{2.4-3}$$

The unit of resistance R was named the ohm in honor of Ohm and is usually abbreviated by the Ω (capital omega) symbol, where $1\ \Omega = 1$ V/A. The resistance of a 10-m length of common TV cable is 2 mΩ.

An element that has a resistance R is called a *resistor*. A resistor is represented by the two-terminal symbol shown in Figure 2.4-2. Ohm's law, Eq. 2.4-3, requires that the i-versus-v relationship be linear. As shown in Figure 2.4-3, a resistor may become nonlinear outside its normal rated range of operation. We will assume that a resistor is linear unless stated otherwise. Thus, we will use a linear model of the resistor as represented by Ohm's law.

In Figure 2.4-4, the element current and element voltage of a resistor are labeled. The relationship between the directions of this current and voltage is important. The voltage direction marks one resistor terminal + and the other −. The current i_a flows from the terminal marked + to the terminal marked −. This relationship between the current and voltage reference directions is a convention called the passive convention. Ohm's law states that when the element voltage and the element current adhere to the passive convention, then

$$v = Ri_a \tag{2.4-4}$$

Photo by Hulton Archive/ Getty Images

FIGURE 2.4-1 Georg Simon Ohm (1787–1854), who determined Ohm's law in 1827. The ohm was chosen as the unit of electrical resistance in his honor.

FIGURE 2.4-2 Symbol for a resistor having a resistance of R ohms.

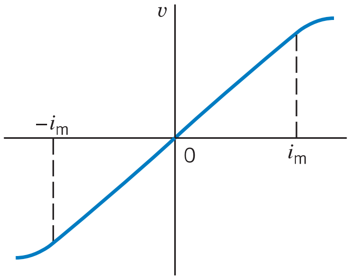


FIGURE 2.4-3 A resistor operating within its specified current range, $\pm i_{\mathrm{m}}$, can be modeled by Ohm's law.

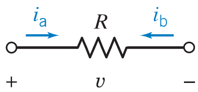


FIGURE 2.4-4 A resistor with element current and element voltage.

Courtesy of Vishay Intertechnology, Inc.

FIGURE 2.4-5 (*a*) Wirewound resistor with an adjustable center tap. (*b*) Wirewound resistor with a fixed tap.

Consider Figure 2.4-4. The element currents i_{a} and i_{b} are the same except for the assigned direction, so

$$i_{\mathrm{a}} = -i_{\mathrm{b}}$$

The element current i_{a} and the element voltage v adhere to the passive convention,

$$v = Ri_{\mathrm{a}}$$

Replacing i_{a} by $-i_{\mathrm{b}}$ gives

$$v = -Ri_{\mathrm{b}}$$

There is a minus sign in this equation because the element current i_{b} and the element voltage v do not adhere to the passive convention. We must pay attention to the current direction so that we don't overlook this minus sign.

Ohm's law, Eq. 2.4-3, can also be written as

$$i = Gv \tag{2.4-5}$$

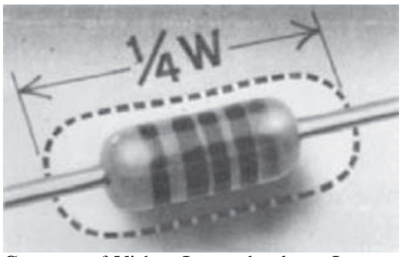
where G denotes the *conductance* in siemens (S) and is the reciprocal of R; that is, $G = 1/R$. Many engineers denote the units of conductance as mhos with the ℧ symbol, which is an inverted omega (mho is *ohm* spelled backward). However, we will use SI units and retain siemens as the units for conductance.

Most discrete resistors fall into one of four basic categories: carbon composition, carbon film, metal film, or wirewound. Carbon composition resistors have been in use for nearly 100 years and are still popular. Carbon film resistors have supplanted carbon composition resistors for many general-purpose uses because of their lower cost and better tolerances. Two wirewound resistors are shown in Figure 2.4-5.

Carbon composition resistors, as shown in Figure 2.4-6, are used in circuits because of their low cost and small size. General-purpose resistors are available in standard values for tolerances of 2, 5, 10, and 20 percent. Carbon composition resistors and some wirewounds have a color code with three to five bands. A color code is a system of standard colors adopted for identification of the resistance of resistors. Figure 2.4-7 shows a metal film resistor with its color bands. This is a 1/4-watt resistor, implying that it should be operated at or below 1/4 watt of power delivered to it. The normal range of resistors is from less than 1 ohm to 10 megohms. Typical values of some commercially available resistors are given in Appendix D.

Courtesy of Hifi Collective.

FIGURE 2.4-6 Carbon composition resistors.



Courtesy of Vishay Intertechnology, Inc.

FIGURE 2.4-7 A 1/4-watt metal film resistor. The body of the resistor is 6 mm long.

The power delivered to a resistor (when the passive convention is used) is

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{2.4-6}$$

Alternatively, because $v = iR$, we can write the equation for power as

$$p = vi = (iR)i = i^2R \tag{2.4-7}$$

Thus, the power is expressed as a nonlinear function of the current i through the resistor or of the voltage v across it.

EXAMPLE 2.4-1 Power Dissipated by a Resistor

Let us devise a model for a car battery when the lights are left on and the engine is off. We have all experienced or seen a car parked with its lights on. If we leave the car for a period, the battery will run down or go dead. An auto battery is a 12-V constant-voltage source, and the lightbulb can be modeled by a resistor of 6 ohms. The circuit is shown in Figure 2.4-8. Let us find the current i, the power p, and the energy supplied by the battery for a four-hour period.

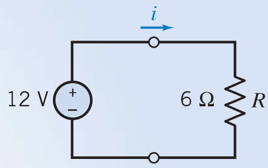


FIGURE 2.4-8 Model of a car battery and the headlight lamp.

Solution

According to Ohm's law, Eq. 2.4-3, we have

$$v = Ri$$

Because $v = 12$ V and $R = 6\ \Omega$, we have $i = 2$ A.

To find the power delivered by the battery, we use

$$p = vi = 12(2) = 24 \text{ W}$$

Finally, the energy delivered in the four-hour period is

$$w = \int_0^t p\,d\tau = 24t = 24(60 \times 60 \times 4) = 3.46 \times 10^5 \text{ J}$$

Because the battery has a finite amount of stored energy, it will deliver this energy and eventually be unable to deliver further energy without recharging. We then say the battery is run down or dead until recharged. A typical auto battery may store 10^6 J in a fully charged condition.

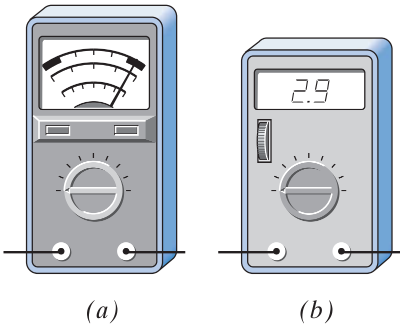


FIGURE 2.6-1 (*a*) A direct-reading (analog) meter. (*b*) A digital meter.

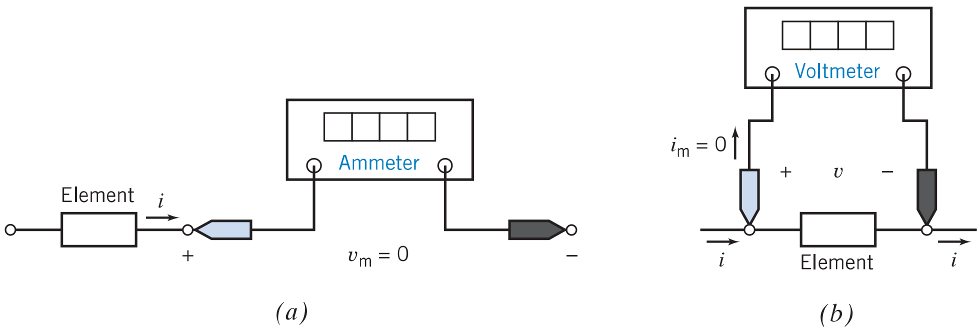


FIGURE 2.6-2 (*a*) Ideal ammeter. (*b*) Ideal voltmeter.

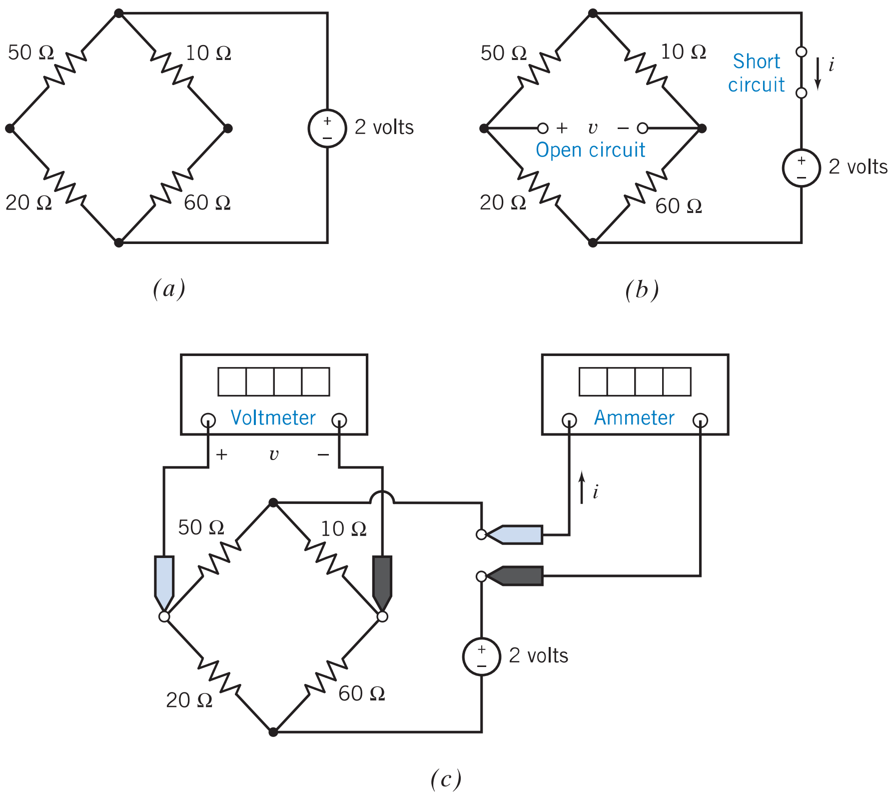
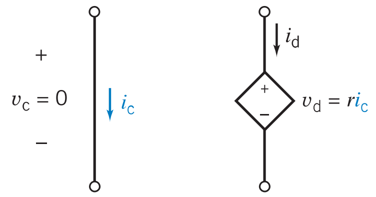
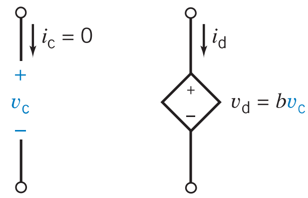
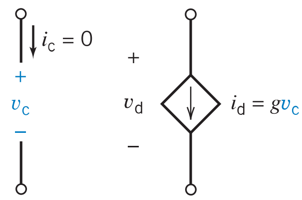
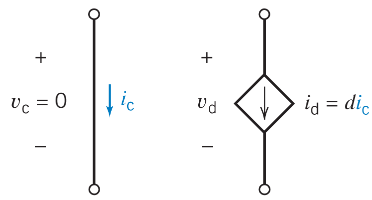


FIGURE 2.6-3 (*a*) An example circuit, (*b*) plus an open circuit and a short circuit. (*c*) The open circuit is replaced by a voltmeter, and the short circuit is replaced by an ammeter.

Table 2.7-1 Dependent Sources

DESCRIPTION	SYMBOL
Current-Controlled Voltage Source (CCVS) r is the gain of the CCVS. r has units of volts/ampere.	
Voltage-Controlled Voltage Source (VCVS) b is the gain of the VCVS. b has units of volts/volt.	
Voltage-Controlled Current Source (VCCS) g is the gain of the VCCS. g has units of amperes/volt.	
Current-Controlled Current Source (CCCS) d is the gain of the CCCS. d has units of amperes/ampere.	

always treat the controlling current of a dependent source as the current in a short circuit. We will use this second point of view to categorize dependent sources in this section.

Figure 2.7-1c shows a circuit that includes a dependent source, represented by the diamond symbol. The arrow inside the diamond identifies the dependent source as a current source and indicates the reference direction of the element current. The label "0.2v" represents the current of this dependent source. This current is a product of two factors, 0.2 and v. The second factor, v, indicates that the current of this dependent source is controlled by the voltage, v, across the 18-Ω resistor. The first factor, 0.2, is the gain of this dependent source. The gain of this dependent source is the ratio of the controlled current, 0.2v, to the controlling voltage, v. This gain has units of A/V. Because this dependent source is a current source and because a voltage controls the current, the dependent source is called a voltage-controlled current source (VCCS).

Figure 2.7-1d shows the circuit from Figure 2.7-1c, using a different point of view. In Figure 2.7-1d, an open circuit has been added in parallel with the 18-Ω resistor. Now we think of the controlling voltage v as the voltage across an open circuit Figure 2.7-1, rather than the voltage across the 18-Ω resistor itself. In this way, we can always treat the controlling voltage of a dependent source as the voltage across an open circuit.

We are now ready to categorize dependent source. Each dependent source consists of two parts: the controlling part and the controlled part. The controlling part is either an open circuit or a short circuit. The controlled part is either a voltage source or a current source. There are four types of dependent source that correspond to the four ways of choosing a controlling part and a controlled part. These four dependent sources are called the voltage-controlled voltage source (VCVS), current-controlled voltage source (CCVS), voltage-controlled current source (VCCS), and current-controlled current source (CCCS). The symbols that represent dependent sources are shown in Table 2.7-1.

Consider the CCVS shown in Table 2.7-1. The controlling element is a short circuit. The element current and voltage of the controlling element are denoted as i_c and v_c. The voltage across a short circuit is zero, so $v_c = 0$. The short-circuit current, i_c, is the controlling signal of this dependent source. The controlled element is a voltage source. The element current and voltage of the controlled element are denoted as i_d and v_d. The voltage v_d is controlled by i_c:

$$v_d = r i_c$$

The constant r is called the gain of the CCVS. The current i_d, like the current in any voltage source, is determined by the rest of the circuit.

Next, consider the VCVS shown in Table 2.7-1. The controlling element is an open circuit. The current in an open circuit is zero, so $i_c = 0$. The open-circuit voltage, v_c, is the controlling signal of this dependent source. The controlled element is a voltage source. The voltage v_d is controlled by v_c:

$$v_d = b v_c$$

The constant b is called the gain of the VCVS. The current i_d is determined by the rest of the circuit.

The controlling element of the VCCS shown in Table 2.7-1 is an open circuit. The current in this open circuit is $i_c = 0$. The open-circuit voltage, v_c, is the controlling signal of this dependent source. The controlled element is a current source. The current i_d is controlled by v_c:

$$i_d = g v_c$$

The constant g is called the gain of the VCCS. The voltage v_d, like the voltage across any current source, is determined by the rest of the circuit.

The controlling element of the CCCS shown in Table 2.7-1 is a short circuit. The voltage across this short circuit is $v_c = 0$. The short-circuit current, i_c, is the controlling signal of this dependent source. The controlled element is a current source. The current i_d is controlled by i_c:

$$i_d = d i_c$$

The constant d is called the gain of the CCCS. The voltage v_d, like the voltage across any current source, is determined by the rest of the circuit.

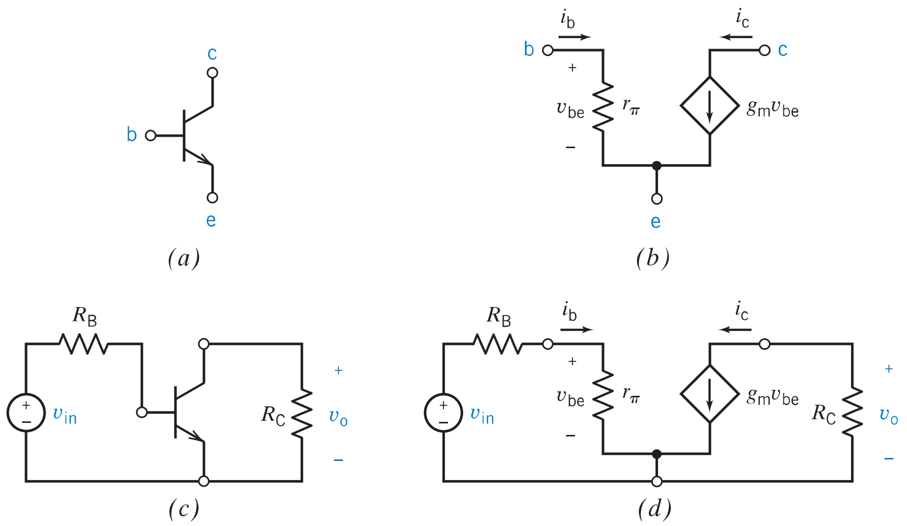


FIGURE 2.7-2 (*a*) A symbol for a transistor. (*b*) A model of the transistor. (*c*) A transistor amplifier. (*d*) A model of the transistor amplifier.

Figure 2.7-2 illustrates the use of dependent sources to model electronic devices. In certain circumstances, the behavior of the transistor shown in Figure 2.7-2*a* can be represented using the model shown in Figure 2.7-2*b*. This model consists of a dependent source and a resistor. The controlling element of the dependent source is an open circuit connected across the resistor. The controlling voltage is v_{be}. The gain of the dependent source is g_m. The dependent source is used in this model to represent a property of the transistor, namely, that the current i_c is proportional to the voltage v_{be}, that is,

$$i_c = g_m v_{be}$$

where g_m has units of amperes/volt. Figures 2.7-2*c* and *d* illustrate the utility of this model. Figure 2.7-2*d* is obtained from Figure 2.7-2*c* by replacing the transistor by the transistor model.

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EXAMPLE 2.7-1 Power and Dependent Sources

Determine the power absorbed by the VCVS in Figure 2.7-3.

Solution

The VCVS consists of an open circuit and a controlled-voltage source. There is no current in the open circuit, so no power is absorbed by the open circuit.

The voltage v_c across the open circuit is the controlling signal of the VCVS. The voltmeter measures v_c to be

$$v_c = 2 \text{ V}$$

The voltage of the controlled voltage source is

$$v_d = 2\, v_c = 4 \text{ V}$$

The ammeter measures the current in the controlled voltage source to be

$$i_d = 1.5 \text{ A}$$

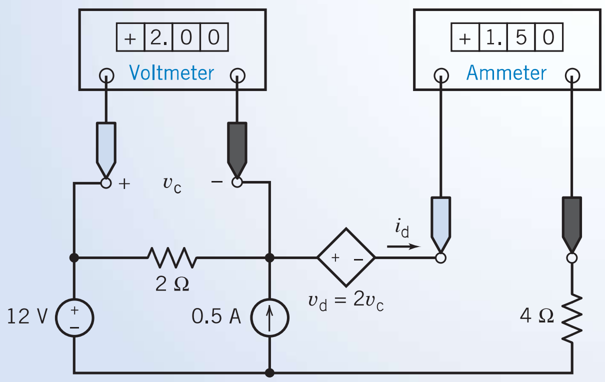

FIGURE 2.7-3 A circuit containing a VCVS. The meters indicate that the voltage of the controlling element is $v_c = 2.0$ volts and that the current of the controlled element is $i_d = 1.5$ amperes.

The element current i_d and voltage v_d adhere to the passive convention. Therefore,

$$p = i_d v_d = (1.5)(4) = 6 \text{ W}$$

is the power absorbed by the VCVS.

EXERCISE 2.7-1 Find the power absorbed by the CCCS in Figure E 2.7-1.

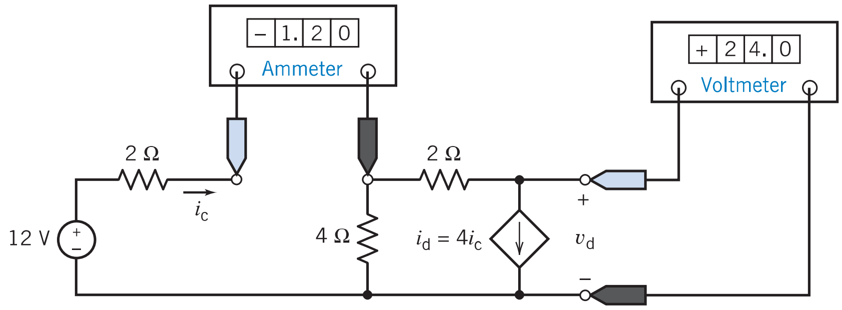

FIGURE E 2.7-1 A circuit containing a CCCS. The meters indicate that the current of the controlling element is $i_c = -1.2$ amperes and that the voltage of the controlled element is $v_d = 24$ volts.

Hint: The controlling element of this dependent source is a short circuit. The voltage across a short circuit is zero. Hence, the power absorbed by the controlling element is zero. How much power is absorbed by the controlled element?

Answer: -115.2 watts are received by the CCCS. (The CCCS supplies $+115.2$ watts to the rest of the circuit.)

2.8 Transducers

Transducers are devices that convert physical quantities to electrical quantities. This section describes two transducers: potentiometers and temperature sensors. Potentiometers convert position to resistance, and temperature sensors convert temperature to current.

Figure 2.8-1*a* shows the symbol for the potentiometer. The potentiometer is a resistor having a third contact, called the wiper, that slides along the resistor. Two parameters, R_p and a, are needed to describe the potentiometer. The parameter R_p specifies the potentiometer resistance ($R_p > 0$). The parameter a represents the wiper position and takes values in the range $0 \le a \le 1$. The values $a = 0$ and $a = 1$ correspond to the extreme positions of the wiper.

Figure 2.8-1*b* shows a model for the potentiometer that consists of two resistors. The resistances of these resistors depend on the potentiometer parameters R_p and a.

Frequently, the position of the wiper corresponds to the angular position of a shaft connected to the potentiometer. Suppose θ is the angle in degrees and $0 \le \theta \le 360$. Then,

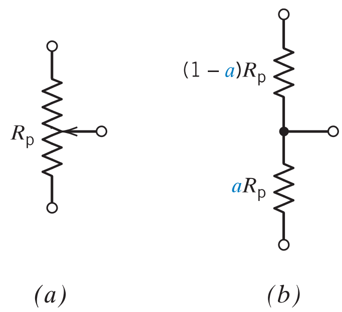

FIGURE 2.8-1 (*a*) The symbol and (*b*) a model for the potentiometer.

$$a = \frac{\theta}{360}$$

Temperature sensors, such as the AD590 manufactured by Analog Devices, are current sources having current proportional to absolute temperature. Figure 2.8-3*a* shows the symbol used to represent the temperature sensor. Figure 2.8-3*b* shows the circuit model of the temperature sensor. For the temperature sensor to operate properly, the branch voltage v must satisfy the condition

$$4 \text{ volts} \le v \le 30 \text{ volts}$$

CHAPTER 3 Resistive Circuits

IN THIS CHAPTER

3.1 *Introduction*

In this chapter, we will do the following:

- Write equations using Kirchhoff's laws.

  Not surprisingly, the behavior of an electric circuit is determined both by the types of elements that comprise the circuit and by the way those elements are connected together. The constitutive equations describe the elements themselves, and Kirchhoff's laws describe the way the elements are connected to each other to form the circuit.
- Analyze simple electric circuits, using only Kirchhoff's laws and the constitutive equations of the circuit elements.
- Analyze two very common circuit configurations: series resistors and parallel resistors.

  We will see that series resistors act like a "voltage divider," and parallel resistors act like a "current divider." Also, series resistors and parallel resistors provide our first examples of an "equivalent circuit." Figure 3.1-1 illustrates this important concept. Here, a circuit has been partitioned into two parts, *A* and *B*. Replacing *B* by an equivalent circuit, B_{eq}, does not change the current or voltage of any circuit element in part *A*. It is in this sense that B_{eq} is equivalent to *B*. We will see how to obtain an equivalent circuit when part *B* consists either of series resistors or of parallel resistors.
- Determine equivalent circuits for series voltage sources and parallel current sources.
- Determine the equivalent resistance of a resistive circuit.

Often, circuits consisting entirely of resistors can be reduced to a single equivalent resistor by repeatedly replacing series and/or parallel resistors by equivalent resistors.

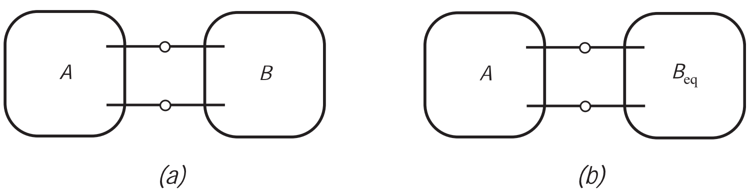

FIGURE 3.1-1 Replacing *B* by an equivalent circuit B_{eq} does not change the current or voltage of any circuit element in *A*.

3.2 Kirchhoff's Laws

An electric circuit consists of circuit elements that are connected together. The places where the elements are connected to each other are called nodes. Figure 3.2-1*a* shows an electric circuit that consists of six elements connected together at four nodes. It is common practice to draw electric circuits using straight lines and to position the elements horizontally or vertically as shown in Figure 3.2-1*b*.

The circuit is shown again in Figure 3.2-1*c*, this time emphasizing the nodes. Notice that redrawing the circuit, using straight lines and horizontal and vertical elements, has changed the way that the nodes are represented. In Figure 3.2-1*a*, nodes are represented as points. In Figures 3.2-1*b*,*c*, nodes are represented using both points and straight-line segments.

The same circuit can be drawn in several ways. One drawing of a circuit might look much different from another drawing of the same circuit. How can we tell when two circuit drawings represent the same circuit? Informally, we say that two circuit drawings represent the same circuit if

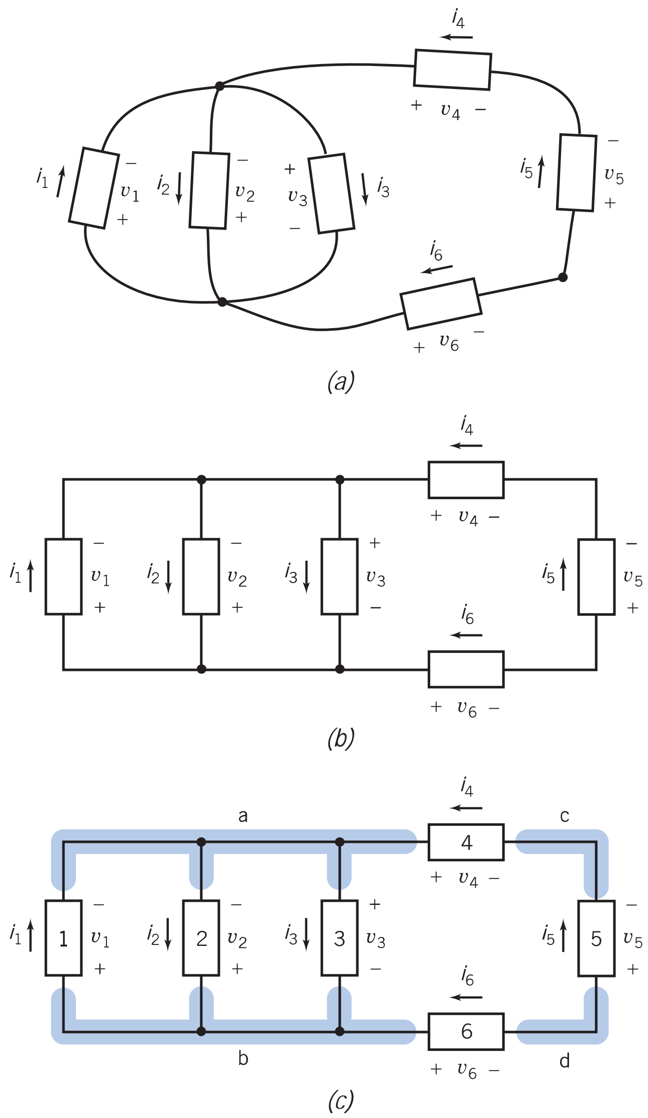

FIGURE 3.2-1 (*a*) An electric circuit. (*b*) The same circuit, redrawn using straight lines and horizontal and vertical elements. (*c*) The circuit after labeling the nodes and elements.

Solution

The circuit drawing shown in Figure 3.2-2*a* has five nodes, labeled r, s, t, u, and v. The circuit drawing in Figure 3.2-1*c* has four nodes. Because the two drawings have different numbers of nodes, there cannot be a one-to-one correspondence between the nodes of the two drawings. Hence, these drawings represent different circuits.

The circuit drawing shown in Figure 3.2-2*b* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. The nodes in Figure 3.2-2*b* have been labeled in the same way as the corresponding nodes in Figure 3.2-1*c*. For example, node c in Figure 3.2-2*b* corresponds to node c in Figure 3.2-1*c*. The elements in Figure 3.2-2*b* have been labeled in the same way as the corresponding elements in Figure 3.2-1*c*. For example, element 5 in Figure 3.2-2*b* corresponds to element 5 in Figure 3.2-1*c*. Corresponding elements are indeed connected to corresponding nodes. For example, element 2 is connected to nodes a and b, in both Figure 3.2-2*b* and in Figure 3.2-1*c*. Consequently, Figure 3.2-2*b* and Figure 3.2-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.2-2*c* has four nodes and six elements, the same number of nodes and elements as the circuit drawing in Figure 3.2-1*c*. The nodes and elements in Figure 3.2-2*c* have been labeled in the same way as the corresponding nodes and elements in Figure 3.2-1*c*. Corresponding elements are indeed connected to corresponding nodes. Therefore, Figure 3.2-2*c* and Figure 3.2-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.2-2*d* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. However, the nodes and elements of Figure 3.2-2*d* cannot be labeled so that corresponding elements of Figure 3.2-1*c* are connected to corresponding nodes. (For example, in Figure 3.2-1*c*, three elements are connected between the same pair of nodes, a and b. That does not happen in Figure 3.2-2*d*.) Consequently, Figure 3.2-2*d* and Figure 3.2-1*c* represent different circuits.

FIGURE 3.2-3 Gustav Robert Kirchhoff (1824–1887). Kirchhoff stated two laws in 1847 regarding the current and voltage in an electrical circuit.

In 1847, Gustav Robert Kirchhoff, a professor at the University of Berlin, formulated two important laws that provide the foundation for analysis of electric circuits. These laws are referred to as *Kirchhoff's current law* (KCL) and *Kirchhoff's voltage law* (KVL) in his honor. Kirchhoff's laws are a consequence of conservation of charge and conservation of energy. Gustav Robert Kirchhoff is pictured in Figure 3.2-3.

Kirchhoff's current law states that the algebraic sum of the currents entering any node is identically zero for all instants of time.

Kirchhoff's current law (KCL): The algebraic sum of the currents into a node at any instant is zero.

The phrase *algebraic sum* indicates that we must take reference directions into account as we add up the currents of elements connected to a particular node. One way to take reference directions into account is to use a plus sign when the current is directed away from the node and a minus sign when the current is directed toward the node. For example, consider the circuit shown in Figure 3.2-1*c*. Four elements of this circuit—elements 1, 2, 3, and 4—are connected to node a. By Kirchhoff's current law, the algebraic sum of the element currents i_1, i_2, i_3, and i_4 must be zero. Currents i_2 and i_3 are directed away from node a, so we will use a plus sign for i_2 and i_3. In contrast, currents i_1 and i_4 are directed toward node a, so we will use a minus sign for i_1 and i_4. The KCL equation for node a of Figure 3.2-1*c* is

$$-i_1 + i_2 + i_3 - i_4 = 0 \tag{3.2-1}$$

An alternate way of obtaining the algebraic sum of the currents into a node is to set the sum of all the currents directed away from the node equal to the sum of all the currents directed toward that node. Using this technique, we find that the KCL equation for node a of Figure 3.2-1*c* is

$$i_2 + i_3 = i_1 + i_4 \tag{3.2-2}$$

Clearly, Eqs. 3.2-1 and 3.2-2 are equivalent.

Similarly, the Kirchhoff's current law equation for node b of Figure 3.2-1*c* is

$$i_1 = i_2 + i_3 + i_6$$

Before we can state Kirchhoff's voltage law, we need the definition of a loop. A *loop* is a closed path through a circuit that does not encounter any intermediate node more than once. For example, starting at node a in Figure 3.2-1*c*, we can move through element 4 to node c, then through element 5 to node d, through element 6 to node b, and finally through element 3 back to node a. We have a closed path, and we did not encounter any of the intermediate nodes—b, c, or d—more than once. Consequently, elements 3, 4, 5, and 6 comprise a loop. Similarly, elements 1, 4, 5, and 6 comprise a loop of the circuit shown in Figure 3.2-1*c*. Elements 1 and 3 comprise yet another loop of this circuit. The circuit has three other loops: elements 1 and 2, elements 2 and 3, and elements 2, 4, 5, and 6.

We are now ready to state Kirchhoff's voltage law.

Kirchhoff's voltage law (KVL): The algebraic sum of the voltages around any loop in a circuit is identically zero for all time.

The phrase *algebraic sum* indicates that we must take polarity into account as we add up the voltages of elements that comprise a loop. One way to take polarity into account is to move around the loop in the clockwise direction while observing the polarities of the element voltages. We write the voltage with a plus sign when we encounter the + of the voltage polarity before the −. In contrast, we write the voltage with a minus sign when we encounter the − of the voltage polarity before the +. For example, consider the circuit shown in Figure 3.2-1*c*. Elements 3, 4, 5, and 6 comprise a loop of the circuit. By Kirchhoff's voltage law, the algebraic sum of the element voltages v_3, v_4, v_5, and v_6 must be zero. As we move around the loop in the clockwise direction, we encounter the + of v_4 before the −, the − of v_5 before the +, the − of v_6 before the +, and the − of v_3 before the +. Consequently, we use a minus sign for v_3, v_5, and v_6 and a plus sign for v_4. The KCL equation for this loop of Figure 3.2-1*c* is

$$v_4 - v_5 - v_6 - v_3 = 0$$

Similarly, the Kirchhoff's voltage law equation for the loop consisting of elements 1, 4, 5, and 6 is

$$v_4 - v_5 - v_6 + v_1 = 0$$

The Kirchhoff's voltage law equation for the loop consisting of elements 1 and 2 is

$$-v_2 + v_1 = 0$$

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EXAMPLE 3.2-2 Kirchhoff's Laws

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 3.2-4*a*. Determine the power supplied by element *C* and the power received by element *D*.

Solution

Figure 3.2-4*a* provides a value for the current in element *C* but not for the voltage *v* across element *C*. The voltage and current of element *C* given in Figure 3.2-4*a* adhere to the passive convention, so the product of this voltage and current is the power *received* by element *C*. Figure 3.2-4*a* provides a value for the voltage across element *D* but not for the current *i* in element *D*. The voltage and current of element *D* given in Figure 3.2-4*a* do not adhere to the passive convention, so the product of this voltage and current is the power *supplied* by element *D*.

We need to determine the voltage *v* across element *C* and the current *i* in element *D*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 3.2-4*b*.

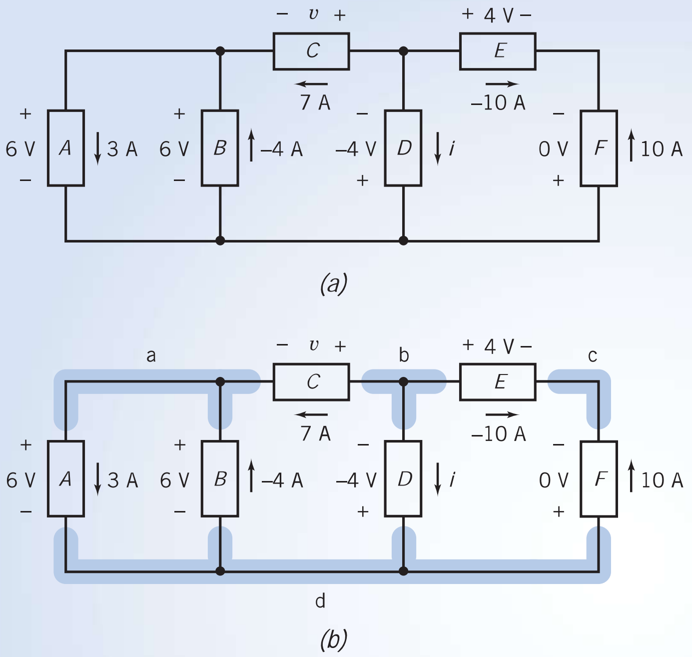

FIGURE 3.2-4 (*a*) The circuit considered in Example 3.2-2 and (*b*) the circuit redrawn to emphasize the nodes.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements C, D, and B to get

$$-v - (-4) - 6 = 0 \Rightarrow v = -2\text{ V}$$

The value of the current in element C in Figure 3.2-4*b* is 7 A. The voltage and current of element C given in Figure 3.2-4*b* adhere to the passive convention, so

$$p_C = v(7) = (-2)(7) = -14\text{ W}$$

is the power *received* by element C. Therefore, element C *supplies* 14 W.

Next, apply Kirchhoff's current law (KCL) at node b to get

$$7 + (-10) + i = 0 \Rightarrow i = 3\text{ A}$$

The value of the voltage across element D in Figure 3.2-4*b* is -4 V. The voltage and current of element D given in Figure 3.2-4*b* do not adhere to the passive convention, so the power *supplied* by element D is given by

$$p_D = (-4)i = (-4)(3) = -12\text{ W}$$

Therefore, element D *receives* 12 W.

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EXAMPLE 3.2-3 Ohm's and Kirchhoff's Laws

Consider the circuit shown in Figure 3.2-5. Notice that the passive convention was used to assign reference directions to the resistor voltages and currents. This anticipates using Ohm's law. Find each current and each voltage when $R_1 = 8\ \Omega$, $v_2 = -10$ V, $i_3 = 2$ A, and $R_3 = 1\ \Omega$. Also, determine the resistance R_2.

Solution

The sum of the currents entering node a is

$$i_1 - i_2 - i_3 = 0$$

Using Ohm's law for R_3, we find that

$$v_3 = R_3 i_3 = 1(2) = 2\text{ V}$$

Kirchhoff's voltage law for the bottom loop incorporating v_1, v_3, and the 10-V source is

$$-10 + v_1 + v_3 = 0$$

Therefore,

$$v_1 = 10 - v_3 = 8\text{ V}$$

Ohm's law for the resistor R_1 is

$$v_1 = R_1 i_1$$

or

$$i_1 = v_1/R_1 = 8/8 = 1\text{ A}$$

Next, apply Kirchhoff's current law at node a to get

$$i_2 = i_1 - i_3 = 1 - 2 = -1\text{ A}$$

We can now find the resistance R_2 from

$$v_2 = R_2 i_2$$

or

$$R_2 = v_2/i_2 = -10/-1 = 10\ \Omega$$

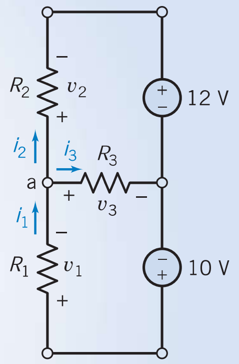

FIGURE 3.2-5 Circuit with two constant-voltage sources.

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EXAMPLE 3.2-4 Ohm's and Kirchhoff's Laws

INTERACTIVE EXAMPLE

Determine the value of the current, in amps, measured by the ammeter in Figure 3.2-6*a*.

Solution

An ideal ammeter is equivalent to a short circuit. The current measured by the ammeter is the current in the short circuit. Figure 3.2-6*b* shows the circuit after replacing the ammeter by the equivalent short circuit.

The circuit has been redrawn in Figure 3.2-7 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent current source, two resistors, and two short circuits. One of the short circuits is the controlling element of the CCCS, and the other short circuit is a model of the ammeter.

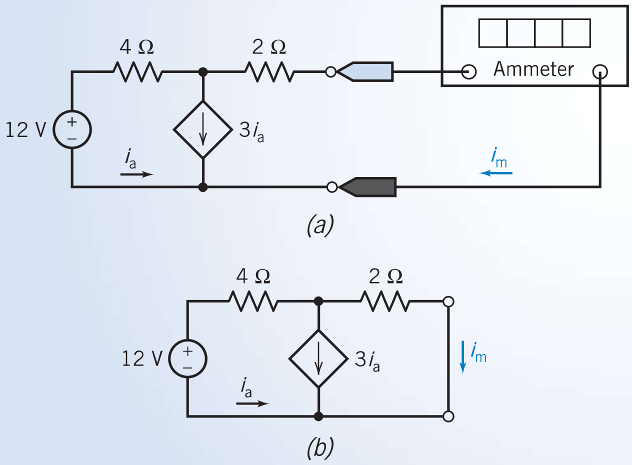

FIGURE 3.2-6 (*a*) A circuit with dependent source and an ammeter. (*b*) The equivalent circuit after replacing the ammeter by a short circuit.

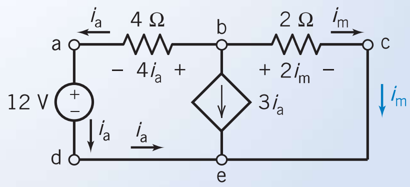

FIGURE 3.2-7 The circuit of Figure 3.2-6 after labeling the nodes and some element currents and voltages.

EXERCISE 3.2-1 Determine the values of i_3, i_4, i_6, v_2, v_4, and v_6 in Figure E 3.2-1.

Answer: $i_3 = -3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = -3$ V, $v_4 = -6$ V, $v_6 = 6$ V

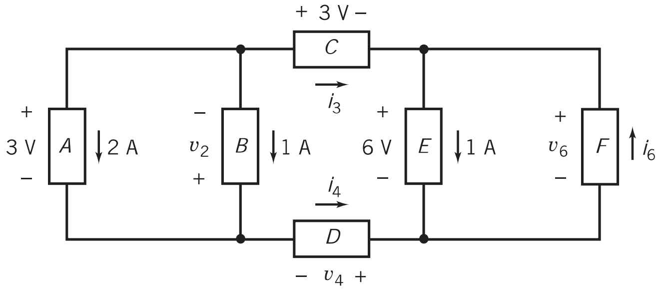

FIGURE E 3.2-1

3.3 *Series Resistors and Voltage Division*

Let us consider a single-loop circuit, as shown in Figure 3.3-1. In anticipation of using Ohm's law, the passive convention has been used to assign reference directions to resistor voltages and currents.

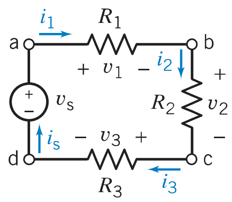

FIGURE 3.3-1
Single-loop circuit with a voltage source v_s.

The connection of resistors in Figure 3.3-1 is said to be a *series* connection because all the elements carry the same current. To identify a pair of series elements, we look for two elements connected to a single node that has no other elements connected to it. Notice, for example, that resistors R_1 and R_2 are both connected to node b and that no other circuit elements are connected to node b. Consequently, $i_1 = i_2$, so both resistors have the same current. A similar argument shows that resistors R_2 and R_3 are also connected in series. Noticing that R_2 is connected in series with both R_1 and R_3, we say that all three resistors are connected in series. The order of series resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.3-1 will not change if we interchange the positions R_2 and R_3.

Using KCL at each node of the circuit in Figure 3.3-1, we obtain

$$\begin{aligned} \text{a:}\ i_s &= i_1 \\ \text{b:}\ i_1 &= i_2 \\ \text{c:}\ i_2 &= i_3 \\ \text{d:}\ i_3 &= i_s \end{aligned}$$

Consequently,

$$i_s = i_1 = i_2 = i_3$$

To determine i_1, we use KVL around the loop to obtain

$$v_1 + v_2 + v_3 - v_s = 0$$

where, for example, v_1 is the voltage across the resistor R_1. Using Ohm's law for each resistor,

$$R_1 i_1 + R_2 i_2 + R_2 i_3 - v_s = 0 \Rightarrow R_1 i_1 + R_2 i_1 + R_2 i_1 = v_s$$

Solving for i_1, we have

$$i_1 = \frac{v_s}{R_1 + R_2 + R_3}$$

Next, let's calculate the power absorbed by the series resistors in Figure 3.3-2a:

$$p = i_s{}^2 R_1 + i_s{}^2 R_2 + i_s{}^2 R_3$$

Doing a little algebra gives

$$p = i_s{}^2(R_1 + R_2 + R_3) = i_s{}^2 R_s$$

which is equal to the power absorbed by the equivalent resistor in Figure 3.3-2b. We conclude that the power absorbed by series resistors is equal to the power absorbed by the equivalent resistor.

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EXAMPLE 3.3-1 Voltage Division

Consider the two similar voltage divider circuits shown in Figure 3.3-3. Use voltage division to determine the values of the voltage v_2 in Figure 3.3-3a and the voltage v_b in Figure 3.3-3b.

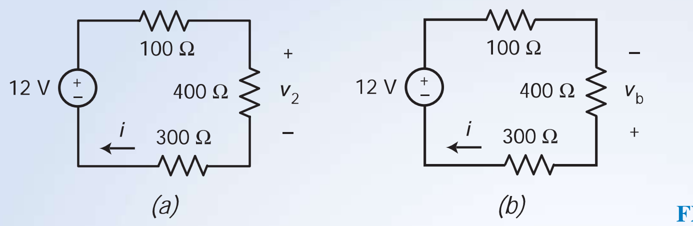

FIGURE 3.3-3 Two similar voltage divider circuits.

Solution

First, consider the circuit shown in Figure 3.3-3a. This circuit is an example of a single loop circuit like the circuit shown in Figure 3.3-1. The 100, 400, and 300-Ω resistors are connected in series. The current in the loop is given by

$$i = \frac{12}{100 + 400 + 300} = 0.015 \text{ A} = 15 \text{ mA}$$

We can calculate the value of v_2 using voltage division:

$$v_2 = \frac{400}{100 + 400 + 300}(12) = 6 \text{ V}$$

As a check, notice that

$$6 = v_2 = 400(i) = 400(0.015)$$

Next, consider the circuit shown in Figure 3.3-3b. This circuit is also an example of a single loop circuit. Again, the current in the loop is given by

$$i = \frac{12}{100 + 400 + 300} = 0.015 \text{ A} = 15 \text{ mA}$$

Notice that the voltage v_b in Figure 3.3-3b is the same voltage as the voltage v_2 in Figure 3.3-3a, ***except for polarity***. Consequently

$$v_2 = -v_b$$

Therefore

$$v_b = \frac{400}{100 + 400 + 300}(12) = -6 \text{ V}$$

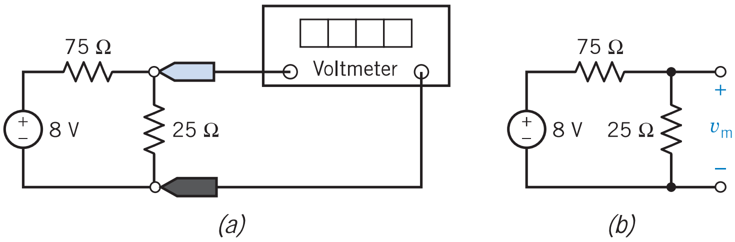

FIGURE E 3.3-1 (*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter v_m.

EXERCISE 3.3-2 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-2*a*.

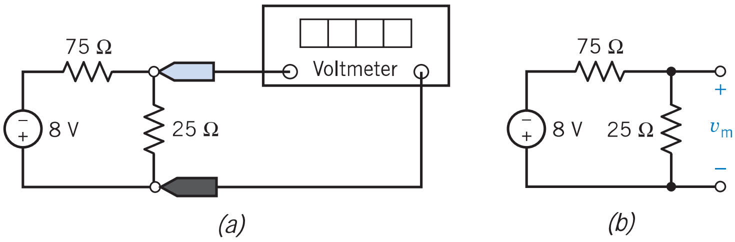

FIGURE E 3.3-2 (*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter v_m.

Hint: Figure E 3.3-2*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter v_m.

Answer: $v_m = -2$ V

3.4 *Parallel Resistors and Current Division*

Circuit elements, such as resistors, are connected in *parallel* when the voltage across each element is identical. The resistors in Figure 3.4-1 are connected in *parallel*. Notice, for example, that resistors R_1 and R_2 are each connected to both node a and node b. Consequently, $v_1 = v_2$, so both resistors have the same voltage. A similar argument shows that resistors R_2 and R_3 are also connected in parallel. Noticing that R_2 is connected in parallel with both R_1 and R_3, we say that all three resistors are connected in parallel. The order of parallel resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.4-1 will not change if we interchange the positions R_2 and R_3.

The defining characteristic of parallel elements is that they have the same voltage. To identify a pair of parallel elements, we look for two elements connected between the same pair of nodes.

Consider the circuit with two resistors and a current source shown in Figure 3.4-2. Note that both resistors are connected to terminals a and b and that the voltage v appears across each parallel

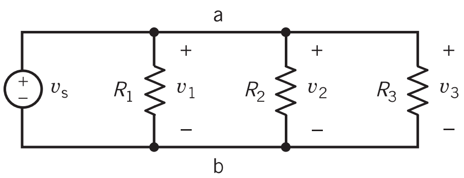

FIGURE 3.4-1 A circuit with parallel resistors.

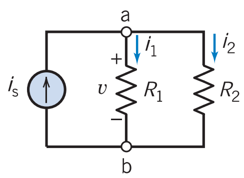

FIGURE 3.4-2 Parallel circuit with a current source.

element. In anticipation of using Ohm's law, the passive convention is used to assign reference directions to the resistor voltages and currents. We may write KCL at node a (or at node b) to obtain

$$i_s - i_1 - i_2 = 0$$

or

$$i_s = i_1 + i_2$$

Next, from Ohm's law

$$i_1 = \frac{v}{R_1} \quad \text{and} \quad i_2 = \frac{v}{R_2}$$

Then

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} \tag{3.4-1}$$

Recall that we defined conductance G as the inverse of resistance R. We may therefore rewrite Eq. 3.4-1 as

$$i_s = G_1 v + G_2 v = (G_1 + G_2)v \tag{3.4-2}$$

Thus, the equivalent circuit for this parallel circuit is a conductance G_p, as shown in Figure 3.4-3, where

$$G_p = G_1 + G_2$$

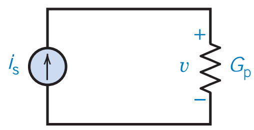

FIGURE 3.4-3 Equivalent circuit for a parallel circuit.

The equivalent resistance for the two-resistor circuit is found from

$$G_p = \frac{1}{R_1} + \frac{1}{R_2}$$

Because $G_p = 1/R_p$, we have

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$\boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}} \tag{3.4-3}$$

Note that the total conductance, G_p, increases as additional parallel elements are added and that the total resistance, R_p, declines as each resistor is added.

The circuit shown in Figure 3.4-2 is called a *current divider* circuit because it divides the source current. Note that

$$i_1 = G_1 v \tag{3.4-4}$$

Also, because $i_s = (G_1 + G_2)v$, we solve for v, obtaining

$$v = \frac{i_s}{G_1 + G_2} \tag{3.4-5}$$

Substituting v from Eq. 3.4-5 into Eq. 3.4-4, we obtain

$$i_1 = \frac{G_1 i_s}{G_1 + G_2} \tag{3.4-6}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_1 + G_2}$$

Note that we may use $G_2 = 1/R_2$ and $G_1 = 1/R_1$ to obtain the current i_2 in terms of two resistances as follows:

$$\boxed{i_2 = \frac{R_1 i_s}{R_1 + R_2}}$$

The current of the source divides between conductances G_1 and G_2 in proportion to their conductance values.

Let us consider the more general case of current division with a set of N parallel conductors as shown in Figure 3.4-4. The KCL gives

$$i_s = i_1 + i_2 + i_3 + \cdots + i_N \tag{3.4-7}$$

for which

$$i_n = G_n v \tag{3.4-8}$$

for $n = 1, \ \ldots, N$. We may write Eq. 3.4-7 as

$$i_s = (G_1 + G_2 + G_3 + \cdots + G_N)v \tag{3.4-9}$$

Therefore,

$$i_s = v \sum_{n=1}^{N} G_n \tag{3.4-10}$$

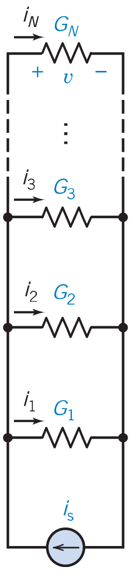

FIGURE 3.4-4 Set of N parallel conductances with a current source i_s.

Because $i_n = G_n v$, we may obtain v from Eq. 3.4-10 and substitute it in Eq. 3.4-8, obtaining

$$i_n = \frac{G_n i_s}{\sum_{n=1}^{N} G_n} \tag{3.4-11}$$

Recall that the equivalent circuit, Figure 3.4-3, has an equivalent conductance G_p such that

$$G_p = \sum_{n=1}^{N} G_n \tag{3.4-12}$$

Therefore,

$$i_n = \frac{G_n i_s}{G_p} \tag{3.4-13}$$

which is the basic equation for the current divider with N conductances. Of course, Eq. 3.4-12 can be rewritten as

$$\boxed{\frac{1}{R_p} = \sum_{n=1}^{N} \frac{1}{R_n}} \tag{3.4-14}$$

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EXAMPLE 3.4-1 Parallel Resistors

For the circuit in Figure 3.4-5, find (a) the current in each branch, (b) the equivalent circuit, and (c) the voltage v. The resistors are

$$R_1 = \frac{1}{2}\ \Omega, \quad R_2 = \frac{1}{4}\ \Omega, \quad R_3 = \frac{1}{8}\ \Omega$$

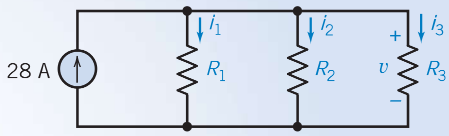

FIGURE 3.4-5 Parallel circuit for Example 3.3-2.

Solution

The current divider follows the equation

$$i_n = \frac{G_n i_s}{G_p}$$

so it is wise to find the equivalent circuit, as shown in Figure 3.4-6, with its equivalent conductance G_p. We have

$$G_p = \sum_{n=1}^{N} G_n = G_1 + G_2 + G_3 = 2 + 4 + 8 = 14\ \text{S}$$

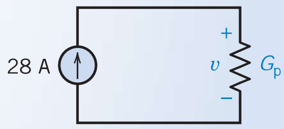

FIGURE 3.4-6 Equivalent circuit for the parallel circuit of Figure 3.4-5.

Recall that the units for conductance are siemens (S). Then

$$i_1 = \frac{G_1 i_s}{G_p} = \frac{2}{14}(28) = 4\text{ A}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_p} = \frac{4(28)}{14} = 8\text{ A}$$

and

$$i_3 = \frac{G_3 i_s}{G_p} = 16\text{ A}$$

Because $i_n = G_n v$, we have

$$v = \frac{i_1}{G_1} = \frac{4}{2} = 2\text{ V}$$

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EXAMPLE 3.4-2 Parallel Resistors

INTERACTIVE EXAMPLE

For the circuit of Figure 3.4-7*a*, find the voltage measured by the voltmeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

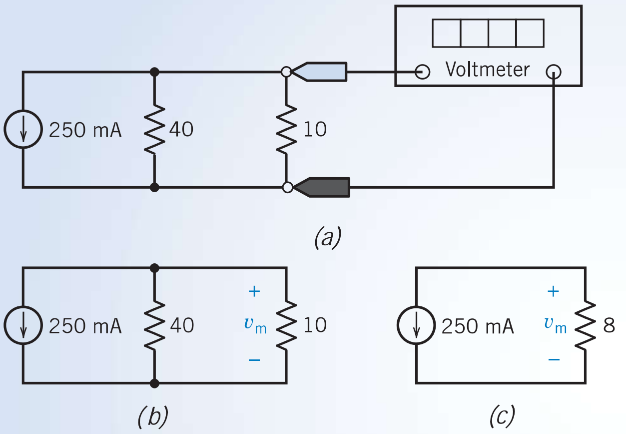


FIGURE 3.4-7 (*a*) A circuit containing parallel resistors. (*b*) The circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter v_m. (*c*) The circuit after the parallel resistors have been replaced by an equivalent resistance.

Solution

Figure 3.4-7*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit, and a label has been added to indicate the voltage measured by the voltmeter v_m. The two resistors are connected in parallel and can be replaced with a single equivalent resistor. The resistance of this equivalent resistor is calculated as

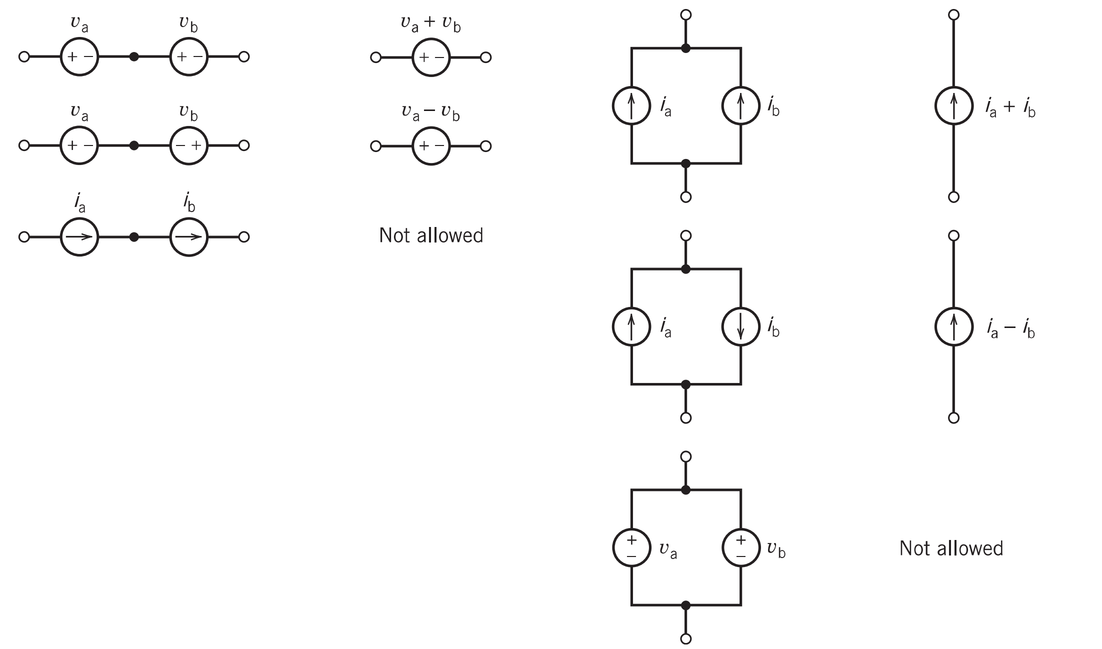
$$\frac{40 \cdot 10}{40 + 10} = 8\ \Omega$$

Figure 3.4-7*c* shows the circuit after the parallel resistors have been replaced by the equivalent resistor. The current in the equivalent resistor is 250 mA, directed upward. This current and the voltage v_m do not adhere to the passive convention. The current in the equivalent resistance can also be expressed as −250 mA, directed downward. This current and the voltage v_m do adhere to the passive convention. Ohm's law gives

$$v_m = 8(-0.25) = -2\text{ V}$$

Table 3.5-1 Parallel and Series Voltage and Current Sources

CIRCUIT	EQUIVALENT CIRCUIT	CIRCUIT	EQUIVALENT CIRCUIT



$$4 = \frac{v_1}{2} + i_s \tag{3.5-8}$$

$$i_s = \frac{v_2}{6} + i_3 \tag{3.5-9}$$

$$v_c = v_1 \tag{3.5-10}$$

$$v_1 = 4 + v_2 \tag{3.5-11}$$

$$v_2 = 3i_3 \tag{3.5-12}$$

The solution to this set of equations is $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V. Eqs. 3.5-8 to 3.5-12 also describe the circuit in Figure 3.5-1*b*. Thus, $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V in both circuits. Replacing series voltage sources by a single, equivalent voltage source does not change the voltage or current of other elements of the circuit.

Figure 3.5-2*a* shows a circuit containing parallel current sources. The circuit in Figure 3.5-2*b* is obtained by replacing these parallel current sources by a single, equivalent current source. The current of the equivalent current source is equal to the algebraic sum of the currents of the parallel current sources.

We are not allowed to connect independent current sources in series. Series elements have the same current. This restriction prevents series current sources from being independent. Similarly, we are not allowed to connect independent voltage sources in parallel.

Table 3.5-1 summarizes the parallel and series connections of current and voltage sources.

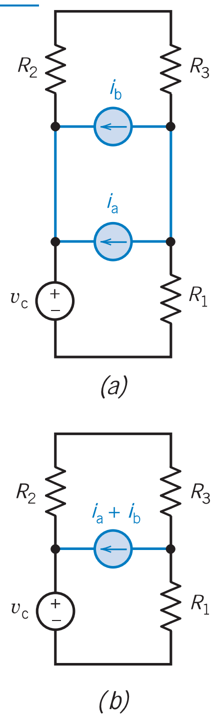


FIGURE 3.5-2 (*a*) A circuit containing parallel current sources and (*b*) an equivalent circuit.

CHAPTER 4 Methods of Analysis of Resistive Circuits

IN THIS CHAPTER

4.1 *Introduction*

To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. The constitutive equations of the circuit elements, such as Ohm's law, provide the remaining equations. The unknown variables are element currents and voltages. Solving the equations provides the values of the element current and voltages.

This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits. A circuit with only 6 elements has 6 element currents and 6 element voltages. We could have 12 equations in 12 unknowns. In this chapter, we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method.
- The mesh current method.

The node voltage method uses a new type of variable called the node voltage. The "node voltage equations" or, more simply, the "node equations," are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the node voltage equations are the node voltages. After solving the node voltage equations, we determine the values of the element currents and voltages from the values of the node voltages.

It's easier to write node voltage equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write node voltage equations for circuits that consist of:

- Resistors and independent current sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

The mesh current method uses a new type of variable called the mesh current. The "mesh current equations" or, more simply, the "mesh equations," are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the mesh current equations are the mesh currents. After solving the mesh current equations, we determine the values of the element currents and voltages from the values of the mesh currents.

It's easier to write mesh current equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write mesh current equations for circuits that consist of:

- Resistors and independent voltage sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

4.2 *Node Voltage Analysis of Circuits with Current Sources*

Consider the circuit shown in Figure 4.2-1*a*. This circuit contains four elements: three resistors and a current source. The *nodes* of a circuit are the places where the elements are connected together. The circuit shown in Figure 4.2-1*a* has three nodes. It is customary to draw the elements horizontally or vertically and to connect these elements by horizontal and vertical lines that represent wires. In other words, nodes are drawn as points or are drawn using horizontal or vertical lines. Figure 4.2-1*b* shows the same circuit, redrawn so that all three nodes are drawn as points rather than lines. In Figure 4.2-1*b*, the nodes are labeled as node a, node b, and node c.

Analyzing a connected circuit containing n. nodes will require $n - 1$ KCL equations. One way to obtain these equations is to apply KCL at each node of the circuit except for one. The node at which KCL is not applied is called the reference node. Any node of the circuit can be selected to be the reference node. We will often choose the node at the bottom of the circuit to be the reference node. (When the circuit contains a grounded power supply, the ground node of the power supply is usually selected as the reference node.) In Figure 4.2-1*b*, node c is selected as the reference node and marked with the symbol used to identify the reference node.

The voltage at any node of the circuit, relative to the reference node, is called a **node voltage**. In Figure 4.2-1*b*, there are two node voltages: the voltage at node a with respect to the reference node, node c, and the voltage at node b, again with respect to the reference node, node c. In Figure 4.2-1*c*, voltmeters are added to measure the node voltages. To measure node voltage at node a, connect the red

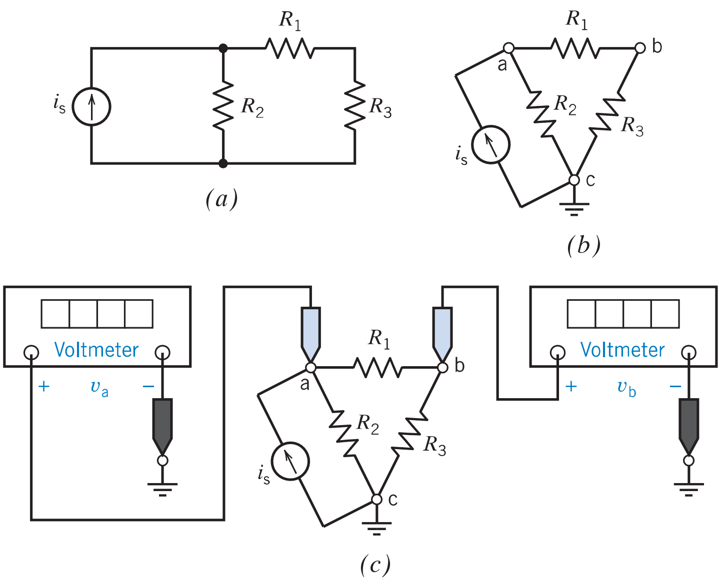


FIGURE 4.2-1 (*a*) A circuit with three nodes. (*b*) The circuit after the nodes have been labeled and a reference node has been selected and marked. (*c*) Using voltmeters to measure the node voltages.

where

$$A = \begin{bmatrix} 0.9 & -0.2 & -0.7 \\ -0.2 & 0.55 & -0.1 \\ -0.7 & 0.1 & 1.3 \end{bmatrix}, \ b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \text{ and, } v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

This matrix equation is solved using MATLAB in Figure 4.2-7.

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 7.1579 \\ 5.0526 \\ 3.4737 \end{bmatrix}$$

Consequently, $v_a = 7.1579$ V, $v_b = 5.0526$ V, and $v_c = 3.4737$ V

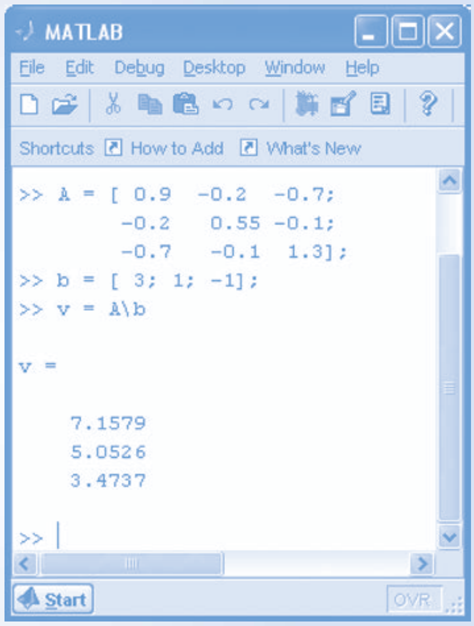


FIGURE 4.2-7 Using MATLAB to solve the node equation in Example 4.2-3.

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EXERCISE 4.2-1 Determine the node voltages v_a and v_b for the circuit of Figure E 4.2-1.

Answer: $v_a = 3$ V and $v_b = 11$ V

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EXERCISE 4.2-2 Determine the node voltages v_a and v_b for the circuit of Figure E 4.2-2.

Answer: $v_a = -4/3$ V and $v_b = 4$ V

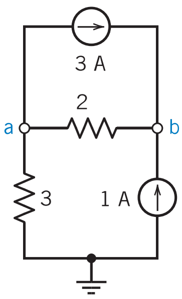

FIGURE E 4.2-1

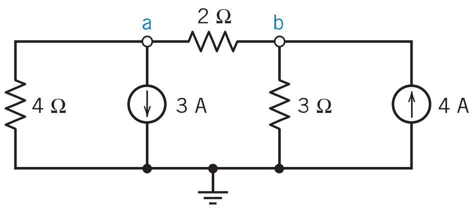

FIGURE E 4.2-2

4.3 *Node Voltage Analysis of Circuits with Current and Voltage Sources*

In the preceding section, we determined the node voltages of circuits with independent current sources only. In this section, we consider circuits with both independent current and voltage sources.

First we consider the circuit with a voltage source between ground and one of the other nodes. Because we are free to select the reference node, this particular arrangement is easily achieved.

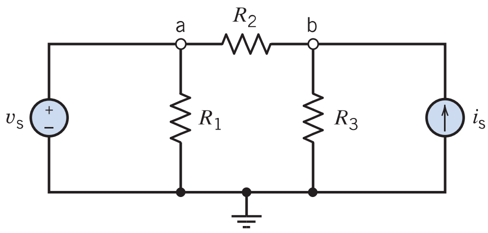

FIGURE 4.3-1 Circuit with an independent voltage source and an independent current source.

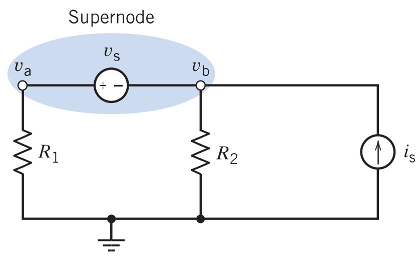

FIGURE 4.3-2 Circuit with a supernode that incorporates v_a and v_b.

Such a circuit is shown in Figure 4.3-1. We immediately note that the source is connected between terminal a and ground and, therefore,

$$v_a = v_s$$

Thus, v_a is known and only v_b is unknown. We write the KCL equation at node b to obtain

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_a}{R_2}$$

However, $v_a = v_s$. Therefore,

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_s}{R_2}$$

Then, solving for the unknown node voltage v_b, we get

$$v_b = \frac{R_2 R_3 i_s + R_3 v_s}{R_2 + R_3}$$

Next, let us consider the circuit of Figure 4.3-2, which includes a voltage source between two nodes. Because the source voltage is known, use KVL to obtain

$$v_a - v_b = v_s$$

or

$$v_a - v_s = v_b$$

To account for the fact that the source voltage is known, we consider both node a and node b as part of one larger node represented by the shaded ellipse shown in Figure 4.3-2. We require a larger node because v_a and v_b are dependent. This larger node is often called a *supernode* or a *generalized node*. KCL says that the algebraic sum of the currents entering a supernode is zero. That means that we apply KCL to a supernode in the same way that we apply KCL to a node.

A **supernode** consists of two nodes connected by an independent or a dependent voltage source.

We then can write the KCL equation at the supernode as

$$\frac{v_a}{R_1} + \frac{v_b}{R_2} = i_s$$

However, because $v_a = v_s + v_b$, we have

$$\frac{v_s + v_b}{R_1} + \frac{v_b}{R_2} = i_s$$

Then, solving for the unknown node voltage v_b, we get

$$v_b = \frac{R_1 R_2 i_s - R_2 v_s}{R_1 + R_2}$$

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EXAMPLE 4.3-1 Node Equations

Determine the values node voltages, v_1 and v_2, in the circuit shown in Figure 4.3-3*a*.

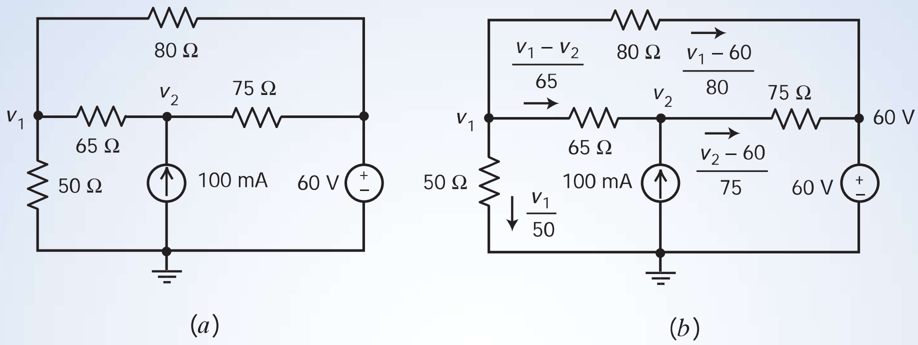

FIGURE 4.3-3 The circuit considered in Example 4.3-1.

Solution

First, represent the resistor currents in terms of the node voltages as shown in Figure 4.3-3*b*. Apply at KCL at node 1 to get

$$\frac{v_1}{50} + \frac{v_1 - v_2}{65} + \frac{v_1 - 60}{80} = 0 \;\Rightarrow\; \left(\frac{1}{50} + \frac{1}{65} + \frac{1}{80}\right) v_1 - \left(\frac{1}{65}\right) v_2 = \frac{60}{80}$$

Apply KCL at node 2 to get

$$0.1 = \frac{v_2 - v_1}{65} + \frac{v_2 - 60}{75} = \;\Rightarrow\; -\left(\frac{1}{65}\right) v_1 + \left(\frac{1}{65} + \frac{1}{75}\right) v_2 = 0.1$$

Organize these equations in matrix form to write

$$\begin{bmatrix} \frac{1}{50} + \frac{1}{65} + \frac{1}{80} & -\frac{1}{65} \\ -\frac{1}{65} & \frac{1}{65} + \frac{1}{75} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{60}{80} \\ 0.1 \end{bmatrix}$$

Solving, we get $\quad v_1 = 30.081\ \text{V}$ and $v_2 = 47.990\ \text{V}$

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EXAMPLE 4.3-2 Supernodes

Determine the values of the node voltages v_a and v_b for the circuit shown in Figure 4.3-4.

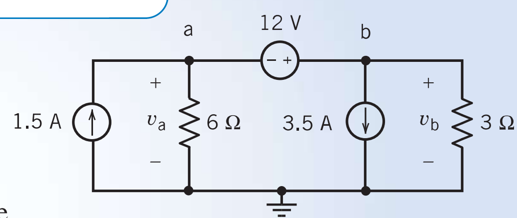

FIGURE 4.3-4 The circuit for Example 4.3-2.

Solution

We can write the first node equation by considering the voltage source. The voltage source voltage is related to the node voltages by

$$v_b - v_a = 12 \;\Rightarrow\; v_b = v_a + 12$$

To write the second node equation, we must decide what to do about the voltage source current. (Notice that there is no easy way to express the voltage source current in terms of the node voltages.) In this example, we illustrate two methods of writing the second node equation.

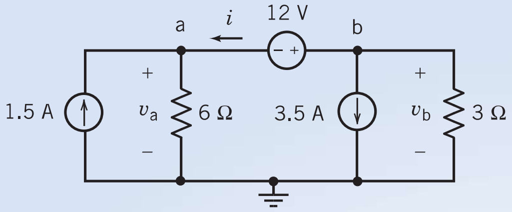

FIGURE 4.3-5 Method 1 For Example 4.3-2.

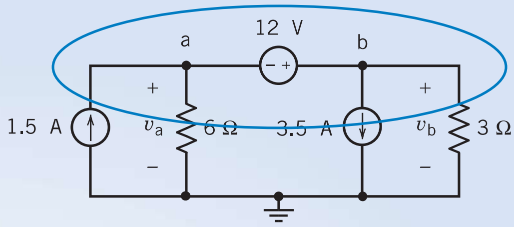

FIGURE 4.3-6 Method 2 for Example 4.3-2.

Method 1: Assign a name to the voltage source current. Apply KCL at both of the voltage source nodes. Eliminate the voltage source current from the KCL equations.

Figure 4.3-5 shows the circuit after labeling the voltage source current. The KCL equation at node a is

$$1.5 + i = \frac{v_a}{6}$$

The KCL equation at node b is

$$i + 3.5 + \frac{v_b}{3} = 0$$

Combining these two equations gives

$$1.5 - \left(3.5 + \frac{v_b}{3}\right) = \frac{v_a}{6} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

Method 2: Apply KCL to the supernode corresponding to the voltage source. Shown in Figure 4.3-6, this supernode separates the voltage source and its nodes from the rest of the circuit. (In this small circuit, the rest of the circuit is just the reference node.)

Apply KCL to the supernode to get

$$1.5 = \frac{v_a}{6} + 3.5 + \frac{v_b}{3} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

This is the same equation that was obtained using method 1. Applying KCL to the supernode is a shortcut for doings three things:

1. Labeling the voltage source current as i.
2. Applying KCL at both nodes of the voltage source.
3. Eliminating i from the KCL equations.

In summary, the node equations are

$$v_b - v_a = 12$$

and

$$\frac{v_a}{6} + \frac{v_b}{3} = -2.0$$

Solving the node equations gives

$$v_a = -12\ \text{V}, \ \text{and} \ v_b = 0\ \text{V}$$

(We might be surprised that v_b is 0 V, but it is easy to check that these values are correct by substituting them into the node equations.)

4.4 Node Voltage Analysis with Dependent Sources

When a circuit contains a dependent source the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.

It is then a simple matter to express the controlled current or voltage as a function of the node voltages. The node equations are then obtained using the techniques described in the previous two sections.

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EXAMPLE 4.4-1 Node Equations for a Circuit Containing a Dependent Source

Determine the node voltages for the circuit shown in Figure 4.4-1.

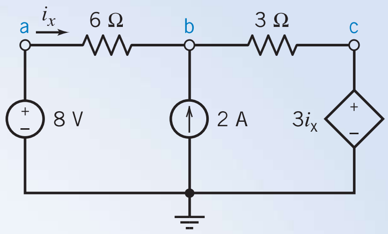

FIGURE 4.4-1 A circuit with a CCVS.

Solution

The controlling current of the dependent source is i_x. Our first task is to express this current as a function of the node voltages:

$$i_x = \frac{v_a - v_b}{6}$$

The value of the node voltage at node a is set by the 8-V voltage source to be

$$v_a = 8 \text{ V}$$

So

$$i_x = \frac{8 - v_b}{6}$$

The node voltage at node c is equal to the voltage of the dependent source, so

$$v_c = 3i_x = 3\left(\frac{8 - v_b}{6}\right) = 4 - \frac{v_b}{2} \qquad (4.4\text{-}1)$$

Next, apply KCL at node b to get

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - v_c}{3} \qquad (4.4\text{-}2)$$

Using Eq. 4.4-1 to eliminate v_c from Eq. 4.4-2 gives

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - \left(4 - \frac{v_b}{2}\right)}{3} = \frac{v_b}{2} - \frac{4}{3}$$

Solving for v_b gives

$$v_b = 7 \text{ V}$$

Then,

$$v_c = 4 - \frac{v_b}{2} = \frac{1}{2} \text{ V}$$

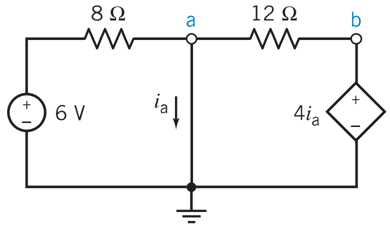

FIGURE E 4.4-1 A circuit with a CCVS.

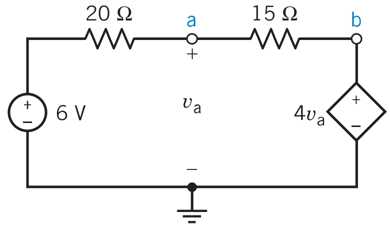

FIGURE E 4.4-2 A circuit with a VCVS.

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EXERCISE 4.4-1 Find the node voltage v_b for the circuit shown in Figure E 4.4-1.

Hint: Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4i_a$ and solve for v_b.

Answer: $-\frac{6}{8} + \frac{v_b}{4} - \frac{v_b}{12} = 0 \quad \Rightarrow \quad v_b = 4.5\text{ V}$

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EXERCISE 4.4-2 Find the node voltages for the circuit shown in Figure E 4.4-2.

Hint: The controlling voltage of the dependent source is a node voltage, so it is already expressed as a function of the node voltages. Apply KCL at node a.

Answer: $\frac{v_a - 6}{20} + \frac{v_a - 4v_a}{15} = 0 \quad \Rightarrow \quad v_a = -2\text{ V}$

4.5 Mesh Current Analysis with Independent Voltage Sources

In this and succeeding sections, we consider the analysis of circuits using Kirchhoff's voltage law (KVL) around a closed path. A *closed path* or a *loop* is drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once.

A mesh is a special case of a loop.

> A **mesh** is a loop that does not contain any other loops within it.

Mesh current analysis is applicable only to planar networks. A planar circuit is one that can be drawn on a plane, without crossovers. An example of a nonplanar circuit is shown in Figure 4.5-1, in which the crossover is identified and cannot be removed by redrawing the circuit. For planar networks, the meshes in the network look like windows. There are four meshes in the circuit shown in Figure 4.5-2. They are identified as M_i. Mesh 2 contains the elements R_3, R_4, and R_5. Note that the resistor R_3 is common to both mesh 1 and mesh 2.

We define a mesh current as the current through the elements constituting the mesh. Figure 4.5-3*a* shows a circuit having two meshes with the mesh currents labeled as i_1 and i_2. We will use the convention of a mesh current in the clockwise direction as shown in Figure 4.5-3*a*. In Figure 4.5-3*b*, ammeters have been inserted into the meshes to measure the mesh currents.

One of the standard methods for analyzing an electric circuit is to write and solve a set of simultaneous equations called the mesh equations. The unknown variables in the mesh equations are the mesh currents of the circuit. We determine the values of the mesh currents by solving the mesh equations.

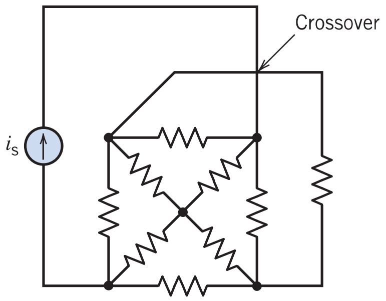

FIGURE 4.5-1 Nonplanar circuit with a crossover.

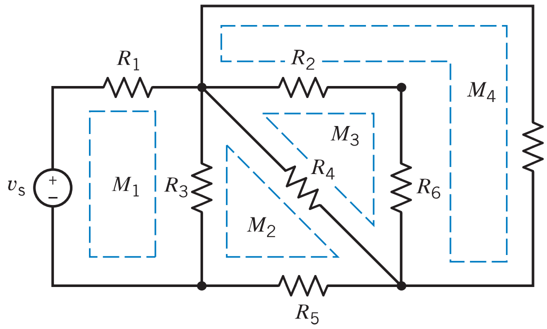

FIGURE 4.5-2 Circuit with four meshes. Each mesh is identified by dashed lines.

To write a set of mesh equations, we do two things:

1. Express element voltages as functions of the mesh currents.
2. Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.

Consider the problem of expressing element voltages as functions of the mesh currents. Although our goal is to express element *voltages* as functions of the mesh currents, we begin by expressing element *currents* as functions of the mesh currents. Figure 4.5-3*b* shows how this is done. The ammeters in Figure 4.5-3*b* measure the mesh currents, i_1 and i_2. Elements C and E are in the right mesh but not in the left mesh. Apply Kirchhoff's current law at node c and then at node f to see that the currents in elements C and E are equal to the mesh current of the right mesh, i_2, as shown in Figure 4.5-3*b*. Similarly, elements A and D are only in the left mesh. The currents in elements A and D are equal to the mesh current of the left mesh, i_1, as shown in Figure 4.5-3*b*.

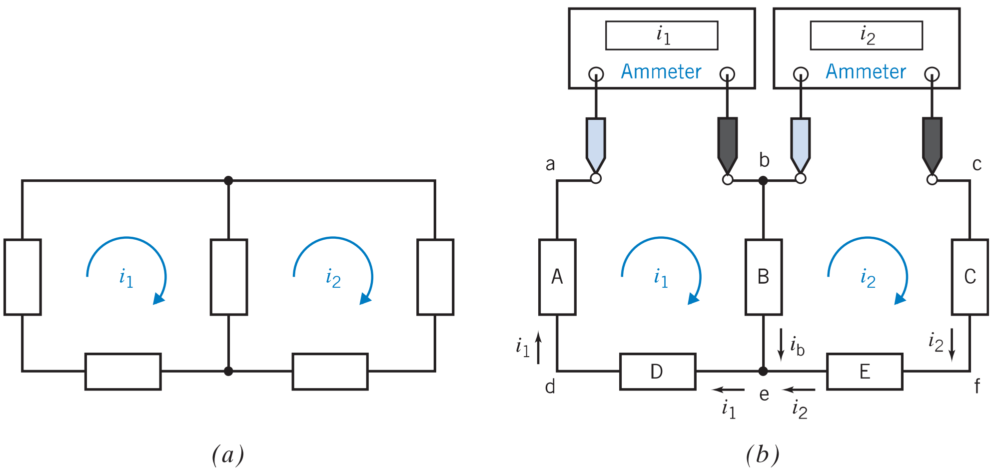

FIGURE 4.5-3 (*a*) A circuit with two meshes. (*b*) Inserting ammeters to measure the mesh currents.

Element B is in both meshes. The current of element B has been labeled as i_b. Applying Kirchhoff's current law at node b in Figure 4.5-3*b* gives

$$i_b = i_1 - i_2$$

This equation expresses the element current i_b as a function of the mesh currents i_1 and i_2.

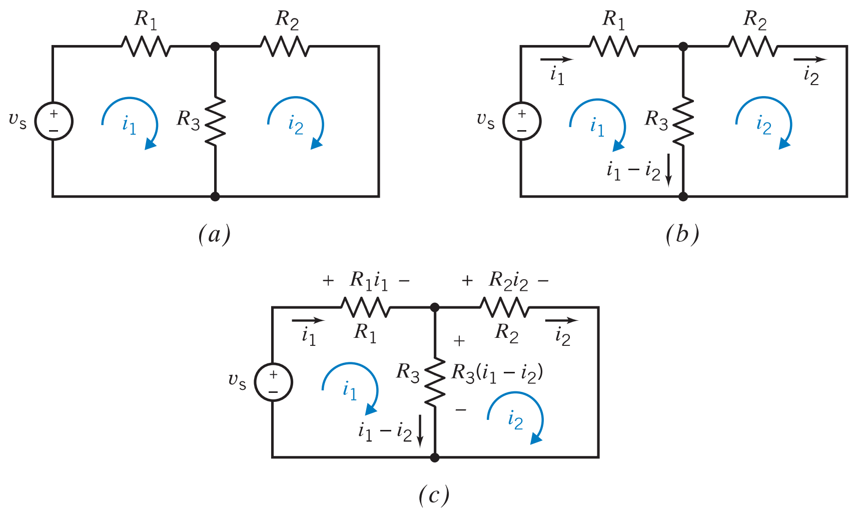

FIGURE 4.5-5 (*a*) A circuit. (*b*) The resistor currents expressed as functions of the mesh currents. (*c*) The resistor voltages expressed as functions of the mesh currents.

$$i_1(R_1 + R_3) - i_2 R_3 = v_s$$

and

$$-i_1 R_3 + i_2(R_3 + R_2) = 0$$

If $R_1 = R_2 = R_3 = 1\ \Omega$, we have

$$2i_1 - i_2 = v_s$$

and

$$-i_1 + 2i_2 = 0$$

Add twice the first equation to the second equation, obtaining $3i_1 = 2v_s$. Then we have

$$i_1 = \frac{2v_s}{3} \text{ and } i_2 = \frac{v_s}{3}$$

Thus, we have obtained two independent mesh current equations that are readily solved for the two unknowns. If we have N meshes and write N mesh equations in terms of N mesh currents, we can obtain N independent mesh equations. This set of N equations is independent and thus guarantees a solution for the N mesh currents.

A circuit that contains only independent voltage sources and resistors results in a specific format of equations that can readily be obtained. Consider a circuit with three meshes, as shown in Figure 4.5-6. Assign the clockwise direction to all of the mesh currents. Using KVL, we obtain the three mesh equations

$$\begin{aligned} &\text{mesh 1: } -v_s + R_1 i_1 + R_4(i_1 - i_2) = 0 \\ &\text{mesh 2: } R_2 i_2 + R_5(i_2 - i_3) + R_4(i_2 - i_1) = 0 \\ &\text{mesh 3: } R_5(i_3 - i_2) + R_3 i_3 + v_g = 0 \end{aligned}$$

These three mesh equations can be rewritten by collecting coefficients for each mesh current as

$$\begin{aligned} &\text{mesh 1: } (R_1 + R_4)i_1 - R_4 i_2 = v_s \\ &\text{mesh 2: } -R_4 i_1 + R_5 + (R_4 + R_2 + R_5)i_2 - R_5 i_3 = 0 \\ &\text{mesh 3: } -R_5 i_2 + (R_3 + R_5)i_3 = -v_g \end{aligned}$$

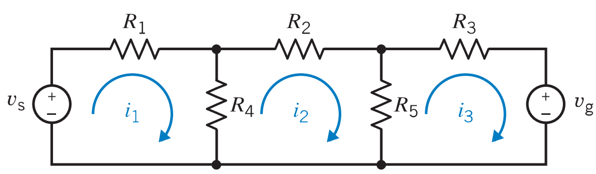

FIGURE 4.5-6 Circuit with three mesh currents and two voltage sources.

We note that if we add Eqs. 4.6-3 and 4.6-4, we eliminate v_{ab}, obtaining

$$R_1 i_1 + (R_2 + R_3) i_2 = v_s$$

However, because $i_2 = i_s + i_1$, we obtain

$$R_1 i_1 + (R_2 + R_3)(i_s + i_1) = v_s$$

or

$$i_1 = \frac{v_s - (R_2 + R_3) i_s}{R_1 + R_2 + R_3} \tag{4.6-5}$$

Thus, we account for independent current sources by recording the relationship between the mesh currents and the current source current. If the current source influences *only one* mesh current, we write the equation that relates that mesh current to the current source current and write the KVL equations for the remaining meshes. If the current source influences two mesh currents, we write the KVL equation for both meshes, assuming a voltage v_{ab} across the terminals of the current source. Then, adding these two mesh equations, we obtain an equation independent of v_{ab}.

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EXAMPLE 4.6-1 Mesh Equations

Consider the circuit of Figure 4.6-3 where $R_1 = R_2 = 1\ \Omega$ and $R_3 = 2\ \Omega$. Find the three mesh currents.

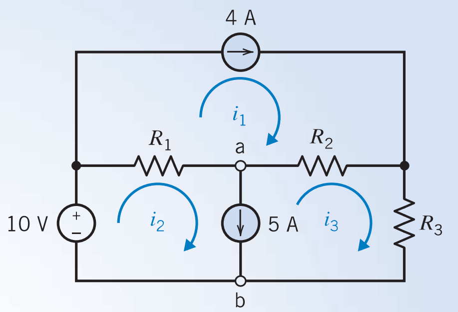

FIGURE 4.6-3 Circuit with two independent current sources.

Solution

Because the 4-A source is in mesh 1 only, we note that

$$i_1 = 4$$

For the 5-A source, we have

$$i_2 - i_3 = 5 \tag{4.6-6}$$

Writing KVL for mesh 2 and mesh 3, we obtain

$$\text{mesh 2: } R_1(i_2 - i_1) + v_{ab} = 10 \tag{4.6-7}$$

$$\text{mesh 3: } R_2(i_3 - i_1) + R_3 i_3 - v_{ab} = 0 \tag{4.6-8}$$

We substitute $i_1 = 4$ and add Eqs. 4.6-7 and 4.6-8 to obtain

$$R_1(i_2 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10 \tag{4.6-9}$$

From Eq. 4.6-6, $i_2 = 5 + i_3$, substituting into Eq. 4.6-9, we have

$$R_1(5 + i_3 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10$$

Using the values for the resistors, we obtain

$$i_3 = \frac{13}{4}\,\text{A} \quad \text{and} \quad i_2 = 5 + i_3 = \frac{33}{4}\,\text{A}$$

Another technique for the mesh analysis method when a current source is common to two meshes involves the concept of a supermesh. A *supermesh* is one mesh created from two meshes that have a current source in common, as shown in Figure 4.6-4.

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EXERCISE 4.6-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.6-1.

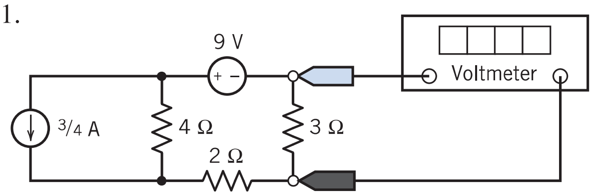

FIGURE E 4.6-1

Hint: Write and solve a single mesh equation to determine the current in the 3-Ω resistor.

Answer: −4 V

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EXERCISE 4.6-2 Determine the value of the current measured by the ammeter in Figure E 4.6-2.

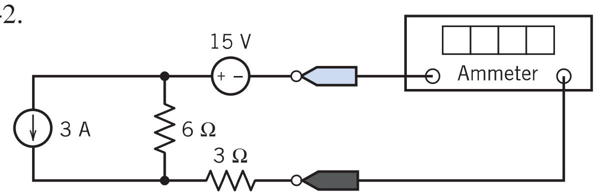

FIGURE E 4.6-2

Hint: Write and solve a single mesh equation.

Answer: −3.67 A

4.7 Mesh Current Analysis with Dependent Sources

When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the mesh currents.

It is then a simple matter to express the controlled current or voltage as a function of the mesh currents. The mesh equations can then be obtained by applying Kirchhoff's voltage law to the meshes of the circuit.

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EXAMPLE 4.7-1 Mesh Equations and Dependent Sources

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.7-1*a*. Find the value of the voltage measured by the voltmeter.

Solution

Figure 4.7-1*b* shows the circuit after replacing the voltmeter by an equivalent open circuit and labeling the voltage, v_m, measured by the voltmeter. Figure 4.7-1*c* shows the circuit after numbering the meshes. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

The controlling current of the dependent source, i_a, is the current in a short circuit. This short circuit is common to meshes 1 and 2. The short-circuit current can be expressed in terms of the mesh currents as

$$i_a = i_1 - i_2$$

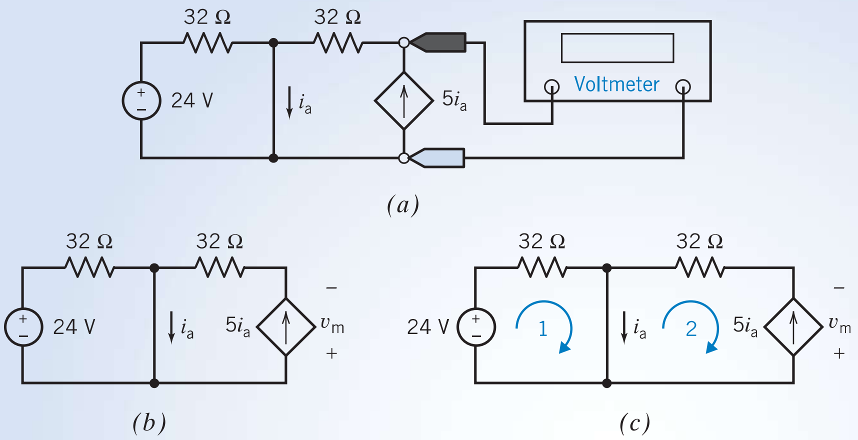

FIGURE 4.7-1 (*a*) The circuit considered in Example 4.7-1. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

The dependent source is in only one mesh, mesh 2. The reference direction of the dependent source current does not agree with the reference direction of i_2. Consequently,

$$5i_a = -i_2$$

Solving for i_2 gives $$i_2 = -5i_a = -5(i_1 - i_2)$$

Therefore, $$-4i_2 = -5i_1 \quad \Rightarrow \quad i_2 = \frac{5}{4} i_1$$

Apply KVL to mesh 1 to get $$32i_1 - 24 = 0 \quad \Rightarrow \quad i_1 = \frac{3}{4}\text{A}$$

Consequently, the value of i_2 is $$i_2 = \frac{5}{4}\left(\frac{3}{4}\right) = \frac{15}{16}\text{A}$$

Apply KVL to mesh 2 to get $$32i_2 - v_m = 0 \quad \Rightarrow \quad v_m = 32i_2$$

Finally, $$v_m = 32\left(\frac{15}{16}\right) = 30\text{ V}$$

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EXAMPLE 4.7-2 Mesh Equations and Dependent Sources

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.7-2*a*. Find the value of the gain A of the CCVS.

Solution

Figure 4 7-2*b* shows the circuit after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter. Figure 4.7-2*c* shows the circuit after numbering the meshes. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

The voltage across the dependent source is represented in two ways. It is Ai_a with the + of reference direction at the bottom and −7.2 V with the + at the top. Consequently,

$$Ai_a = -(-7.2) = 7.2\text{ V}$$

The controlling current of the dependent source, i_a, is the current in a short circuit. This short circuit is common to meshes 1 and 2. The short-circuit current can be expressed in terms of the mesh currents as

$$i_a = i_1 - i_2$$

CHAPTER 5 Circuit Theorems

IN THIS CHAPTER

5.1 Introduction

In this chapter, we consider five circuit theorems:

- A **source transformation** allows us to replace a voltage source and series resistor by a current source and parallel resistor. Doing so does not change the element current or voltage of any other element of the circuit.
- **Superposition** says that the response of a linear circuit to several inputs working together is equal to the sum of the responses to each of the inputs working separately.
- **Thévenin's theorem** allows us to replace part of a circuit by a voltage source and series resistor. Doing so does not change the element current or voltage of any element in the rest of the circuit.
- **Norton's theorem** allows us to replace part of a circuit by a current source and parallel resistor. Doing so does not change the element current or voltage of any element in the rest of the circuit.
- The **maximum power transfer theorem** describes the condition under which one circuit transfers as much power as possible to another circuit.

Each of these circuit theorems can be thought of as a shortcut, a way to reduce the complexity of an electric circuit so that it can be analyzed more easily. More important, these theorems provide insight into the nature of linear electric circuits.

5.2 Source Transformations

The ideal voltage source is the simplest model of a voltage source, but occasionally we need a more accurate model. Figure 5.2-1*a* shows a more accurate but more complicated model of a voltage source. The circuit shown in Figure 5.2-1 is sometimes called a nonideal voltage source. (The voltage of a practical voltage source decreases as the voltage source supplies more power. The nonideal voltage source models this behavior, whereas the ideal voltage source does not. The nonideal voltage source is a more accurate model of a practical voltage source than the ideal voltage source, but it is also more complicated. We will usually use ideal voltage sources to model practical voltage sources but will occasionally need to use a nonideal voltage source.) Figure 5.2-1*b* shows a nonideal current source. It is a more accurate but more complicated model of a practical current source.

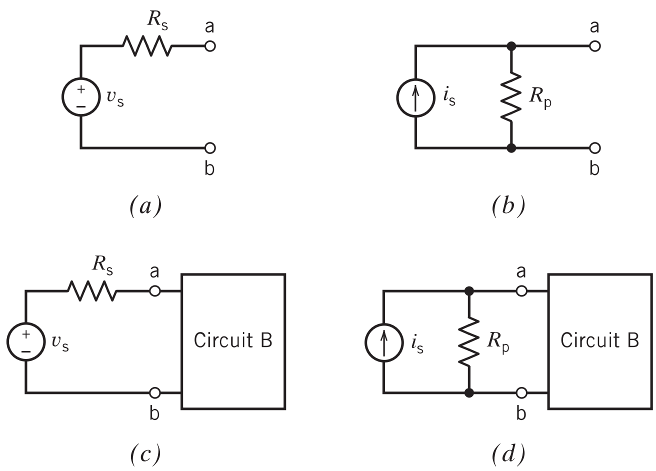


FIGURE 5.2-1 (*a*) A nonideal voltage source. (*b*) A nonideal current source. (*c*) Circuit B connected to the nonideal voltage source. (*d*) Circuit B connected to the nonideal current source.

Under certain conditions ($R_p = R_s$ and $v_s = R_s i_s$), the nonideal voltage source and the nonideal current source are equivalent to each other. Figure 5.2-1 illustrates the meaning of "equivalent." In Figure 5.2-1*c*, a nonideal voltage source is connected to circuit B. In Figure 5.2-1*d*, a nonideal current source is connected to that same circuit B. Perhaps Figure 5.2-1*d* was obtained from Figure 5.2-1*c*, by replacing the nonideal voltage source with a nonideal current source. Replacing the nonideal voltage source by the *equivalent* nonideal current source does not change the voltage or current of any element in circuit B. That means that if you looked at a list of the values of the currents and voltages of all the circuit elements in circuit B, you could not tell whether circuit B was connected to a nonideal voltage source or to an equivalent nonideal current source. Similarly, we can imagine that Figure 5.2-1*c* was obtained from Figure 5.2-1*d* by replacing the nonideal current source with a nonideal voltage source. Replacing the nonideal current source by the *equivalent* nonideal voltage source does not change the voltage or current of any element in circuit B. The process of transforming Figure 5.2-1*c* into Figure 5.2-1*d*, or vice versa, is called a source transformation.

To see why the source transformation works, we will perform an experiment using the test circuit shown in Figure 5.2-2. This test circuit contains a device called an "operational amplifier." We will learn about operational amplifiers in Chapter 6, so we aren't ready to analyze this circuit yet. Instead, imagine building the circuit and making some measurements to learn how it works.

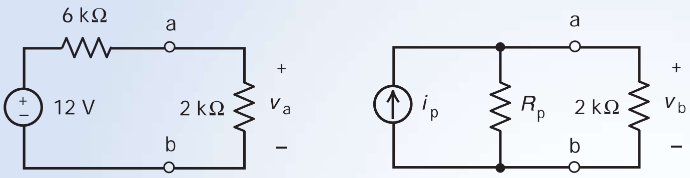
Consider the following experiment. We connect a resistor having resistance R to the terminals of the test circuit as shown in Figure 5.2-2 and measure the resistor voltage v and resistor current i. Next, we change the resistor and measure the new values of the resistor voltage and current. After some trial and error, we collect the following data:

R, kΩ	0	1	2	5	10	20	50	∞
i, mA	3	2.667	2.4	1.846	1.33	0.857	0.414	0
v, V	0	2.667	4.8	9.231	13.33	17.143	20.69	24

Two of these data points deserve special attention. The resistor acts like an open circuit when $R = \infty$ so we connect an open circuit across the terminals of the test circuit in this case. As expected, $i = 0$. The resistor voltage is referred to as the "open circuit voltage," denoted as v_{oc}. We have measured $v_{oc} = 24$ V. The resistor acts like a short circuit when $R = 0$, so we connect a short circuit across the terminals of the test circuit. As expected, $v = 0$. The resistor current is referred to as the "short-circuit current," denoted as i_{sc}. We have measured $i_{sc} = 3$ mA.

EXAMPLE 5.2-1 Source Transformations

First, determine the values of i_p and R_p that cause the part of the circuit connected to the 2-kΩ resistor in Figure 5.2-7*b* to be equivalent to part of the circuit connected to the 2-kΩ resistor in Figure 5.2-7*a*. Next, determine the values of v_a and v_b.



(*a*) (*b*)

FIGURE 5.2-7 The circuit considered in Example 5.2-1.

Solution

We can use a source transformation to determine the required values of i_p and R_p. Referring to Figure 5.2-6 we get

$$i_p = \frac{12}{6000} = 0.002 \text{ A} = 2 \text{ mA and } R_p = 6 \text{ k}\Omega$$

Using voltage division in Figure 5.2-7*a*, we calculate

$$v_a = \frac{2000}{2000 + 6000}(12) = 3 \text{ V}$$

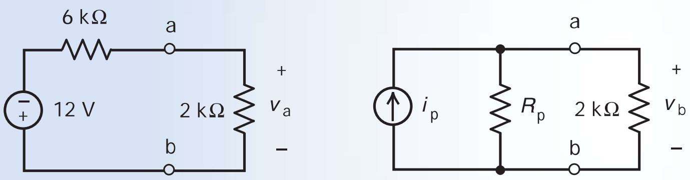
The voltage across the parallel resistors in Figure 5.2-7*b* is given by

$$v_b = \frac{2000 R_p}{2000 + R_p} i_p = \frac{2000(6000)}{2000 + 6000}(0.002) = 1500(0.002) = 3 \text{ V}$$

As expected, the source transformation did not change the value of the voltage across the 2-kΩ resistor.

EXAMPLE 5.2-2 Source Transformations

First, determine the values of i_p and R_p that cause the part of the circuit connected to the 2-kΩ resistor in Figure 5.2-8*b* to be equivalent to part of the circuit connected to the 2-kΩ resistor in Figure 5.2-8*a*. Next, determine the values of v_a and v_b.



(*a*) (*b*)

FIGURE 5.2-8 The circuit considered in Example 5.2-2.

Solution

This example is very similar to the previous example. The difference between these examples is the polarity of the voltage source in part (a) of the figures. Reversing both the polarity of voltage source and the sign of the source voltage produces an equivalent circuit. Consequently, we can redraw Figure 5.2-8 as shown in Figure 5.2-9.

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EXERCISE 5.2-4 Determine values of R and v_s so that the circuits shown in Figures E 5.2-4a,b are equivalent to each other due to a source transformation.

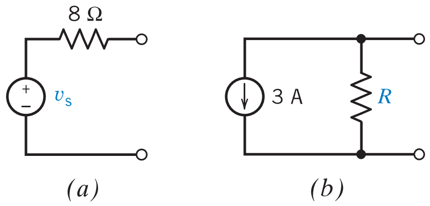

FIGURE E 5.2-4

Hint: Notice that the reference direction of the current source in Figure E 5.2-4b is not the same as in Figure E 5.2-3b.

Answer: $R = 8\ \Omega$ and $v_s = -24$ V

5.3 Superposition

The output of a linear circuit can be expressed as a linear combination of its inputs. For example, consider any circuit having the following three properties:

1. The circuit consists entirely of resistors and dependent and independent sources.
2. The circuit inputs are the voltages of all the independent voltage sources and the currents of all the independent current sources.
3. The output is the voltage or current of any element of the circuit.

Such a circuit is a linear circuit. Consequently, the circuit output can be expressed as a linear combination of the circuit inputs. For example,

$$v_o = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n \tag{5.3-1}$$

where v_0 is the output of the circuit (it could be a current instead of a voltage) and $v_1, v_2, \ldots, v_n$ are the inputs to the circuit (any or all the inputs could be currents instead of voltages). The coefficients $a_1, a_2, \ldots, a_n$ of the linear combination are real constants called gains.

Next, consider what would happen if we set all but one input to zero. Let v_{oi} denote output when all inputs except the ith input have been set to zero. For example, suppose we set $v_2, v_3, \ldots, v_n$ to zero. Then

$$v_{o1} = a_1 v_1 \tag{5.3-2}$$

We can interpret $v_{o1} = a_1 v_1$ as the circuit output due to input $v_1$acting separately. In contrast, the v_o in Eq 5.3-1 is the circuit output due to all the inputs working together. We now have the following important interpretation of Eq. 5.3-1:

> The output of a linear circuit due to several inputs working together is equal to the sum of the outputs due to each input working separately.

The inputs to our circuit are voltages of independent voltage sources and the currents of independent current sources. When we set all but one input to zero, the other inputs become 0-V voltage sources and 0-A current sources. Because 0-V voltage sources are equivalent to short circuits and 0-A current sources are equivalent to open circuits, we replace the sources corresponding to the other inputs by short or open circuits.

Equation 5.3-2 suggests a method for determining the values of the coefficients $a_1, a_2, \ldots, a_n$ of the linear combination. For example, to determine a_1, set $v_2, v_3, \ldots, v_n$ to zero. Then, dividing both sides of Eq. 5.5-2 by v_1, we get

$$a_1 = \frac{v_{o1}}{v_1}$$

The other gains are determined similarly.

EXAMPLE 5.3-1 Superposition

The circuit shown in Figure 5.3-1 has one output, v_o, and three inputs, v_1, i_2, and v_3. (As expected, the inputs are voltages of independent voltage sources and the currents of independent current sources.) Express the output as a linear combination of the inputs.

Solution

Let's analyze the circuit using node equations. Label the node voltage at the top node of the current source and identify the supernode corresponding to the horizontal voltage source as shown in Figure 5.3-2. Apply KCL to the supernode to get

$$\frac{v_1 - (v_3 + v_o)}{40} + i_2 = \frac{v_o}{10}$$

Multiply both sides of this equation by 40 to eliminate the fractions. Then we have

$$v_1 - (v_3 + v_o) + 40i_2 = 4v_o \quad \Rightarrow \quad v_1 + 40i_2 - v_3 = 5v_o$$

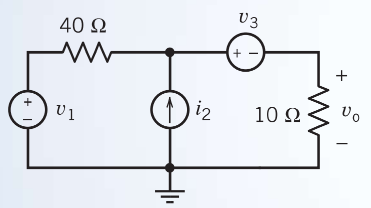

FIGURE 5.3-1 The linear circuit for Example 5.3-1.

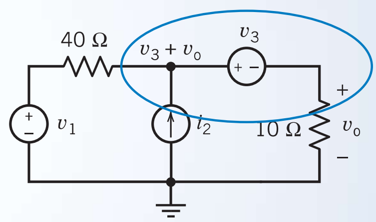

FIGURE 5.3-2 A supernode.

Dividing both sides by 5 expresses the output as a linear combination of the inputs:

$$v_o = \frac{v_1}{5} + 8i_2 - \frac{v_3}{5}$$

Also, the coefficients of the linear combination can now be determined to be

$$a_1 = \frac{v_{o1}}{v_1} = \frac{1}{5}\,\text{V/V}, a_2 = \frac{v_{o2}}{i_2} = 8\text{V/A, and } a_3 = \frac{v_{o3}}{v_3} = -\frac{1}{5}\,\text{V/V}$$

Alternate Solution

Figure 5.3-3 shows the circuit from Figure 5.3-1 when $i_2 = 0$ A and $v_3 = 0$ V. (A zero current source is equivalent to an open circuit, and a zero voltage source is equivalent to a short circuit.)

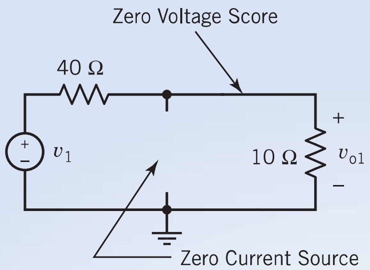

FIGURE 5.3-3 Output due to the first input.

Using voltage division

$$v_{o1} = \frac{10}{40+10} v_1 = \frac{1}{5} v_1$$

In other words,

$$a_1 = \frac{v_{o1}}{v_1} = \frac{1}{5} \text{ V/V}$$

Next, Figure 5.3-4 shows the circuit when $v_1 = 0$ V and $v_3 = 0$ V. The resistors are connected in parallel. Applying Ohm's law to the equivalent resistance gives

$$v_{o2} = \frac{40 \times 10}{40+10} i_2 = 8 i_2$$

In other words,

$$a_2 = \frac{v_{o2}}{i_2} = 8 \text{ V/A}$$

Finally, Figure 5.3-5 shows the circuit when $v_1 = 0$ V and $i_2 = 0$ A. Using voltage division,

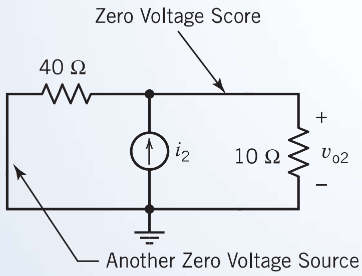

FIGURE 5.3-4 Output due to the second input.

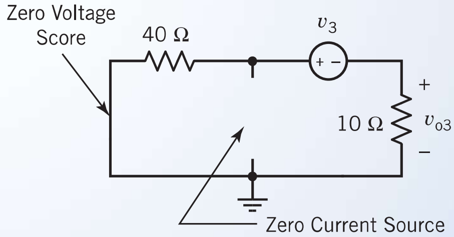

FIGURE 5.3-5 Output due to the third input.

$$v_{o3} = \frac{10}{40+10}(-v_3) = -\frac{1}{5} v_3$$

In other words,

$$a_3 = \frac{v_{o3}}{v_3} = -\frac{1}{5} \text{V/V}$$

Now the output can be expressed as a linear combination of the inputs

$$v_o = a_1 v_1 + a_2 i_2 + a_3 v_3 = \frac{1}{5} v_1 + 8 i_2 + \left(-\frac{1}{5}\right) v_3$$

as before.

Next, apply Kirchhoff's current law at node a to get

$$i_2 + 7 = \frac{v_a - 3i_2}{2} \quad \Rightarrow \quad i_2 + 7 = \frac{-3i_2 - 3i_2}{2} \quad \Rightarrow \quad i_2 = -\frac{7}{4}\ \text{A}$$

Step 3: The current, i, caused by the two independent sources acting together is equal to the sum of the currents, i_1 and i_2, caused by each source acting separately:

$$i = i_1 + i_2 = 3 - \frac{7}{4} = \frac{5}{4}\ \text{A}$$

5.4 Thévenin's Theorem

In this section, we introduce the Thévenin equivalent circuit, based on a theorem developed by M. L. Thévenin, a French engineer, who first published the principle in 1883. Thévenin, who is credited with the theorem, probably based his work on earlier work by Hermann von Helmholtz (see Figure 5.4-1).

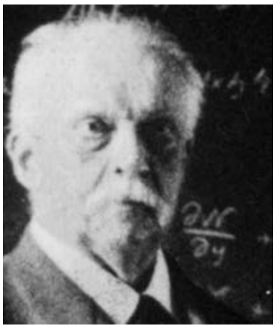

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FIGURE 5.4-1 Hermann von Helmholtz (1821–1894), who is often credited with the basic work leading to Thévenin's theorem.

Figure 5.4-2 illustrates the use of the Thévenin equivalent circuit. In Figure 5.4-2*a*, a circuit is partitioned into two parts—circuit A and circuit B—that are connected at a single pair of terminals. (This is the only connection between circuits A and B. In particular, if the overall circuit contains a dependent source, then either both parts of that dependent source must be in circuit A or both parts must be in circuit B.) In Figure 5.4-2*b*, circuit A is replaced by its Thévenin equivalent circuit, which consists of an ideal voltage source in series with a resistor. Replacing circuit A by its Thévenin equivalent circuit does not change the voltage or current of any element in circuit B. This means that if you looked at a list of the values of the currents and voltages of all the circuit elements in circuit B, you could not tell whether circuit B was connected to circuit A or connected to its Thévenin equivalent circuit.

Finding the Thévenin equivalent circuit of circuit A involves three parameters: the open-circuit voltage, v_{oc}, the short-circuit current, i_{sc}, and the Thévenin resistance, R_t. Figure 5.4-3 illustrates the meaning of these three parameters. In Figure 5.4-3*a*, an open circuit is connected across the terminals of circuit A. The voltage across that open circuit is the open-circuit voltage, v_{oc}. In Figure 5.4-3*b*, a short circuit is connected across the terminals of circuit A. The current in that short circuit is the short-circuit current, i_{sc}.

Figure 5.4-3*c* indicates that the Thévenin resistance, R_t, is the equivalent resistance of circuit A*. Circuit A* is formed from circuit A by replacing all the *independent* voltage sources by short circuits and replacing all the *independent* current sources by open circuits. (*Dependent* current and voltage sources are not replaced with open circuits or short circuits.) Frequently, the Thévenin resistance, R_t, can be determined by repeatedly replacing series or parallel resistors by equivalent resistors. Sometimes, a more formal method is required. Figure 5.4-4 illustrates a formal method for determining the value of the Thévenin resistance. A current source having current i_t is connected across the terminals of circuit A*. The voltage, v_t, across the current source is calculated or measured. The Thévenin

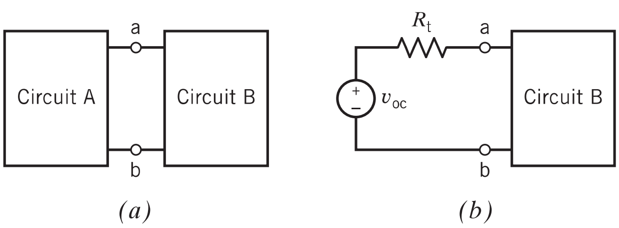

FIGURE 5.4-2 (*a*) A circuit partitioned into two parts: circuit A and circuit B. (*b*) Replacing circuit A by its Thévenin equivalent circuit.

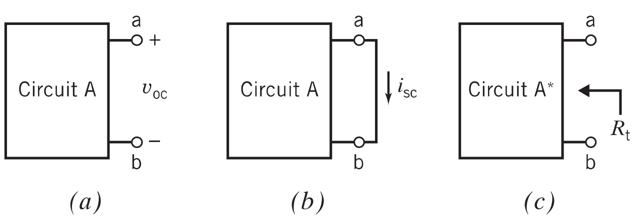

FIGURE 5.4-3 The Thévenin equivalent circuit involves three parameters: (*a*) the open-circuit voltage, v_{oc}, (*b*) the short-circuit current, i_{sc}, and (*c*) the Thévenin resistance, R_t.

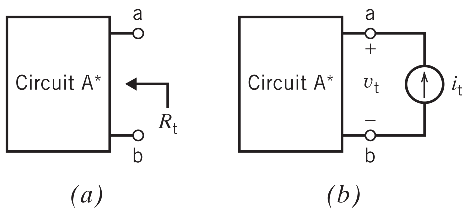

FIGURE 5.4-4 (*a*) The Thévenin resistance, R_t, and (*b*) a method for measuring or calculating the Thévenin resistance, R_t.

resistance is determined from the values of i_t and v_t, using

$$R_t = \frac{v_t}{i_t} \tag{5.4-1}$$

The open-circuit voltage, v_{oc}, the short-circuit current, i_{sc}, and the Thévenin resistance, R_t, are related by the equation

$$v_{oc} = R_t i_{sc} \tag{5.4-2}$$

Consequently, the Thévenin resistance can be calculated from the open-circuit voltage and the short-circuit current.

In summary, the Thévenin equivalent circuit for circuit A consists of an ideal voltage source, having voltage v_{oc}, in series with a resistor, having resistance R_t. Replacing circuit A by its Thévenin equivalent circuit does not change the voltage or current of any element in circuit B.

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Example 5.4-1 Thévenin Equivalent Circuit

Determine the Thévenin equivalent circuit for the circuit shown in Figure 5.4-5.

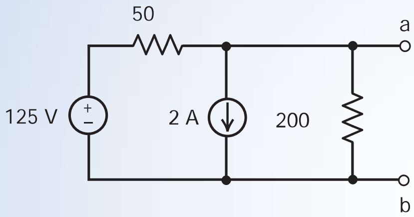

FIGURE 5.4-5 The circuit considered in Example 5.4-1.

First Solution

Referring to Figure 5.4-2, we see that we can draw a Thévenin equivalent circuit once we have found the open-circuit voltage v_{oc} and Thévenin resistance, R_t. Figure 5.4-3 shows how to determine the open-circuit voltage, the Thévenin resistance, and also the short-circuit current i_{sc}. After we determine the values of v_{oc}, R_t, and i_{sc} we will use Eq. 5.4-2 to check that our values are correct.

To determine the open-circuit voltage of the circuit shown in Figure 5.4-5, we connect an open circuit between terminals a and b as shown in Figure 5.4-6*a*. As the name suggests, the voltage across that open circuit is the open-circuit voltage, v_{oc}. After taking node b in Figure 5.4-6*a* to be the reference node, we see that the node voltage at node a is equal to v_{oc}. Applying KCL at node a, we obtain the node equation

Second Solution

Often we can simplify a circuit using source transformations and equivalent circuits. In this solution we will transform a circuit into an equivalent circuit repeatedly. We will start at the left side of the circuit in Figure 5.4-5, away from terminals a-b. If it is possible to continue these transformations until the equivalent circuit consists of the series connection of a voltage source and a resistor, connected between terminals a-b, then that series circuit is the Thévenin equivalent circuit. Figure 5.4-7 illustrates this procedure.

The circuit in Figure 5.4-6 contains a voltage source connected in series with a 50-Ω resistor. Using a source transformation, these circuit elements are replaced by the parallel connection of a 2.5-A current source and 50-Ω resistor in Figure 5.4-7*a*. The circuit in Figure 5.4-7*a* contains both parallel current sources and parallel resistors. In Figure 5.4-7*b* the parallel current sources are replaced by an equivalent current source and the parallel resistors are replaced by an equivalent resistor. A final source transformation converts the parallel connection of a current source and resistor in Figure 5.4-7*b* to the series connection of a voltage source and resistor in Figure 5.4-7*c*. We recognize Figure 5.4-7*c* as a Thévenin circuit that is equivalent to the circuit shown in Figure 5.4-5 and conclude that Figure 5.4-7*c* is the Thévenin equivalent of the circuit shown in Figure 5.4-5.

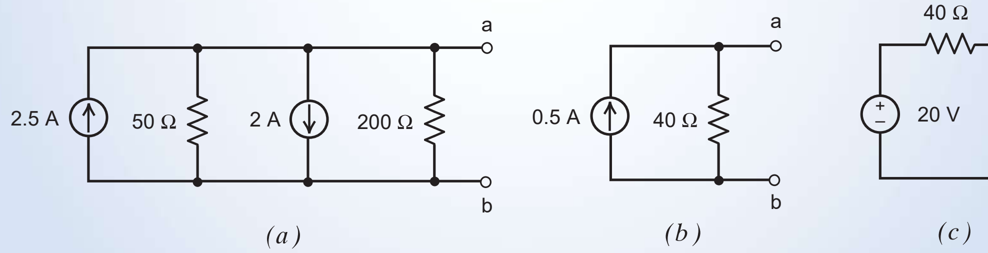

FIGURE 5.4-7 Using source transformations and equivalent circuits to determine the Thévenin equivalent circuit of the circuit shown in Figure 5.4-5.

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EXAMPLE 5.4-2 Thévenin Equivalent Circuit of a Circuit Containing a Dependent Source

Determine the Thévenin equivalent circuit for the circuit shown in Figure 5.4-8.

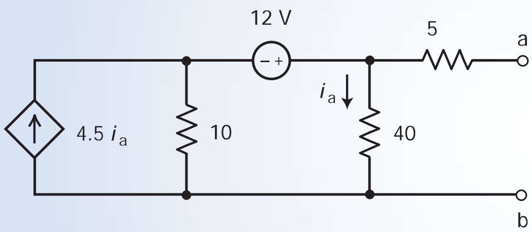

FIGURE 5.4-8 The circuit considered in Example 5.4-2.

Solution

We will determine the values of v_{oc}, R_t, and i_{sc} and use Eq. 5.4-2 to check that our values are correct.

To determine the open-circuit voltage of the circuit shown in Figure 5.4-8, we connect an open circuit between terminals a and b and label the voltage across that open circuit as v_{oc}. Figure 5.4-9 shows the resulting circuit after using KCL to label the element currents.

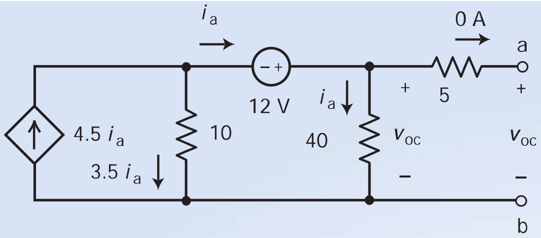

FIGURE 5.4-9 The circuit used to find the open-circuit voltage.

The open circuit causes the current in the 5-Ω resistor to be zero. The voltage across that resistor is also zero, so the voltage across the 40-Ω resistor is v_{oc} as labeled in Figure 5.4-9.

Using Ohm's law
$$i_a = \frac{v_{oc}}{40}$$

Applying KVL to the loop consisting the 12-V source, 10-Ω resistor, and 40-Ω resistor gives

$$0 = -12 + v_{oc} - 10(3.5 i_a)$$

Solving these equations for v_{oc} gives
$$v_{oc} = 96 \text{ V}$$

To determine the short-circuit current of the circuit shown in Figure 5.4-8, we connect a short circuit between terminals a and b and label the current across that short circuit as i_{sc}. Figure 5.4-10 shows the resulting circuit after using KCL to label the element currents.

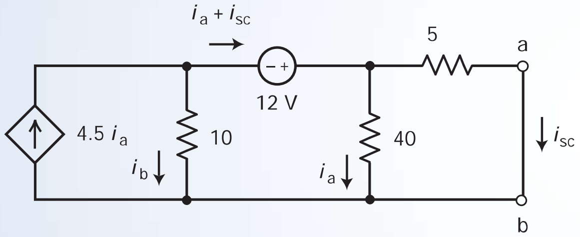

FIGURE 5.4-10 The circuit used to find the short-circuit current.

Applying KVL to the loop consisting of the 5-Ω and 40-Ω resistors gives

$$5 i_{sc} - 40 i_a = 0 \quad \Rightarrow \quad i_a = \frac{i_{sc}}{8}$$

Apply KCL at the top node of the 10-Ω resistor to write

$$4.5 i_a = i_b + (i_a + i_{sc}) \quad \Rightarrow \quad i_b = 3.5 i_a - i_{sc} = -\frac{9}{16} i_{sc}$$

Apply KVL to the loop consisting of the voltage source and the 5-Ω and 10-Ω resistors to write

$$-12 + 5 i_{sc} - 10\left(-\frac{9}{16} i_{sc}\right) = 0$$

Solving this equation for i_{sc} gives
$$i_{sc} = \frac{12}{5 + \frac{90}{16}} = 1.1294 \text{ A}$$

Referring to Figure 5.4-4, we'll determine the Thévenin resistance of the circuit by replacing the independent voltage source by a short circuit and connecting a current source to terminal a-b as shown in Figure 5.4-11. (Circuit A* in Figure 5.4-4 is obtained from Circuit A by replacing the independent voltage sources by short circuits and the independent current sources by open circuits.)

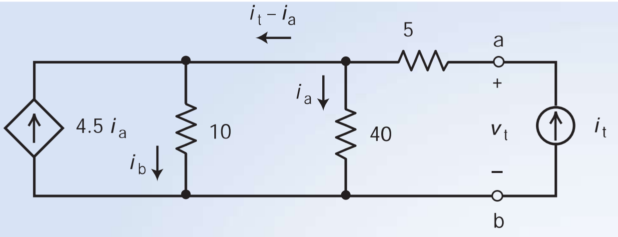

FIGURE 5.4-11 The circuit used to find the Thévenin resistance.

Apply KCL at the top node of the 10-Ω resistor to write

$$4.5\,i_a + (i_t - i_a) = i_b \quad \Rightarrow \quad i_b = 3.5\,i_a + i_t$$

Applying KVL to the loop consisting of the 10-Ω and 40-Ω resistors gives

$$40\,i_a = 10\,i_b = 10\,(3.5\,i_a + i_t) \quad \Rightarrow \quad i_a = 2\,i_t$$

Applying KVL to the loop consisting of the independent current source and the 10-Ω and 5-Ω resistors gives

$$v_t = 5\,i_t + 10\,i_b = 5\,i_t + 10(3.5\,i_a + i_t) = 15\,i_t + 35\,i_a = 15\,i_t + 35\,(2\,i_t) = 85\,i_t$$

The Thévenin resistance is $$R_t = \frac{v_t}{i_t} = 85\ \Omega$$

Our values of v_{oc}, R_t, and i_{sc} satisfy Eq. 5.4-2, so we're confident that they are correct. Finally, the Thévenin equivalent circuit is shown in Figure 5.4-12.

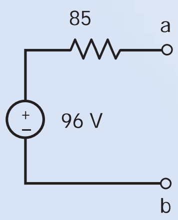

FIGURE 5.4-12 The Thévenin equivalent circuit for the circuit shown in Figure 5.4-8.

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EXAMPLE 5.4-3 An Application of the Thévenin Equivalent Circuit

Consider the circuit shown in Figure 5.4-13.

(a) Determine the current, i, when $R = 2\ \Omega$.
(b) Determine the value of the resistance R required to cause $i = 5$ A.
(c) Determine the value of the resistance R required to cause $i = 8$ A.

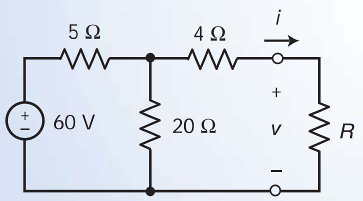

FIGURE 5.4-13 The circuit considered in Example 5.4-3.

5.6 Maximum Power Transfer

Many applications of circuits require the maximum power available from a source to be transferred to a load resistor R_L. Consider the circuit A shown in Figure 5.6-1, terminated with a load R_L. As demonstrated in Section 5.4, circuit A can be reduced to its Thévenin equivalent, as shown in Figure 5.6-2.

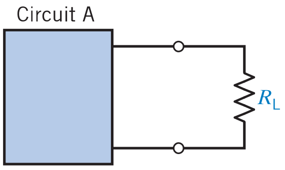

FIGURE 5.6-1 Circuit A contains resistors and independent and dependent sources. The load is the resistor R_L.

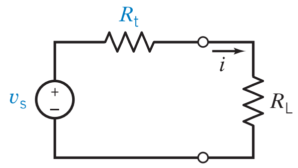

FIGURE 5.6-2 The Thévenin equivalent is substituted for circuit A. Here we use v_s for the Thévenin source voltage.

The general problem of power transfer can be discussed in terms of efficiency and effectiveness. Power utility systems are designed to transport the power to the load with the greatest efficiency by reducing the losses on the power lines. Thus, the effort is concentrated on reducing R_t, which would represent the resistance of the source plus the line resistance. Clearly, the idea of using superconducting lines that would exhibit no line resistance is exciting to power engineers.

In the case of signal transmission, as in the electronics and communications industries, the problem is to attain the maximum signal strength at the load. Consider the signal received at the antenna of an FM radio receiver from a distant station. It is the engineer's goal to design a receiver circuit so that the maximum power ultimately ends up at the output of the amplifier circuit connected to the antenna of your FM radio. Thus, we may represent the FM antenna and amplifier by the Thévenin equivalent circuit shown in Figure 5.6-2.

Let us consider the general circuit of Figure 5.6-2. We wish to find the value of the load resistance R_L such that maximum power is delivered to it. First, we need to find the power from

$$p = i^2 R_L$$

Because the current i is

$$i = \frac{v_s}{R_L + R_t}$$

we find that the power is

$$p = \left(\frac{v_s}{R_L + R_t}\right)^2 R_L \tag{5.6-1}$$

Assuming that v_s and R_t are fixed for a given source, the maximum power is a function of R_L. To find the value of R_L that maximizes the power, we use the differential calculus to find where the derivative dp/dR_L equals zero. Taking the derivative, we obtain

$$\frac{dp}{dR_L} = {v_s}^2 \frac{(R_t + R_L)^2 - 2(R_t + R_L)R_L}{(R_L + R_t)^4}$$

The derivative is zero when

$$(R_t + R_L)^2 - 2(R_t + R_L)R_L = 0 \tag{5.6-2}$$

or

$$(R_t + R_L)(R_t + R_L - 2R_L) = 0 \tag{5.6-3}$$

Solving Eq. 5.6-3, we obtain

$$R_L = R_t \tag{5.6-4}$$

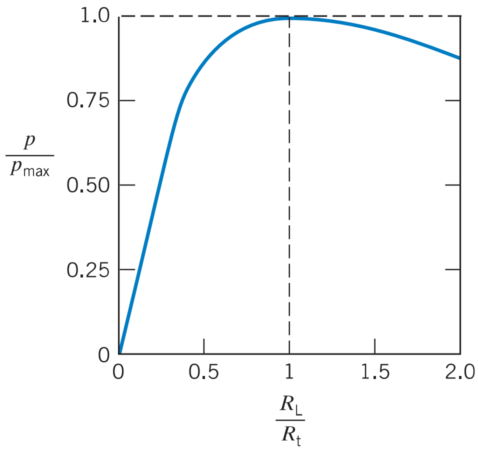

FIGURE 5.6-3 Power actually attained as R_L varies in relation to R_t.

To confirm that Eq. 5.6-4 corresponds to a maximum, it should be shown that $d^2p/dR_L^2 < 0$. Therefore, the maximum power is transferred to the load when R_L is equal to the Thévenin equivalent resistance R_t.

The maximum power, when $R_L = R_t$, is then obtained by substituting $R_L = R_t$ in Eq. 5.6-1 to yield

$$p_{max} = \frac{v_s^2 R_t}{(2R_t)^2} = \frac{v_s^2}{4R_t}$$

The power delivered to the load will differ from the maximum attainable as the load resistance R_L departs from $R_L = R_t$. The power attained as R_L varies from R_t is portrayed in Figure 5.6-3.

The **maximum power transfer** theorem states that the maximum power delivered to a load by a source is attained when the load resistance, R_L, is equal to the Thévenin resistance, R_t, of the source.

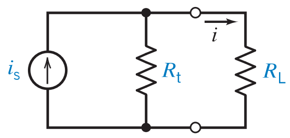

FIGURE 5.6-4 Norton's equivalent circuit representing the source circuit and a load resistor R_L. Here we use i_s as the Norton source current.

We may also use Norton's equivalent circuit to represent circuit A in Figure 5.6.1. We then have a circuit with a load resistor R_L as shown in Figure 5.6-4. The current i may be obtained from the current divider principle to yield

$$i = \frac{R_t}{R_t + R_L} i_s$$

Therefore, the power p is

$$p = i^2 R_L = \frac{i_s^2 R_t^2 R_L}{(R_t + R_L)^2} \tag{5.6-5}$$

Using calculus, we can show that the maximum power occurs when

$$R_L = R_t \tag{5.6-6}$$

Then the maximum power delivered to the load is

$$p_{max} = \frac{R_t i_s^2}{4} \tag{5.6-7}$$

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EXAMPLE 5.6-1 Maximum Power Transfer

Find the load resistance R_L that will result in maximum power delivered to the load for the circuit of Figure 5.6-5. Also, determine the maximum power delivered to the load resistor.

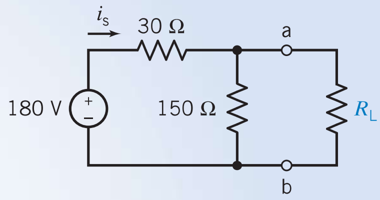

FIGURE 5.6-5 Circuit for Example 5.6-1. Resistances in ohms.

Solution

First, we determine the Thévenin equivalent circuit for the circuit to the left of terminals a–b. Disconnect the load resistor. The Thévenin voltage source v_{oc} is

$$v_{oc} = \frac{150}{180} \times 180 = 150 \text{ V}$$

The Thévenin resistance R_t is

$$R_t = \frac{30 \times 150}{30 + 150} = 25 \ \Omega$$

The Thévenin circuit connected to the load resistor is shown in Figure 5.6-6. Maximum power transfer is obtained when $R_L = R_t = 25 \ \Omega$.

Then the maximum power is

$$p_{max} = \frac{{v_{oc}}^2}{4R_L} = \frac{(150)^2}{4 \times 25} = 225 \text{ W}$$

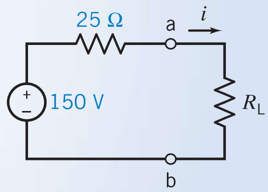

FIGURE 5.6-6 Thévenin equivalent circuit connected to R_L for Example 5.6-1.

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EXAMPLE 5.6-2 Maximum Power Transfer

Find the load R_L that will result in maximum power delivered to the load of the circuit of Figure 5.6-7*a*. Also, determine p_{max} delivered.

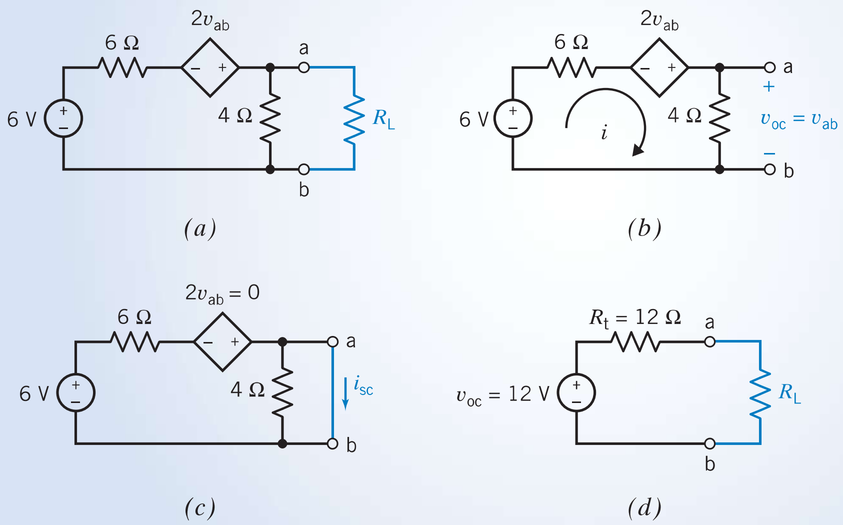

FIGURE 5.6-7 Determination of maximum power transfer to a load R_L.

Solution

We will obtain the Thévenin equivalent circuit for the part of the circuit to the left of terminals a,b in Figure 5.6-7*a*. First, we find v_{oc} as shown in Figure 5.6-7*b*. The KVL gives

$$-6 + 10i - 2v_{ab} = 0$$

Also, we note that $v_{ab} = v_{oc} = 4i$. Therefore,

$$10i - 8i = 6$$

or $i = 3$ A. Therefore, $v_{oc} = 4i = 12$ V.

To determine the short-circuit current, we add a short circuit as shown in Figure 5.6-7*c*. The 4-Ω resistor is short circuited and can be ignored. Writing KVL, we have

$$-6 + 6i_{sc} = 0$$

Hence, $i_{sc} = 1$ A.

Therefore, $R_t = v_{oc}/i_{sc} = 12\ \Omega$. The Thévenin equivalent circuit is shown in Figure 5.6-7*d* with the load resistor. Maximum load power is achieved when $R_L = R_t = 12\ \Omega$. Then,

$$p_{max} = \frac{v_{oc}^2}{4R_L} = \frac{12^2}{4(12)} = 3\ \text{W}$$

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EXERCISE 5.6-1 Find the maximum power that can be delivered to R_L for the circuit of Figure E 5.6-1, using a Thévenin equivalent circuit.

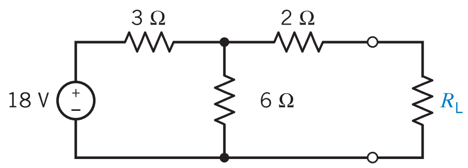

FIGURE E 5.6-1

Answer: 9 W when $R_L = 4\ \Omega$

5.7 Using MATLAB to Determine the Thévenin Equivalent Circuit

MATLAB can be used to reduce the work required to determine the Thévenin equivalent of a circuit such as the one shown in Figure 5.7-1*a*. First, connect a resistor, *R*, across the terminals of the network, as shown in Figure 5.7-1*b*. Next, write node or mesh equations to describe the circuit with the resistor connected across its terminals. In this case, the circuit in Figure 5.7-1*b* is represented by the mesh equations

$$\begin{aligned} 12 &= 28i_1 - 10i_2 - 8i_3 \\ 12 &= -10i_1 + 28i_2 - 8i_3 \\ 0 &= -8i_1 - 8i_2 + (16 + R)i_3 \end{aligned} \tag{5.7-1}$$

CHAPTER 6 The Operational Amplifier

IN THIS CHAPTER

6.1 *Introduction*

This chapter introduces another circuit element, the operational amplifier, or op amp. We will learn how to analyze and design electric circuits that contain op amps. In particular, we will see that:

- Several models, of varying accuracy and complexity, are available for operational amplifiers. Simple models are easy to use. Accurate models are more complicated. The simplest model of the operational amplifier is the ideal operational amplifier.
- Circuits that contain ideal operational amplifiers are analyzed by writing and solving node equations.
- Operational amplifiers can be used to build circuits that perform mathematical operations. Many of these circuits are widely used and have been named. Figure 6.5-1 provides a catalog of some useful operational amplifier circuits.
- Practical operational amplifiers have properties that are not included in the ideal operational amplifier. These include the input offset voltage, bias current, dc gain, input resistance, and output resistance. More complicated models are needed to account for these properties.

6.2 *The Operational Amplifier*

The *operational amplifier* is an electronic circuit element designed to be used with other circuit elements to perform a specified signal-processing operation. The μA741 operational amplifier is shown in Figure 6.2-1*a*. It has eight pin connections, whose functions are indicated in Figure 6.2-1*b*.

The operational amplifier shown in Figure 6.2-2 has five terminals. The names of these terminals are shown in both Figure 6.2-1*b* and Figure 6.2-2. Notice the plus and minus signs in the triangular part of the symbol of the operational amplifier. The plus sign identifies the noninverting input, and the minus sign identifies the inverting input.

The power supplies are used to bias the operational amplifier. In other words, the power supplies cause certain conditions that are required for the operational amplifier to function properly. It is

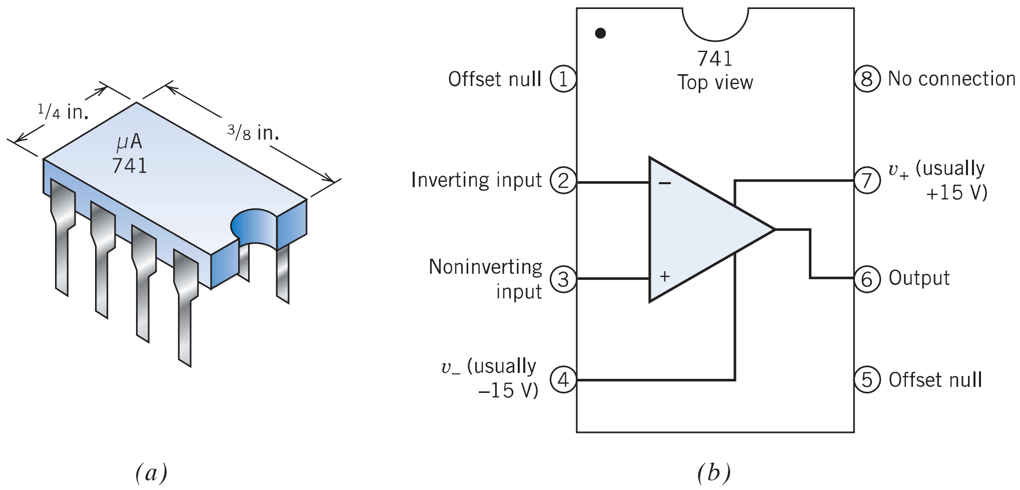

FIGURE 6.2-1 (*a*) A μA741 integrated circuit has eight connecting pins. (*b*) The correspondence between the circled pin numbers of the integrated circuit and the nodes of the operational amplifier.

inconvenient to include the power supplies in drawings of operational amplifier circuits. These power supplies tend to clutter drawings of operational amplifier circuits, making them harder to read. Consequently, the power supplies are frequently omitted from drawings that accompany explanations of the function of operational amplifier circuits, such as the drawings found in textbooks. It is understood that power supplies are part of the circuit even though they are not shown. (Schematics, the drawings used to describe how to assemble a circuit, are a different matter.) The power supply voltages are shown in Figure 6.2-2, denoted as v_+ and v_-.

Because the power supplies are frequently omitted from the drawing of an operational amplifier circuit, it is easy to overlook the power supply currents. This mistake is avoided by careful application of Kirchhoff's current law (KCL). As a general rule, it is not helpful to apply KCL in a way that involves any power supply current. Two specific cases are of particular importance. First, the ground node in Figure 6.2-2 is a terminal of both power supplies. Both power supply currents would be involved if KCL were applied to the ground node. These currents must not be overlooked. It is best simply to refrain from applying KCL at the ground node of an operational amplifier circuit. Second, KCL requires that the sum of all currents into the operational amplifier be zero:

$$i_1 + i_2 + i_o + i_+ + i_- = 0$$

Both power supply currents are involved in this equation. Once again, these currents must not be overlooked. It is best simply to refrain from applying KCL to sum the currents into an operational amplifier when the power supplies are omitted from the circuit diagram.

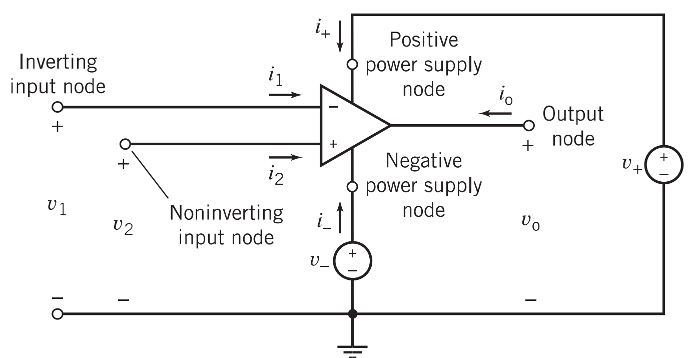

FIGURE 6.2-2 An op amp, including power supplies v_+ and v_-.

6.3 *The Ideal Operational Amplifier*

Operational amplifiers are complicated devices that exhibit both linear and nonlinear behavior. The operational amplifier output voltage and current, v_o and i_o, must satisfy three conditions for an operational amplifier to be linear, that is:

$$|v_o| \leq v_{sat}$$
$$|i_o| \leq i_{sat}$$
$$\left|\frac{dv_o(t)}{dt}\right| \leq SR \tag{6.3-1}$$

The saturation voltage v_{sat}, the saturation current i_{sat}, and the slew rate limit SR are all parameters of an operational amplifier. For example, if a μA741 operational amplifier is biased using +15-V and −15-V power supplies, then

$$v_{sat} = 14\text{ V}, \quad i_{sat} = 2\text{ mA}, \quad \text{and} \quad SR = 500{,}000\ \frac{\text{V}}{\text{s}} \tag{6.3-2}$$

These restrictions reflect the fact that operational amplifiers cannot produce arbitrarily large voltages or arbitrarily large currents or change output voltage arbitrarily quickly.

Figure 6.3-1 describes the *ideal operational amplifier*. The ideal operational amplifier is a simple model of an operational amplifier that is linear. The ideal operational amplifier is characterized by restrictions on its input currents and voltages. The currents into the input terminals of an ideal operational amplifier are zero. Consequently, in Figure 6.3-1,

$$i_1 = 0 \quad \text{and} \quad i_2 = 0$$

The node voltages at the input nodes of an ideal operational amplifier are equal. Consequently, in Figure 6.3-1,

$$v_2 = v_1$$

The ideal operational amplifier is a model of a linear operational amplifier, so the operational amplifier output current and voltage must satisfy the restrictions in Eq. 6.3-1. If they do not, then the ideal operational amplifier is not an appropriate model of the real operational amplifier. The output current and voltage depend on the circuit in which the operational amplifier is used. The ideal op amp conditions are summarized in Table 6.3-1.

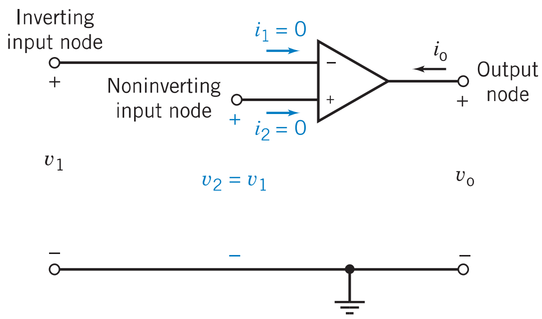

FIGURE 6.3-1 The ideal operational amplifier.

Table 6.3-1 Operating Conditions for an Ideal Operational Amplifier

VARIABLE	IDEAL CONDITION
Inverting node input current	$i_1 = 0$
Noninverting node input current	$i_2 = 0$
Voltage difference between inverting node voltage v_1 and noninverting node voltage v_2	$v_2 - v_1 = 0$

2. The currents in the input leads of an ideal operational amplifier are zero. These currents are involved in the KCL equations at the input nodes of the operational amplifier.
3. The output current of the operational amplifier is not zero. This current is involved in the KCL equations at the output node of the operational amplifier. Applying KCL at this node adds another unknown to the node equations. If the output current of the operational amplifier is not to be determined, then it is not necessary to apply KCL at the output node of the operational amplifier.

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EXAMPLE 6.4-1 Difference Amplifier

The circuit shown in Figure 6.4-1 is called a difference amplifier. The operational amplifier has been modeled as an ideal operational amplifier. Use node equations to analyze this circuit and determine v_o in terms of the two source voltages v_a and v_b.

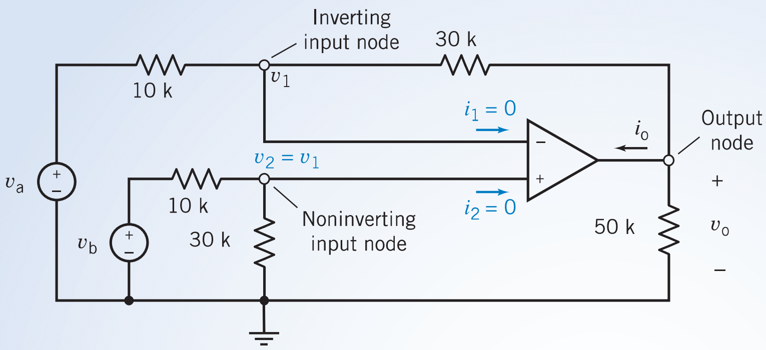


FIGURE 6.4-1 Circuit of Example 6.4-1.

Solution

The node equation at the noninverting node of the ideal operational amplifier is

$$\frac{v_2}{30{,}000} + \frac{v_2 - v_b}{10{,}000} + i_2 = 0$$

Because $v_2 = v_1$ and $i_2 = 0$, this equation becomes

$$\frac{v_1}{30{,}000} + \frac{v_1 - v_b}{10{,}000} = 0$$

Solving for v_1, we have

$$v_1 = 0.75 \cdot v_b$$

The node equation at the inverting node of the ideal operational amplifier is

$$\frac{v_1 - v_a}{10{,}000} + \frac{v_1 - v_o}{30{,}000} + i_1 = 0$$

Because $v_1 = 0.75 v_b$ and $i_1 = 0$, this equation becomes

$$\frac{0.75 \cdot v_b - v_a}{10{,}000} + \frac{0.75 \cdot v_b - v_o}{30{,}000} = 0$$

Solving for v_o, we have

$$v_o = 3(v_b - v_a)$$

The difference amplifier takes its name from the fact that the output voltage v_o is a function of the difference, $v_b - v_a$, of the input voltages.

Apply KCL to node 3 to get

$$\frac{v_2 - v_3}{40{,}000} = \frac{v_3}{10{,}000} + \frac{v_3 - v_4}{8000} \quad \Rightarrow \quad 5v_4 = -v_2 + 10v_3 = 10v_3$$

Combining these equations gives

$$v_4 = 2v_3 = -4v_1$$

Using $v_m = v_4$ and $v_1 = -3.35$ V gives the value of the voltage measured by the voltmeter to be

$$v_m = -4(-3.35) = 13.4 \text{ V}$$

6.5 Design Using Operational Amplifiers

One of the early applications of operational amplifiers was to build circuits that performed mathematical operations. Indeed, the operational amplifier takes its name from this important application. Many of the operational amplifier circuits that perform mathematical operations are used so often that they have been given names. These names are part of an electrical engineer's vocabulary. Figure 6.5-1 shows several standard operational amplifier circuits. The next several examples show how to use Figure 6.5-1 to design simple operational amplifier circuits.

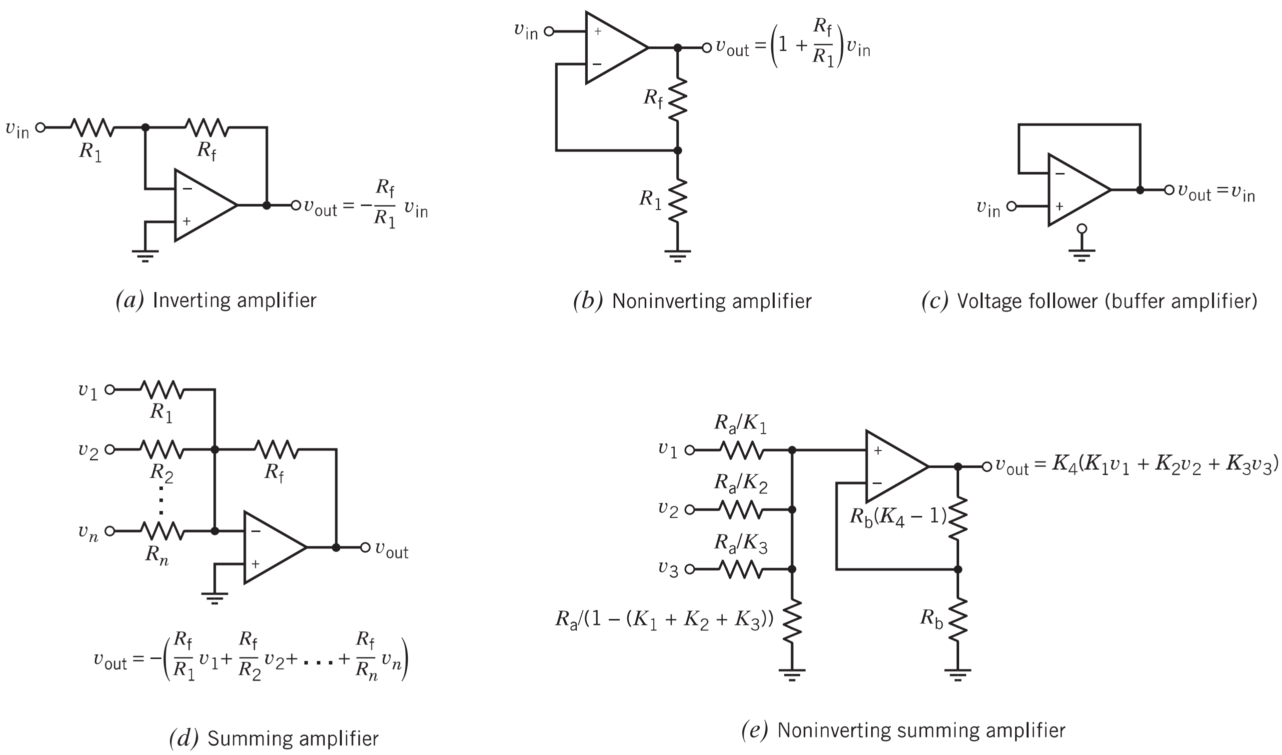
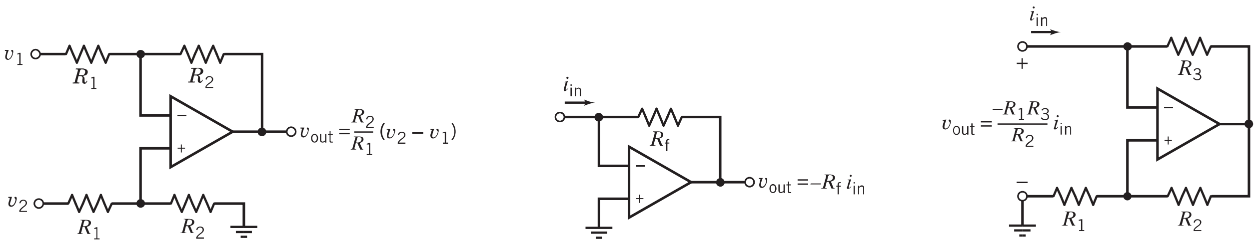
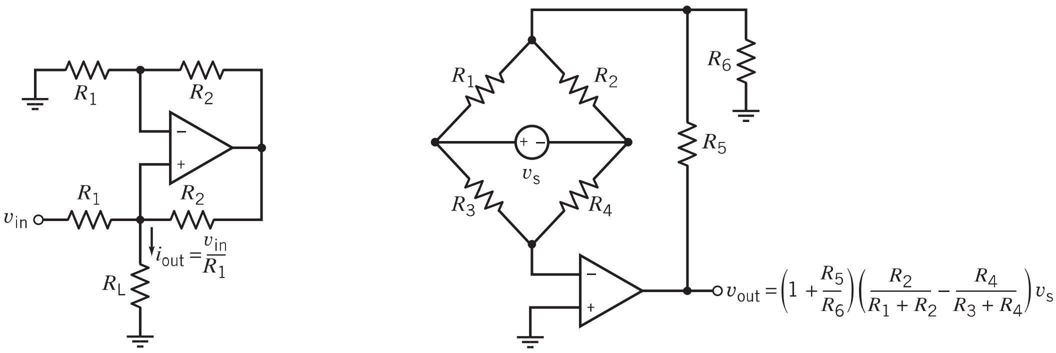


FIGURE 6.5-1 A brief catalog of operational amplifier circuits. Note that all node voltages are referenced to the ground node.



(*f*) Difference amplifier (*g*) Current-to-voltage converter (*h*) Negative resistance convertor



(*i*) Voltage-controlled current source (VCCS) (*j*) Bridge amplifier

FIGURE 6.5-1 (*Continued*)

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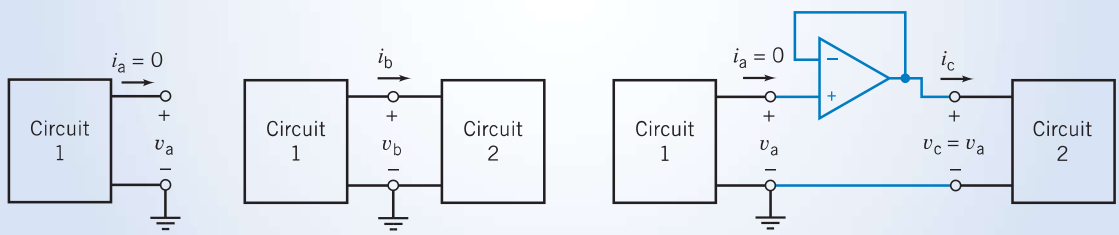
EXAMPLE 6.5-1 Preventing Loading Using a Voltage Follower

This example illustrates the use of a voltage follower to prevent loading. The voltage follower is shown in Figure 6.5-1*c*. Loading can occur when two circuits are connected. Consider Figure 6.5-2. In Figure 6.5-2*a*, the output of circuit 1 is the voltage v_a. In Figure 6.5-2*b*, circuit 2 is connected to circuit 1. The output of circuit 1 is used as the input to circuit 2. Unfortunately, connecting circuit 2 to circuit 1 can change the output of circuit 1. This is called *loading*. Referring again to Figure 6.5-2, circuit 2 is said to load circuit 1 if $v_b \neq v_a$. The current i_b is called the load current. Circuit 1 is required to provide this current in Figure 6.5-2*b* but not in Figure 6.5-2*a*. This is the cause of the loading. The load current can be eliminated using a voltage follower as shown in Figure 6.5-2*c*. The voltage follower copies voltage v_a from the output of circuit 1 to the input of circuit 2 without disturbing circuit 1.



(*a*) (*b*) (*c*)

FIGURE 6.5-2 Circuit 1 (*a*) before and (*b*) after circuit 2 is connected. (*c*) Preventing loading, using a voltage follower.

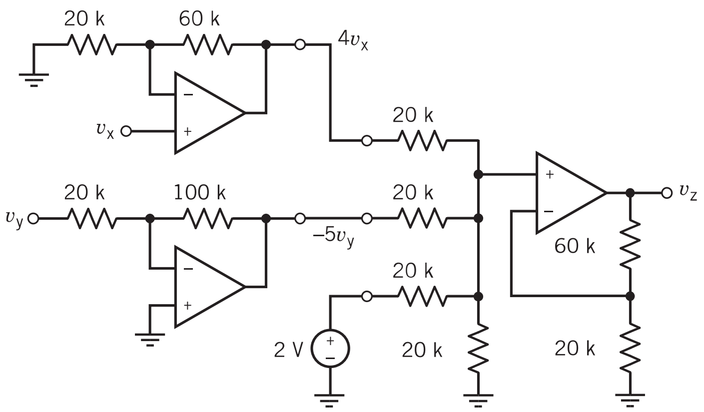


FIGURE 6.6-5 An operational amplifier circuit that implements Eq. 6.6-2.

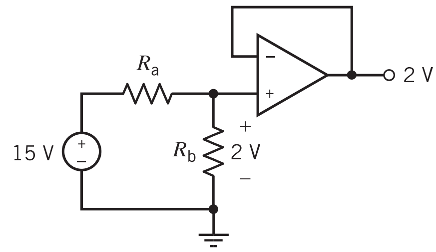


FIGURE 6.6-6 Using the operational amplifier power supply to obtain a 2-V signal.

with a voltage divider and a voltage follower to obtain the 2-V input for the summer. Figure 6.6-6 illustrates the situation. The voltage divider produces a constant voltage equal to 2 V. The voltage follower prevents loading (see Example 6.5-1).

Applying the voltage division rule in Figure 6.6-6 requires that

$$\frac{R_b}{R_a + R_b} = \frac{2}{15} = 0.133 \quad \Rightarrow \quad R_a = 6.5\, R_b$$

The solution to this equation is not unique. One solution is $R_a = 130\ \text{k}\Omega$ and $R_b = 20\ \text{k}\Omega$. Figure 6.6-7 shows the improved operational amplifier circuit. We can verify, perhaps by writing node equations, that

$$v_z = 4v_x - 5v_y + 2$$

Voltage saturation of the operational amplifiers should be considered when defining the relationship between the signals v_x, v_y, and v_z and the variables x, y, and z. The output voltage of an operational

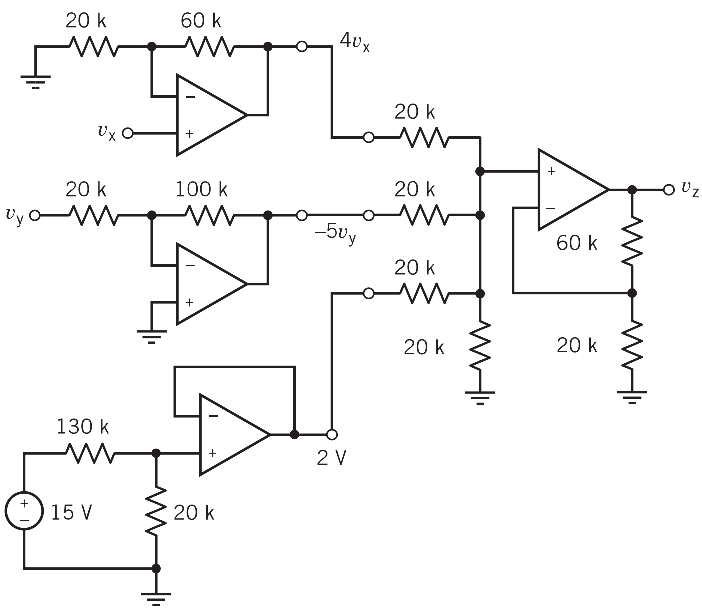


FIGURE 6.6-7 An improved operational amplifier circuit that implements Eq. 6.6-2.

amplifier is restricted by $|v_o| \leq v_{sat}$. Typically, v_{sat} is approximately equal to the magnitude of the voltages of the power supplies used to bias the operational amplifier. That is, v_{sat} is approximately 15 V when ±15-V voltage sources are used to bias the operational amplifier. In Figure 6.6.7, v_z, $4v_x$, and $-5v_y$ are each output voltages of one of the operational amplifiers. Consequently,

$$|v_x| \leq \frac{v_{sat}}{4} \approx \frac{15}{4} = 3.75\text{ V}, \quad |v_y| \leq \frac{v_{sat}}{5} \approx \frac{15}{5} = 3\text{ V}, \quad \text{and} \quad |v_z| \leq v_{sat} \approx 15\text{ V} \tag{6.6-4}$$

The simple encoding of x, y, and z by v_x, v_y, and v_z is

$$v_x = x, \quad v_y = y, \quad \text{and} \quad v_z = z \tag{6.6-5}$$

This is convenient because, for example, $v_z = 4.5$ V indicates that $z = 4.5$. However, using Eq. 6.6-3 to replace v_x, v_y, and v_z in Eq. 6.6-4 with x, y, and z gives

$$|x| \leq 3.75, \quad |y| \leq 3.0, \quad \text{and} \quad |z| \leq 15$$

Should these conditions be too restrictive, consider defining the relationship between the signals v_x, v_y, and v_z and the variables x, y, and z differently. For example, suppose

$$v_x = \frac{x}{10}, \quad v_y = \frac{y}{10}, \quad \text{and} \quad v_z = \frac{z}{10} \tag{6.6-6}$$

Now we need to multiply the value of v_z by 10 to get the value of z. For example, $v_z = 4.5$ V indicates that $z = 45$. On the other hand, the circuit can accommodate larger values of x, y, and z. Equations 6.6-4 and 6.6-6 imply that

$$|x| \leq 37.5, \quad |y| \leq 30.0, \quad \text{and} \quad |z| \leq 150.0$$

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EXERCISE 6.6-1 Specify the values of R_1 and R_2 in Figure E 6.6-1 that are required to cause v_3 to be related to v_1 and v_2 by the equation $v_3 = (4)v_1 - \left(\frac{1}{5}\right)v_2$.

Answer: $R_1 = 10\text{ k}\Omega$ and $R_2 = 2.5\text{ k}\Omega$

EXERCISE 6.6-2 Specify the values of R_1 and R_2 in Figure E 6.6-1 that are required to cause v_3 to be related to v_1 and v_2 by the equation $v_3 = (6)v_1 - \left(\frac{4}{5}\right)v_2$.

Answer: $R_1 = 20\text{ k}\Omega$ and $R_2 = 40\text{ k}\Omega$

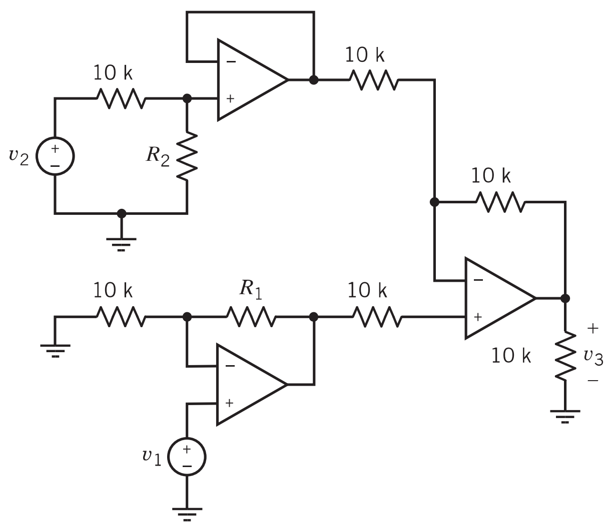

FIGURE E 6.6-1