G12 ECZ MATHEMATICS 2021 PAPER 2 SOLUTIONS

Examinations Council of Zambia — School Certificate Ordinary Level (15 November 2021)

Worked solutions — Section A (answer all) and Section B (any four)

The full question paper appears first — worked answers are at the end.

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EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Mathematics 4024/2

Paper 2

Monday **15 NOVEMBER 2021**

Additional materials:
Answer Booklet
Silent Electronic Calculator (non programmable)
Geometrical instruments
Graph paper (3 sheets)
Plain paper (1 sheet)

Time: 2 hours 30 minutes **Marks: 100**

Instructions to Candidates

1 Write the **centre number** and your **examination number** on **every page** of the separate **Answer Booklet** provided.

2 Write your answers and working in the separate **Answer Booklet provided.**

3 If you use more than one Answer Booklet, fasten the Answer Booklets together.

4 Omission of essential working will result in loss of marks.

5 There are **twelve** questions in this paper.

 (i) Section A
 Answer **all** questions.

 (ii) Section B
 Answer any **four** questions.

6 **Silent non programmable Calculators may be used.**

Information for Candidates

1 The number of marks is given in brackets [ ] at the end of each question or part question.

2 If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

3 Cell phones are **not allowed** in the examination room.

Mathematical Formulae

1 **ALGEBRA**

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 **SERIES**

Geometric Progression

$$S_n = \frac{a(1-r^n)}{1-r}, (r < 1)$$

$$S_n = \frac{a(r^n - 1)}{r-1}, (r > 1)$$

$$S\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 **TRIGONOMETRY**

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A$$

4 **STATISTICS**

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\Sigma x}{n}, \text{ SD} = \sqrt{\left\{\frac{\Sigma(x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\Sigma x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\Sigma fx}{\Sigma f}, \text{ SD} = \sqrt{\left\{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}\right\}} = \sqrt{\left\{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2\right\}}$$

Section A (52 Marks)

Answer all questions in this section

1 **(a)** Simplify $\frac{a-12}{a^2-144}$. [2]

(b) A box contains 3 black and 2 white marbles of the same size. A marble is taken out at random from the box and not replaced. A second marble is then drawn. Calculate the probability that both marbles are

(i) white, [2]

(ii) of different colours. [3]

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2 **(a)** Evaluate $\int_{-1}^{1}\left(1-4x+3x^2\right)dx$. [3]

(b) At a certain secondary school, all the learners in Grade twelve take at least one of the following optional subjects: French (F), Home Economics (HE) and Geography (G). 20 learners take French only, 25 learners take Home Economics only and 22 learners take Geography only. Furthermore, 9 learners take French and Geography, 12 learners take French and Home Economics, 15 learners take Geography and Home Economics and 4 learners take all the three subjects.

(i) Illustrate this information in a Venn diagram. [2]

(ii) How many Grade twelve learners

(a) are at this school, [1]

(b) take Home Economics and Geography but not French, [1]

(c) take two optional subjects only? [1]

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3 **(a)** The second and the fifth terms of a geometric progression are 16 and 2 respectively. Find

(i) the common ratio and first term, [3]

(ii) the 12th term, [2]

(iii) the sum of the first 10 terms. [2]

(b) Express $\frac{5}{1-x}-\frac{3}{x-2}$ as a single fraction in its lowest terms. [3]

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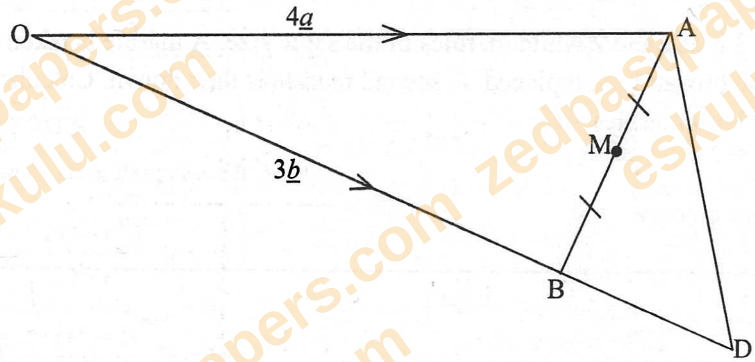
4 **(a)** Solve the equation $2x^2 - 13x + 14 = 0$, giving your answers correct to 2 decimal places. [5]

(b) In the diagram below, $\overrightarrow{OA} = 4\underline{a}$, $\overrightarrow{OB} = 3\underline{b}$, M is the midpoint of AB and OB = 3BD.



Express in terms of $\underline{a}$ and/or $\underline{b}$

(i) $\overrightarrow{AB}$, [1]

(ii) $\overrightarrow{OM}$, [1]

(iii) $\overrightarrow{AD}$, [1]

(iv) $\overrightarrow{MD}$. [2]

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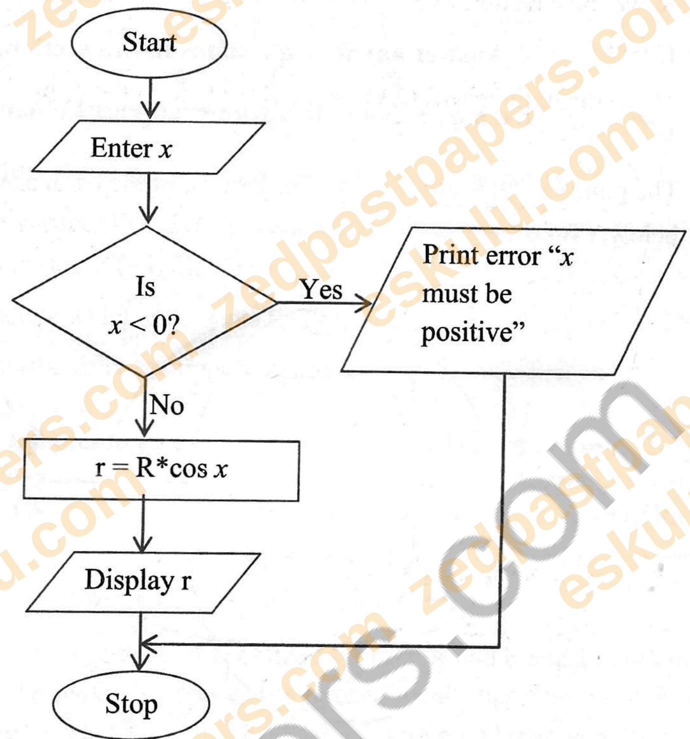
5 **(a)** Given that matrix $P = \begin{pmatrix} 1 & 2 \\ 2 & -x \end{pmatrix}$,

(i) find the value of x for which the determinant of P is -3, [2]

(ii) hence, find the inverse of P. [2]

(b) Study the flowchart below.



Write a pseudocode corresponding to the flowchart programme. [5]

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6 Answer the whole of this question on a sheet of plain paper.

(a) (i) Construct triangle ABC in which AB = 6cm, BC = 9cm and AC = 5cm. [1]

(ii) Measure and write the size of angle ABC. [1]

(b) Within the triangle ABC, draw the locus of points which are

(i) equidistant from A and C, [1]

(ii) 3cm from A, [1]

(iii) equidistant from AC and BC. [2]

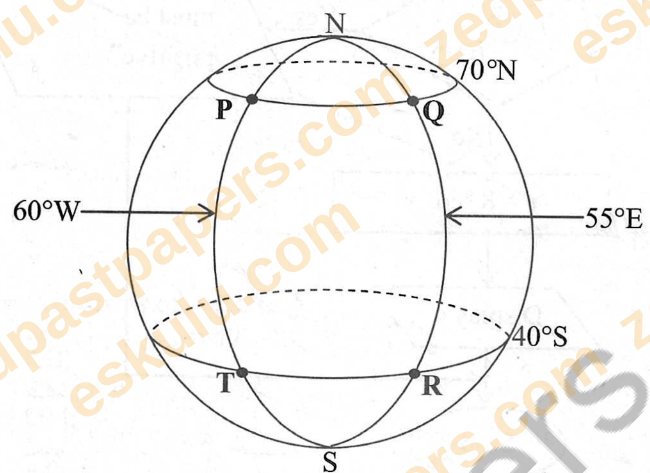
(c) A point P, within triangle ABC, is such that it is less than or equal to 3cm from A, nearer to BC than AC and closer to A than to C. Indicate clearly by shading, the region in which P must lie. [2]

Section B [48 marks]

Answer any four questions in this section.

Each question in this section carries 12 marks.

7 (a) The points P, Q, R and T are on the surface of the earth as shown in the diagram below. (Take $\pi = 3.142$ and $R = 3\ 437$nm)

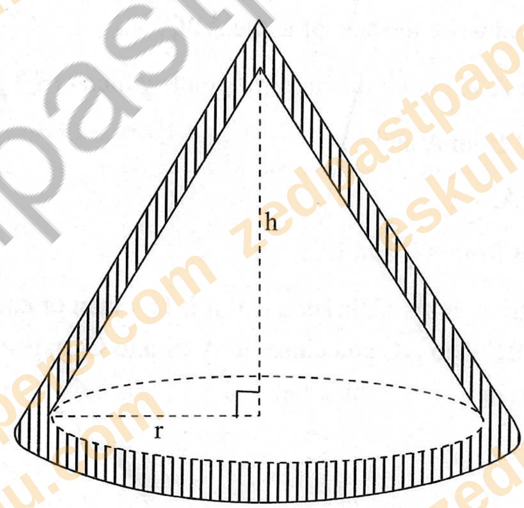


(i) Determine the difference in longitudes between points Q and R. [2]

(ii) Find the length of the circle of latitude 40°S in nautical miles. [2]

(iii) Calculate the distance TR in nautical miles. [2]

(b) The diagram below shows a cone. The shaded part is the thickness of the cone. The internal volume is 34 650cm^3. The internal base area is 3 850cm^2. [Take $\pi = 3.142$]



(i) Calculate the internal height (h) and radius (r). [4]

(ii) Given that the cone is 0.7cm thick, calculate the external volume of the cone. [2]

8 **(a)** A point C is mapped onto a point C′(–2, 3) by a single transformation whose matrix is $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

(i) Find the coordinates of the point C. [2]

(ii) Describe fully the transformation represented by the matrix above. [3]

(b) Triangle PQR with vertices P(2, 2), Q(2, 0) and R(0, 1) is mapped onto triangle P′Q′R′ whose vertices are P′(–4, 2), Q′(–4, 0) and R′(0, 1).

(i) Find the matrix which represents this transformation. [3]

(ii) Hence, describe fully the transformation. [3]

(c) F is a transformation represented by the matrix $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$. The images of A(2, 2) and B(–2, 4) under F are A′(5, 5) and B′(–5, 10) respectively. Find the value of k. [1]

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9 **Answer the whole of this question on a sheet of graph paper.**

A traditional drinks dealer stocks two brands of drinks, brand A and brand B, both of which are produced in bottles of the same size. He wishes to order fresh supplies and finds that he has room for up to 900 bottles. He knows that brand A is more popular and so decides to order at least twice as many bottles of brand A as brand B. He wishes, however, to have at least 100 bottles of brand B and not more than 700 bottles of brand A.

(a) Let x be the number of bottles of brand A and y the number of bottles of brand B. Write four inequalities to represent the information above. [4]

(b) Using a scale of 2cm to represent 100 bottles on each axis, draw x and y axes for $0 \leq x \leq 900$ and $0 \leq y \leq 900$ respectively and shade the unwanted region to indicate clearly the region where the solution of the inequalities lie. [4]

(c) Given that the profit on a bottle of brand A is K3.00 and on a bottle of brand B is K2.00, find

(i) the number of bottles of each brand that gives maximum profit, [2]

(ii) the maximum profit. [2]

10 The table below shows the heights of trees in a plantation.

Height (cm)	$0<x\leq10$	$10<x\leq20$	$20<x\leq30$	$30<x\leq40$	$40<x\leq50$	$50<x\leq60$	$60<x\leq70$
Frequency	30	70	200	200	160	30	10

(a) Calculate the standard deviation. [6]

(b) **Answer this part of the question on a sheet of graph paper.**

(i) Using the table above, copy and complete the relative cumulative frequency table below.

Height (cm)	≤ 0	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Cumulative frequency	0	30	100	300	500	660	690	700
Relative cumulative frequency	0.00	0.04	0.14	0.43				1.00

[1]

(ii) Using a scale of 2cm to represent 10 units on the x-axis for $0 \leq x \leq 70$ and 2cm to represent 0.1 units on the y-axis for $0.00 \leq y \leq 1.00$, draw a smooth relative cumulative frequency curve. [3]

(iii) Showing your method clearly, use your graph to estimate the 80th percentile. [2]

11 **(a)** **Answer this part of the question on a sheet of graph paper.**

The values of x and y are connected by the equation $y = x(x^2 - 4)$. Some corresponding values of x and y are given in the table below.

x	-3	-2	-1	0	1	2	3
y	-15	r	3	0	-3	0	15

(i) Calculate the value of r. [1]

(ii) Using a scale of 2cm to represent 1 unit on the x-axis for $-3 \leq x \leq 3$ and 2cm to represent 10 units on the y-axis for $-20 \leq y \leq 20$, draw the graph of $y = x(x^2 - 4)$. [3]

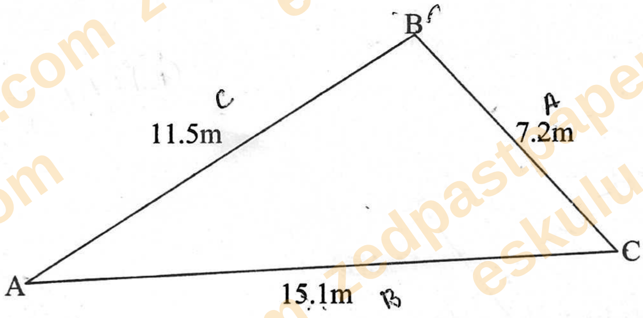
(iii) Use your graph to solve the equations

(a) $x(x^2 - 4) = 0$, [2]

(b) $x(x^2 - 4) = 2$. [3]

(b) Find the equation of the normal to the curve $y = x^2 - x + 4$ at the point $(-1, 6)$. [3]

12 **(a)** The diagram below shows triangle ABC in which AB = 11.5m, BC = 7.2m and AC = 15.1m.



Calculate

(i) angle ABC, [5]

(ii) the area of triangle ABC, [2]

(iii) the shortest distance from B to AC. [2]

(b) Solve the equation $15\tan\theta = 14$ for $180° \leq \theta \leq 270°$. [1]

(c) Simplify $\dfrac{39x^3}{28y^4} \div \dfrac{65x^5}{56y^5}$. [2]

Question 1. (a) Simplify $(a - 12)/(a^2 - 144)$. (b) A box has 3 black and 2 white marbles; a marble is drawn at random and NOT replaced, then a second is drawn. Find the probability that both marbles are (i) white, (ii) of different colours.

(a) Factorise the denominator (difference of two squares)

$a^2 - 144 = (a - 12)(a + 12)$

$(a - 12) / [(a - 12)(a + 12)]$

Answer: $= 1/(a + 12)$

(b)(i) P(both white) — 5 marbles (3 B, 2 W), no replacement

$P(WW) = 2/5 \times 1/4 = 2/20$

Answer: P(both white) = 1/10

(b)(ii) P(different colours) = P(BW) + P(WB)

$= (3/5 \times 2/4) + (2/5 \times 3/4) = 6/20 + 6/20 = 12/20$

Answer: P(different colours) = 3/5

Question 2. (a) Evaluate ∫ from −1 to 1 of $(1 - 4x + 3x^2)$ dx. (b) Grade 12 learners each take at least one of French (F), Home Economics (HE), Geography (G). F only = 20, HE only = 25, G only = 22, F∩G = 9, F∩HE = 12, G∩HE = 15, all three = 4. (i) Venn diagram; (ii) how many (a) are at the school, (b) take HE and G but not French, (c) take exactly two optional subjects.

(a) Integrate term by term

$\int(1 - 4x + 3x^2)\,dx = x - 2x^2 + x^3$

At $x = 1$: $1 - 2 + 1 = 0$

At $x = -1$: $-1 - 2 - 1 = -4$

Value $= 0 - (-4)$

Answer: = 4

(b) Complete the Venn (the pair totals include the centre 4)

F∩G only = 9 − 4 = 5

F∩HE only = 12 − 4 = 8

G∩HE only = 15 − 4 = 11

(b)(ii)(a) Number at the school (sum of all regions)

= 20 + 25 + 22 + 5 + 8 + 11 + 4

Answer: = 95 learners

(b)(ii)(b) HE and G but not French

Answer: G∩HE only = 11 learners

(b)(ii)(c) Exactly two optional subjects

= (F∩G only) + (F∩HE only) + (G∩HE only) = 5 + 8 + 11

Answer: = 24 learners

Question 3. (a) The 2nd and 5th terms of a GP are 16 and 2. Find (i) the common ratio and first term, (ii) the 12th term, (iii) the sum of the first 10 terms. (b) Express 5/(1 – x) – 3/(x – 2) as a single fraction in its lowest terms.

(a)(i) Use ar = 16 and ar■ = 2

ar■ / ar = r^3 = 2/16 = 1/8 ⇒ r = 1/2

a = 16/r = 16 ÷ (1/2) = 32

Answer: r = 1/2, first term a = 32

(a)(ii) 12th term = a r^{11}

T■■ = 32 × $(1/2)^{11}$ = 32/2048

Answer: T■■ = 1/64

(a)(iii) Sum of first 10 terms (r < 1)

S■■ = a(1 – r^1■)/(1 – r) = 32(1 – $(1/2)^1$■)/(1/2)

= 64(1 – 1/1024) = 64 × 1023/1024

Answer: S■■ = 1023/16 ≈ 63.94

(b) 5/(1 – x) – 3/(x – 2)

Common denominator (1 – x)(x – 2):

= [5(x – 2) – 3(1 – x)] / [(1 – x)(x – 2)]

= [5x – 10 – 3 + 3x] / [(1 – x)(x – 2)]

Answer: = (8x – 13) / [(1 – x)(x – 2)]

Question 4. (a) Solve $2x^2$ – 13x + 14 = 0, answers to 2 decimal places. (b) OA■ = 4a, OB■ = 3b, M is the midpoint of AB and OB = 3BD. Express in terms of a and/or b: (i) AB■, (ii) OM■, (iii) AD■, (iv) MD■.

(a) Quadratic formula, a = 2, b = –13, c = 14

x = [13 ± √(169 – 112)] / 4 = [13 ± √57] / 4

√57 = 7.54983

x = (13 + 7.54983)/4 or x = (13 – 7.54983)/4

Answer: x = 5.14 or x = 1.36

(b) Note OB = 3BD with O, B, D in line ⇒ BD■ = b, so OD■ = 3b + b = 4b

(b)(i) AB■

AB■ = OB■ – OA■ = 3b – 4a

Answer: AB■ = 3b – 4a

(b)(ii) OM■ (M midpoint of AB)

OM■ = OA■ + ½AB■ = 4a + ½(3b – 4a) = 4a + 1.5b – 2a

Answer: OM■ = 2a + (3/2)b

(b)(iii) AD■

AD■ = OD■ – OA■ = 4b – 4a

Answer: AD■ = 4b – 4a

(b)(iv) MD■

MD■ = OD■ – OM■ = 4b – (2a + 1.5b)

Answer: MD■ = (5/2)b – 2a

Question 5. (a) Given matrix P = (1 2 ; 2 –x): (i) find x for which det P = –3; (ii) hence find the inverse of P. (b) Write a pseudocode corresponding to the given flowchart (reads x, validates x ≥ 0, computes r = R·cos x).

(a)(i) det P = –3

det P = (1)(–x) – (2)(2) = –x – 4

–x – 4 = –3 ⇒ –x = 1

Answer: x = –1

(a)(ii) Inverse (x = –1 ⇒ P = (1 2 ; 2 1), det = –3)

P■^{1} = (1/det)(d –b ; –c a) = (1/–3)(1 –2 ; –2 1)

Answer: P■^{1} = (–1/3)(1 –2 ; –2 1) = (–1/3 2/3 ; 2/3 –1/3)

(b) Pseudocode

Start

Enter x

If x < 0 THEN

Print "x must be positive"

Else

r = R * cos x

Display r

End If

Answer: Stop

Question 6. Construct triangle ABC with AB = 6 cm, BC = 9 cm, AC = 5 cm. (a)(ii) Measure ∠ABC. (b) Within ABC draw loci: (i) equidistant from A and C, (ii) 3 cm from A, (iii) equidistant from AC and BC. (c) Shade the region for P: ≤ 3 cm from A, nearer BC than AC, and closer to A than to C.

(a) Construction & (ii) ∠ABC (cosine rule, AC opposite B)

cos B = (AB^2 + BC^2 – AC^2)/(2·AB·BC)

= (36 + 81 – 25)/(2 × 6 × 9) = 92/108 = 0.8519

Answer: ∠ABC ≈ 31.6° (accept your measured value ≈ 32°)

(b) Loci

(i) Equidistant from A and C → perpendicular bisector of AC.

(ii) 3 cm from A → arc of a circle, centre A, radius 3 cm.

(iii) Equidistant from AC and BC → bisector of angle C (∠ACB).

(c) Region for P

≤ 3 cm from A → inside the 3 cm arc of A;

nearer BC than AC → the BC-side of the angle-C bisector;

closer to A than C → the A-side of the perpendicular bisector of AC.

Answer: Shade the overlap of these three regions inside triangle ABC.

Question 7. (a) P, Q at 70°N and T, R at 40°S on the earth (π = 3.142, R = 3437 nm); the meridians involved are 60°W and 55°E. (i) Difference in longitude between Q and R; (ii) length of the circle of latitude 40°S in nm; (iii) distance TR in nm. (b) A cone has internal volume 34 650 cm³ and internal base area 3850 cm² (π = 3.142). (i) Find internal height h and radius r; (ii) given the cone is 0.7 cm thick, find the external volume.

(a)(i) Difference in longitude between Q and R

Q and R lie on the meridians 55°E and 60°W (opposite sides of 0°), so add:

= 60° + 55°

Answer: Difference in longitude = 115°

(a)(ii) Length of circle of latitude 40°S

= 2πR cos 40° = 2 × 3.142 × 3437 × cos 40°

= 21598 × 0.76604

Answer: ≈ 16 545 nm

(a)(iii) Distance TR along latitude 40°S (longitude difference 115°)

TR = (115/360) × 2πR cos 40° = (115/360) × 16545

Answer: TR ≈ 5285 nm

(b)(i) Internal radius and height

Base area πr² = 3850 ⇒ r² = 3850/3.142 = 1225 ⇒ r = 35 cm

Volume = ■ × (base area) × h = ■ × 3850 × h = 34650

h = (34650 × 3)/3850 = 27 cm

Answer: r = 35 cm, h = 27 cm

(b)(ii) External volume (add 0.7 cm thickness)

External radius = 35 + 0.7 = 35.7 cm; external height = 27 + 0.7 = 27.7 cm

V = ■$\pi(35.7^2)(27.7)$ = ■ × 3.142 × 1274.49 × 27.7

Answer: External volume ≈ 36 970 cm³

Question 8. (a) Point C maps to C′(–2, 3) under matrix (1 –2 ; 0 1). (i) Find C; (ii) describe the transformation fully. (b) Triangle P(2,2)Q(2,0)R(0,1) maps to P′(–4,2)Q′(–4,0)R′(0,1). (i) Find the matrix; (ii) describe the transformation fully. (c) F = (k 0 ; 0 k) maps A(2,2)→A′(5,5) and B(–2,4)→B′(–5,10). Find k.

(a)(i) Find C from (1 –2 ; 0 1)(x ; y) = (–2 ; 3)

Row 2: $y = 3$. Row 1: $x - 2y = -2 \Rightarrow x - 6 = -2 \Rightarrow x = 4$

Answer: C = (4, 3)

(a)(ii) Describe matrix (1 –2 ; 0 1)

$(x, y) \to (x - 2y, y)$: the x-axis ($y = 0$) is unchanged; points move parallel

to the x-axis by –2 per unit of y.

Answer: Shear, x-axis (y = 0) invariant, shear factor –2

(b)(i) Matrix M with M(2;0)=(–4;0), M(0;1)=(0;1)

From Q: $2a = -4 \Rightarrow a = -2$; $2c = 0 \Rightarrow c = 0$

From R: $b = 0$, $d = 1$. Check P: M(2;2) = (–4; 2) ✓

Answer: M = (–2 0 ; 0 1)

(b)(ii) Describe M

$(x, y) \to (-2x, y)$: the y-axis ($x = 0$) is invariant; x-distances scale by –2.

Answer: One-way stretch parallel to the x-axis, factor –2, y-axis (x = 0) invariant

(c) F = (k 0 ; 0 k): A(2,2) → (2k, 2k) = (5, 5)

$2k = 5 \Rightarrow k = 5/2$; check B(–2,4) → (–2k, 4k) = (–5, 10) ✓

Answer: k = 5/2 = 2.5

Question 9. A dealer stocks brand A (x bottles) and brand B (y bottles): room for up to 900 bottles; at least twice as many A as B; at least 100 of B; not more than 700 of A. (a) Write four inequalities; (b) draw and shade the unwanted region; (c) profit K3 per A and K2 per B — find (i) the numbers for maximum profit, (ii) the maximum profit.

(a) Four inequalities

Room for up to 900: $x + y \le 900$

At least twice as many A as B: $x \ge 2y$

At least 100 of B: $y \ge 100$

Not more than 700 of A: $x \le 700$

(b) Graph

Draw $x + y = 900$, $x = 2y$, $y = 100$, $x = 700$; shade OUT the unwanted side of each.

(c)(i) Best combination — test corner points, $P = 3x + 2y$

(200, 100): $P = 600 + 200 = K800$

(700, 100): $P = 2100 + 200 = K2300$

(600, 300): $[x + y = 900 \cap x = 2y]$ $P = 1800 + 600 = K2400$

(700, 200): $[x = 700 \cap x + y = 900]$ $P = 2100 + 400 = K2500$

Answer: Maximum at (700, 200): 700 bottles of A and 200 bottles of B

(c)(ii) Maximum profit

$P = 3(700) + 2(200) = 2100 + 400$

Answer: Maximum profit = K2500

Question 10. Heights of trees: classes $0 < x \le 10$ (f=30), $10 < x \le 20$ (70), $20 < x \le 30$ (200), $30 < x \le 40$ (200), $40 < x \le 50$ (160), $50 < x \le 60$ (30), $60 < x \le 70$ (10). (a) Calculate the standard deviation. (b)(i) Complete the relative cumulative frequency table; (ii) draw the relative cumulative frequency curve; (iii) estimate the 80th percentile.

(a) Standard deviation (mid-values 5,15,…,65; $\Sigma f = 700$)

$\Sigma fx = 150 + 1050 + 5000 + 7000 + 7200 + 1650 + 650 = 22700$

Mean = 22700/700 = 32.43

$\Sigma fx^2 = 750 + 15750 + 125000 + 245000 + 324000 + 90750 + 42250 = 843500$

$SD = \sqrt{(843500/700 - 32.43^2)} = \sqrt{(1205.0 - 1051.6)} = \sqrt{153.4}$

Answer: SD ≈ 12.4 cm

(b)(i) Cumulative and relative cumulative frequency (÷ 700)

CF: ≤0: 0, ≤10: 30, ≤20: 100, ≤30: 300, ≤40: 500, ≤50: 660, ≤60: 690, ≤70: 700

Rel CF: 0.00, 0.04, 0.14, 0.43, 0.71, 0.94, 0.99, 1.00

(b)(iii) 80th percentile (relative cf = 0.80)

Between (40, 0.71) and (50, 0.94):

$x = 40 + ((0.80 - 0.71)/(0.94 - 0.71)) \times 10 \approx 40 + 3.8$

Answer: 80th percentile ≈ 43.8 cm (read from the graph)

Question 11. (a) $y = x(x^2 - 4)$, table for $-3 \le x \le 3$. (i) Calculate r (value at $x = -2$); (ii) draw the graph; (iii) use it to solve (a) $x(x^2 - 4) = 0$ and (b) $x(x^2 - 4) = 2$. (b) Find the equation of the normal to $y = x^2 - x + 4$ at the point (–1, 6).

(a)(i) r = value of y at x = –2

$r = (-2)((-2)^2 - 4) = (-2)(4 - 4) = (-2)(0)$

Answer: r = 0

(a)(ii) Graph

Plot (–3,–15), (–2,0), (–1,3), (0,0), (1,–3), (2,0), (3,15); smooth curve.

(a)(iii)(a) Solve $x(x^2 - 4) = 0$

$x = 0$ or $x^2 = 4 \Rightarrow x = \pm 2$ (where the curve crosses the x-axis)

Answer: $x = -2, 0, 2$

(a)(iii)(b) Solve $x(x^2 - 4) = 2$

Draw the line $y = 2$ and read the intersections ($x^3 - 4x - 2 = 0$):

Answer: $x \approx -1.7$, $x \approx -0.5$, $x \approx 2.2$ (read from the graph)

(b) Normal to $y = x^2 - x + 4$ at (–1, 6)

$dy/dx = 2x - 1$; at $x = -1$: gradient $= -2 - 1 = -3$

Normal gradient $= -1/(-3) = 1/3$

$y - 6 = (1/3)(x + 1)$

Answer: $x - 3y + 19 = 0$ (i.e. $y = (1/3)x + 19/3$)

Question 12. Triangle ABC: AB = 11.5 m, BC = 7.2 m, AC = 15.1 m. (a)(i) Calculate angle ABC, (ii) the area of triangle ABC, (iii) the shortest distance from B to AC. (b) Solve 15 tan θ = 14 for 180° ≤ θ ≤ 270°. (c) Simplify (39x³)/(28y■) ÷ (65x■)/(56y■).

(a)(i) Angle ABC (opposite AC = 15.1) — cosine rule

$\cos B = (AB^2 + BC^2 - AC^2)/(2 \cdot AB \cdot BC)$

$= (132.25 + 51.84 - 228.01)/(2 \times 11.5 \times 7.2) = -43.92/165.6 = -0.2652$

Answer: $\angle ABC \approx 105.4°$

(a)(ii) Area = ½·AB·BC·sin B

$= \frac{1}{2} \times 11.5 \times 7.2 \times \sin 105.4° = \frac{1}{2} \times 82.8 \times 0.9642$

Answer: Area ≈ 39.9 m²

(a)(iii) Shortest distance from B to AC (perpendicular height)

Area $= \frac{1}{2} \times AC \times h \Rightarrow 39.92 = \frac{1}{2} \times 15.1 \times h$

$h = 79.83/15.1$

Answer: Shortest distance ≈ 5.29 m

(b) $15 \tan\theta = 14 \Rightarrow \tan\theta = 14/15 = 0.9333$

Reference angle = 43.0°; tan is positive in the 3rd quadrant (180°–270°)

$\theta = 180° + 43.0°$

Answer: $\theta = 223.0°$

(c) (39x³)/(28y■) ÷ (65x■)/(56y■)

= (39x³)/(28y■) × (56y■)/(65x■)

Numbers: (39/65)(56/28) = (3/5)(2) = 6/5

Letters: x^(3–5) = x■^{2}; y^(5–4) = y

Answer: = 6y/(5x^{2})

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