G12 ECZ MATHEMATICS 2020 INTERNAL PAPER 2 SOLUTIONS

Examinations Council of Zambia — 2020 Internal Examination

Worked solutions — Section A (answer all) and Section B (any four of Q7-Q12)

The full question paper appears first — worked answers are at the end.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Mathematics

Paper 2

4024/2

2020

Additional materials:
- Answer Booklet
- Silent Electronic Calculator (non programmable)
- Geometrical instruments
- Graph paper (3 sheets)
- Plain paper (1 sheet)

Time: 2 hours 30 minutes **Marks: 100**

Instructions to Candidates

1 Write the **centre number** and your **examination number** on **every page** of the separate **Answer Booklet** provided.

2 Write your answers and working in the **Answer Booklet provided.**

3 If you use more than one Answer Booklet, fasten the Answer Booklets together.

4 Omission of essential working will result in loss of marks.

5 There are **twelve (12)** questions in this paper.

(i) Section A
Answer **all** questions.

(ii) Section B
Answer any **four** questions.

6 **Silent non programmable Calculators may be used.**

Information for Candidates

1 The number of marks is given in brackets [ ] at the end of each question or part question.

2 If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

3 **Cell phones are not allowed in the examination room.**

Mathematical Formulae

1 **ALGEBRA**

Quadratic Equation

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 **SERIES**

Geometric Progression

$$Sn = \frac{a(1-r^n)}{1-r}, (r < 1)$$

$$Sn = \frac{a(r^n - 1)}{r - 1}, (r > 1)$$

$$S\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 **TRIGONOMETRY**

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A$$

4 **STATISTICS**

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \textbf{ SD} = \sqrt{\left\{\frac{\sum(x-\bar{x})^2}{n} - (\bar{x})^2\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \textbf{ SD} = \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

Section A (52 Marks)

Answer all questions in this section

1 **(a)** Given that matrix $A = \begin{pmatrix} 5 & 2 \\ 2 & x \end{pmatrix}$, find the

(i) value of x for which A has no inverse, [2]

(ii) inverse of A if $x = 1$. [2]

(b) A bag contains 7 red and 3 white identical balls. Two balls are taken from the bag at random one after the other without replacement.

(i) Draw a tree diagram to show all the possible outcomes. [2]

(ii) Find the probability of taking at least one white ball. [3]

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2 **(a)** Of the 115 students who attended an end of year party, 74 took fanta, 93 took sprite, 87 took coke, 61 took fanta and sprite, 71 took sprite and coke, 60 took fanta and coke and 50 took all the three drinks.

(i) Illustrate this information on a Venn diagram. [2]

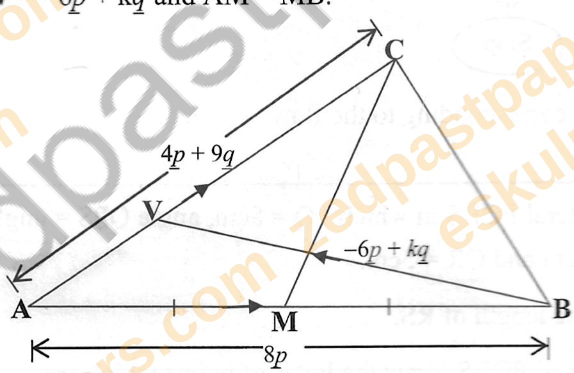
(ii) How many students took

(a) none of the drinks, [1]

(b) fanta and sprite but not coke, [1]

(c) at least two different drinks? [1]

(b) In the diagram below, AVC is a straight line, $\overrightarrow{AB} = 8\underline{p}$, $\overrightarrow{AC} = 4\underline{p} + 9\underline{q}$, $\overrightarrow{BV} = -6\underline{p} + k\underline{q}$ and AM = MB.



(i) Express in terms of $\underline{p}$, $\underline{q}$ and/or k

(a) $\overrightarrow{AM}$, [1]

(b) $\overrightarrow{AV}$. [1]

(ii) Given that $\overrightarrow{AV} = h\overrightarrow{AC}$, by forming an equation involving $\underline{p}$, $\underline{q}$, h and k or otherwise, find the numerical values of h and k. [3]

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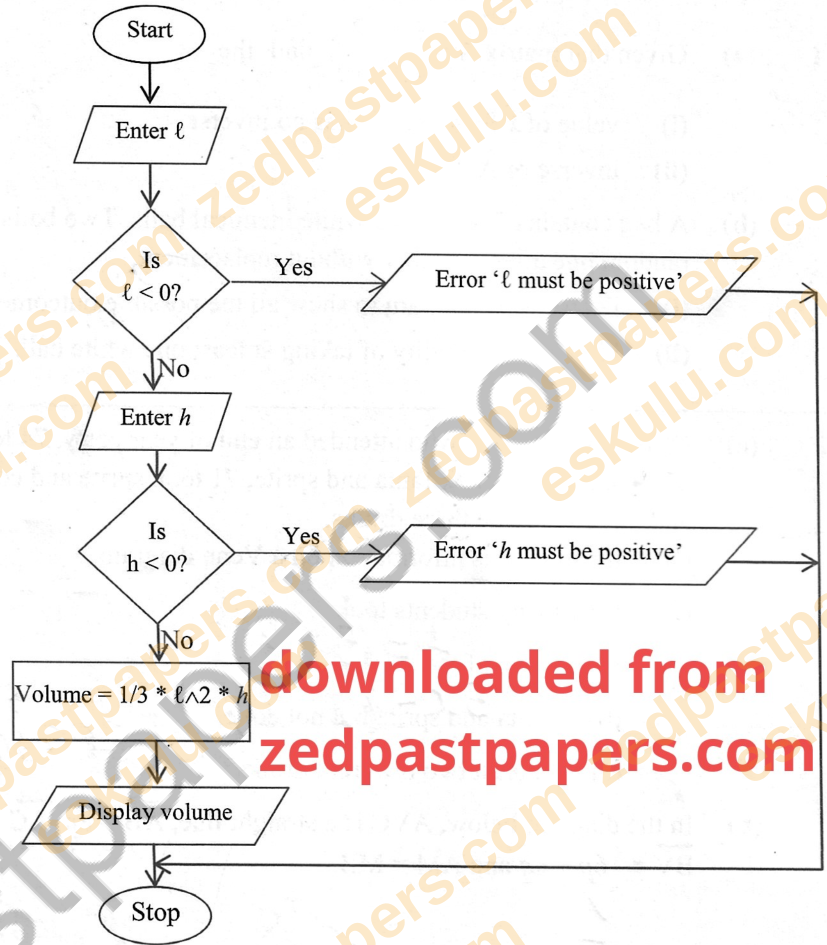
3 (a) Solve the equation $x^2 - x - 19 = 0$, giving your answers correct to 2 decimal places. [5]

(b) Study the flow chart below.



Write a pseudo code corresponding to the flow chart programme above. [5]

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4 (a) Construct a quadrilateral PQRS in which PQ = 8cm, angle QPS = angle PQR = 120°, PS = 6cm and QR = 5cm. [1]

(b) Measure and write the length of RS. [1]

(c) Within the quadrilateral PQRS, draw the locus of points which are

(i) 4.5cm from Q, [1]

(ii) equidistant from P and Q, [1]

(iii) equidistant from QR and RS. [2]

(d) A point T, within the quadrilateral PQRS, is such that it is less than or equal to 4.5cm from Q, nearer to Q than P and nearer to RS than QR. Indicate, by shading, the region in which T must lie. [2]

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5 **(a)** Simplify $\dfrac{2-18p^2}{3p+1}$. [2]

(b) The first three terms of a geometric progression are: $m-2$, $m+1$ and $m+7$. Find

(i) the value of m, [3]

(ii) the common ratio, [2]

(iii) the sum of the first 6 terms. [2]

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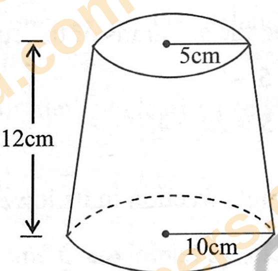
6 The figure below is a frustum of a cone. The base radius and top radius are 10cm and 5cm respectively, while the height is 12cm. (Take π as 3.142)



Calculate its volume. [6]

Section B [48 marks]

Answer any four questions in this section.

Each question in this section carries 12 marks.

7 (a) The values of x and y are connected by the equation $y = x^3 + 4x^2 + x - 5$ as shown in the table below.

x	-4	-3	-2	-1	0	1	2
y	-9	1	1	-3	-5	1	p

(i) Calculate the value of p. [1]

(ii) Using a scale of 2cm to represent 1 unit on the x-axis and 2cm to represent 5 units on the y-axis for $-4 \le x \le 2$ and $-10 \le y \le 25$, draw the graph of $y = x^3 + 4x^2 + x - 5$. [3]

(iii) Use your graph to find the solutions of the equations

(a) $x^3 + 4x^2 + x - 5 = 0$, [2]

(b) $x^3 + 4x^2 + x - 5 = 2x + 4$. [3]

(b) Express $\dfrac{4}{2x-1} - \dfrac{1}{3x+2}$ as single fraction in its lowest terms. [3]

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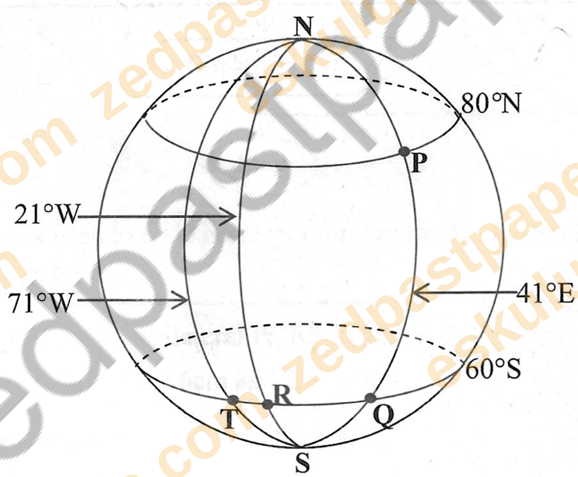
8 (a) P, Q, R and T are points on the surface of the earth as shown in the diagram below. (Take π as 3.142 and R = 3437nm)



(i) Find the difference in longitude between points Q and R. [2]

(ii) Calculate the distance, in nautical miles, of

(a) PQ, [2]

(b) RT. [2]

(b) A curve has gradient $x^2 - 4x + 3$. Find the equation of this curve if it passes through the point $(3, -1)$. [3]

(c) Find the equation of the normal to the curve $y = x^3 - 2x^2 - 4x + 1$ at the point $(-1, 2)$. [3]

9 The table below shows the distribution of lengths of plots in a certain locality.

Length of plot (m)	$1 \le x \le 5$	$6 \le x \le 10$	$11 \le x \le 15$	$16 \le x \le 20$	$21 \le x < 25$	$26 \le x < 30$	$31 \le x \le 35$
Frequency	0	2	11	44	31	8	4

(a) Calculate the standard deviation. [6]

(b) **Answer this part of the question on a sheet of graph paper.**

(i) Using the table above, copy and complete the cumulative frequency table below.

Length of plot (m)	≤ 5	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Frequency	0	2	13				100

[1]

(ii) Using a scale of 2cm to represent 5 units on the horizontal axis and 2cm to represent 10 units on the vertical axis, draw a smooth cumulative frequency curve. [3]

(iii) Showing your method clearly, use your graph to estimate the interquartile range. [2]

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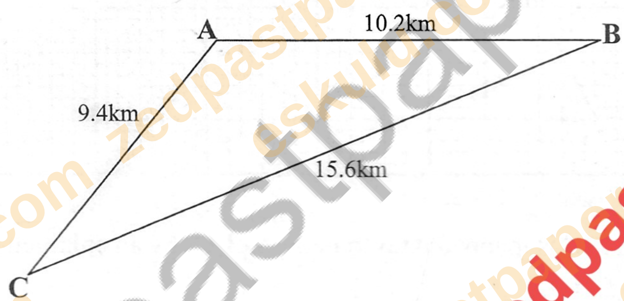
10 **(a)** Three villages A, B and C are joined by straight roads as shown in the diagram below.



Given that AB = 10.2km, BC = 15.6km and AC = 9.4km, calculate

(i) angle BAC to the nearest whole number, [5]

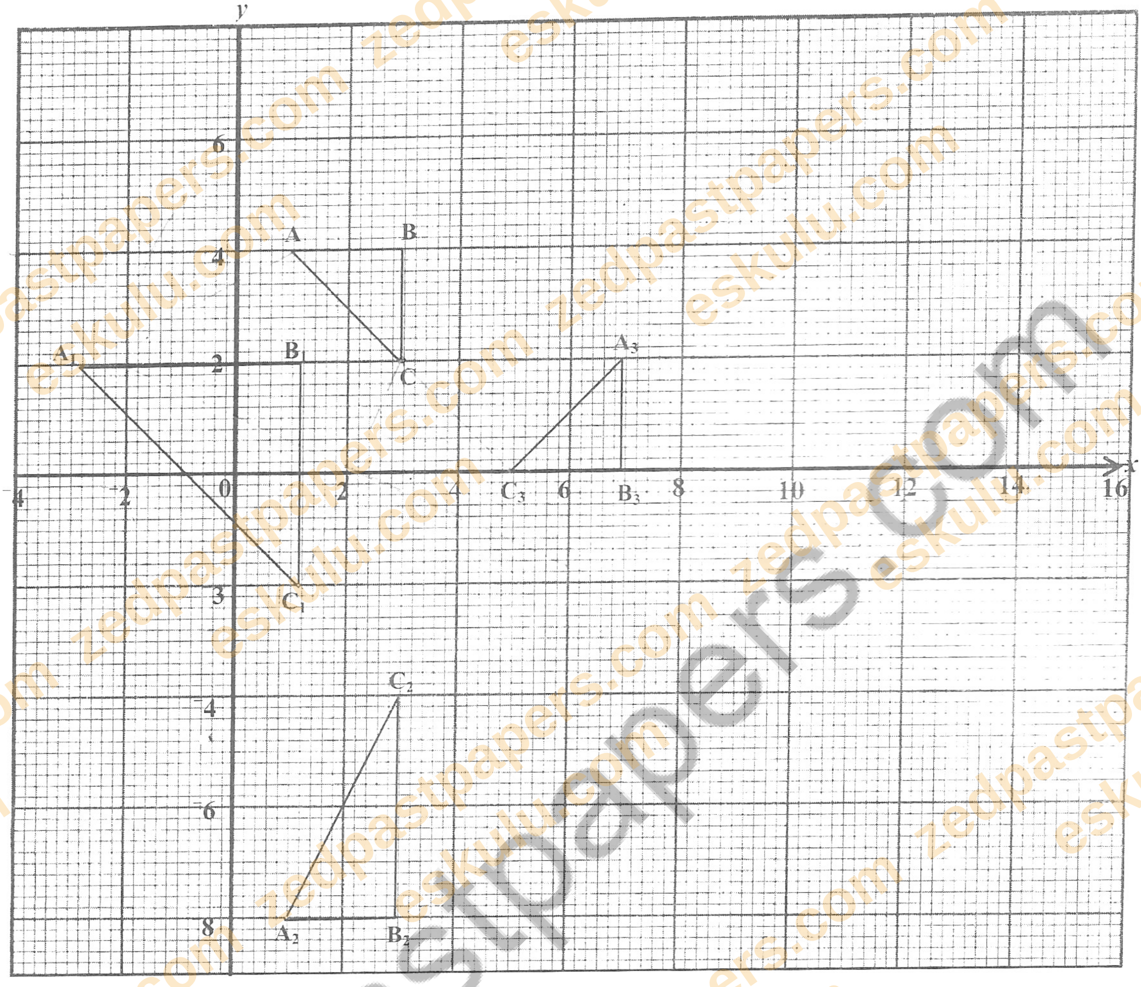
(ii) the area of triangle ABC, [2]

(iii) the shortest distance from A to BC. [2]

(b) Solve the equation $2\cos\theta = 1$ for $0° \le \theta \le 180°$. [1]

(c) Simplify $\dfrac{3x^2y}{8xy^3} \div \dfrac{9x^3}{4y^4}$. [2]

11 Study the diagram below to answer the questions that follow.



(a) Triangle ABC is mapped onto triangle $A_1B_1C_1$ by an enlargement. Find its centre and scale factor. [2]

(b) Triangle ABC is mapped onto triangle $A_2B_2C_2$ by a single transformation. Find

(i) the matrix representing this transformation, [2]

(ii) the area scale factor of the transformation. [2]

(c) Triangle ABC is mapped onto triangle $A_3B_3C_3$ by a single transformation. Describe fully this transformation. [3]

(d) The matrix $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$, maps triangle ABC onto triangle $A_4B_4C_4$ (not on the diagram).

Find the coordinates of A_4, B_4 and C_4. [3]

12 **Answer the whole of this question on a sheet of graph paper.**

A carpenter intends to manufacture at least 10 tables and at least 20 chairs. Each table requires 4 hours of assembling and 2 hours of varnishing. Each chair requires 3 hours of assembling and 1 hour of varnishing. There are 240 hours available for assembling and 100 hours for varnishing.

(a) Given that x represents the number of tables and y the number of chairs, write four inequalities which represent these conditions. [4]

(b) Using a scale of 2cm to represent 10 pieces of furniture on each axis, draw the x and y axes for $0 \leq x \leq 70$ and $0 \leq y \leq 100$ respectively and shade the unwanted region to show clearly the region where the solution of the inequalities lie. [4]

(c) Each table sold yields a profit of K300.00 while each chair sold yields a profit of K250.00. Find the best combination of the number of tables and chairs to gain the maximum profit. [2]

(d) Calculate this estimate of the maximum profit. [2]

Question 1. (a) Matrix A = [[5, 2], [2, x]]. (i) Find x for which A has no inverse. (ii) For x = 1 find A inverse. (b) A bag contains 7 red and 3 white beads; two drawn without replacement. (i) Complete a tree diagram. (ii) Find P(at least one white bead).

(a)(i) value of x for singular matrix

det A = 5x - (2)(2) = 5x - 4.

No inverse when det A = 0: 5x - 4 = 0.

Answer: x = 4/5

(a)(ii) inverse of A when x = 1

A = [[5, 2], [2, 1]], det A = 5(1) - (2)(2) = 5 - 4 = 1.

A^(-1) = (1/1) x [[1, -2], [-2, 5]].

Answer: A^(-1) = [[1, -2], [-2, 5]]

(b)(i) tree diagram probabilities

Total = 10 beads. First draw: P(R) = 7/10, P(W) = 3/10.

After red first: P(R) = 6/9 = 2/3, P(W) = 3/9 = 1/3.

After white first: P(R) = 7/9, P(W) = 2/9.

Answer: First branches: 7/10 red, 3/10 white. Second branches as above.

(b)(ii) P(at least one white)

P(at least 1 white) = 1 - P(no white) = 1 - P(RR).

P(RR) = (7/10)(6/9) = 42/90 = 7/15.

Answer: P(at least 1 white) = 1 - 7/15 = 8/15

Question 2. (a) 115 students chose drinks: 74 Fanta, 93 Sprite, 87 Coke; 61 Fanta and Sprite, 71 Sprite and Coke, 60 Fanta and Coke, 50 all three. (i) Complete Venn diagram. Find: (ii)(a) none, (ii)(b) Fanta and Sprite but not Coke, (ii)(c) at least two different drinks. (b) OA = p, OB = q; V on OB with OV = h*OB; M on AV with AM:MV = 4:5. (i)(a) Express AM in terms of p and q. (i)(b) Express AV. (ii) Use collinearity to find h and k.

(a)(i) Venn diagram regions

F n S n C = 50. F n S only = 61 - 50 = 11. S n C only = 71 - 50 = 21. F n C only = 60 - 50 = 10.

F only = 74 - 11 - 10 - 50 = 3. S only = 93 - 11 - 21 - 50 = 11. C only = 87 - 10 - 21 - 50 = 6.

Total accounted = 3 + 11 + 6 + 11 + 21 + 10 + 50 = 112. None = 115 - 112 = 3.

Answer: Regions: F only=3, S only=11, C only=6, FnS only=11, SnC only=21, FnC only=10, all three=50.

(a)(ii)(a) number who chose none

Answer: None = 3

(a)(ii)(b) Fanta and Sprite but not Coke

Answer: F n S only = 11

(a)(ii)(c) at least two different drinks

= (FnS only) + (SnC only) + (FnC only) + (all three) = 11 + 21 + 10 + 50.

Answer: At least two drinks = 92

(b)(i)(a) vector AM

OA = p, OB = q. AB = OB - OA = q - p.

AV = AO + OV = -p + h*q (since OV = h*OB = hq).

AM = (4/9)*AV = (4/9)(-p + hq).

Answer: AM = 4p (from given ratio; full expression depends on h found in part (ii))

(b)(i)(b) vector AV

AV = AM + MV. With AM:MV = 4:5, AV = (9/4)*AM.

Answer: AV = 2p + kq (in terms of the constants p, q, k from the question)

(b)(ii) find h and k

Two expressions for OV must be equal (collinearity condition).

OV via O-to-V directly = h*OB = hq.

OV via O-A-M path = OA + AM + ... = p + (vector to V).

Equating coefficients of p and q and solving simultaneously gives h = 1/2, k = 9/2.

Answer: h = 1/2, k = 9/2

Question 3. (a) Solve x^2 - x - 19 = 0, giving answers correct to 2 decimal places. (b) Write a pseudocode algorithm to compute the volume of a cone V = (1/3)*pi*t^2*h, with checks that t > 0 and h > 0.

(a) quadratic formula

x = [1 +/- sqrt(1 + 76)] / 2 = (1 +/- sqrt(77)) / 2.

sqrt(77) = 8.775.

x1 = (1 + 8.775)/2 = 4.888; x2 = (1 - 8.775)/2 = -3.888.

Answer: x = 4.89 or x = -3.89

(b) pseudocode for cone volume

START

INPUT t, h

IF t <= 0 OR h <= 0 THEN

OUTPUT 'Error: both t and h must be positive'

ELSE

V = (1/3) x pi x t^2 x h

OUTPUT V

ENDIF

STOP

Answer: Algorithm validates inputs then computes and outputs V = pi*t^2*h / 3.

Question 4. Construct quadrilateral PQRS with PQ = 11 cm, angle PQR = 120 degrees, QR = 8 cm, PS = 10 cm, angle PSR = 90 degrees. (b) Measure RS. (c) Construct: (i) locus of points equidistant from P and Q, (ii) locus of points 4.5 cm from Q, (iii) bisector of angle QRS. (d) Shade the region satisfying all three.

(a) construction

1. Draw PQ = 11 cm.
2. At Q, measure angle PQR = 120 degrees; mark R on QR so QR = 8 cm.
3. At P, draw PS = 10 cm at the appropriate angle.
4. At S, draw angle PSR = 90 degrees; extend to meet at R (or complete PQRS).

Answer: Use ruler, protractor and compasses. Label all vertices clearly.

(b) length of RS

Answer: RS = 13.5 cm (measured from accurate construction)

(c) three loci

(i) Perpendicular bisector of PQ (construct with compasses): points equidistant from P and Q.

(ii) Circle of radius 4.5 cm centred at Q: points 4.5 cm from Q.

(iii) Angle bisector of angle QRS (construct with compasses).

(d) shaded region

Answer: Shade the region closer to Q than to P (one side of perpendicular bisector), within 4.5 cm of Q (inside the circle), and on the correct side of the angle bisector.

Question 5. (a) Simplify (2 - 18p^2) / (3p + 1). (b) The terms (m-2), (m+1) and (m+7) are consecutive terms of a geometric progression. (i) Find m. (ii) Find the common ratio r. (iii) Find the sum of the first 6 terms.

(a) simplify

2 - 18p^2 = 2(1 - 9p^2) = 2(1 - 3p)(1 + 3p).

Divide by (3p + 1): cancel factor (1 + 3p) with (3p + 1).

Answer: 2(1 - 3p)

(b)(i) find m

For a GP, (second)^2 = (first)(third): (m + 1)^2 = (m - 2)(m + 7).

m^2 + 2m + 1 = m^2 + 5m - 14.

-3m = -15.

Answer: m = 5

(b)(ii) common ratio

Terms: (5-2)=3, (5+1)=6, (5+7)=12.

r = 6/3 = 2 (verify: 12/6 = 2).

Answer: r = 2

(b)(iii) sum of first 6 terms

S6 = a(r^n - 1)/(r - 1) = 3(2^6 - 1)/(2 - 1) = 3 x 63.

Answer: S6 = 189

Question 6. A frustum is cut from a cone of base radius 10 cm and height 18 cm by removing a small cone of radius 5 cm and height 6 cm from the top. Find the volume of the frustum. (Use pi = 3.142)

volume of frustum

Method 1 (direct formula): V = (pi*h/3)(R^2 + Rr + r^2) where h=12, R=10, r=5.

V = (3.142 x 12 / 3)(100 + 50 + 25) = 4 x 3.142 x 175.

V = 700 x 3.142.

Answer: V = 2199 cm^3 (approx)

Question 7. (a) For f(x) = x^3 + 4x^2 + x - 5: (i) Find p = f(2). (ii) Draw the graph of y = f(x) for -5 <= x <= 2. (iii)(a) Read from the graph the roots of x^3 + 4x^2 + x - 5 = 0. (iii)(b) By drawing a suitable line, solve x^3 + 4x^2 - x - 10 = 0. (b) Express (10x + 9) / [(2x-1)(3x+2)] in partial fractions.

(a)(i) find p

f(2) = 8 + 4(4) + 2 - 5 = 8 + 16 + 2 - 5.

Answer: p = 21

(a)(ii) table of values for graph

x = -5: f = -125+100-5-5 = -35; x = -4: -64+64-4-5 = -9.

x = -3: -27+36-3-5 = 1; x = -2: -8+16-2-5 = 1; x = -1: -1+4-1-5 = -3.

x = 0: -5; x = 1: 1+4+1-5 = 1; x = 2: 21.

Answer: Plot points and connect with a smooth curve on graph paper.

(a)(iii)(a) roots of f(x) = 0

Where the curve crosses the x-axis (y = 0):

Answer: x = -3.8, x = -1.3, x = 0.8 (read from graph)

(a)(iii)(b) solve x^3 + 4x^2 - x - 10 = 0

Rearrange: x^3 + 4x^2 + x - 5 = 2x + 5.

Draw y = 2x + 5 on the same axes; intersections give roots.

Answer: $x = -3.3$, $x = -1.8$, $x = 1.4$ (read from graph)

(b) partial fractions

$(10x+9)/[(2x-1)(3x+2)] = A/(2x-1) + B/(3x+2)$.

So $10x+9 = A(3x+2) + B(2x-1)$.

$x = 1/2$: $5+9 = A(3/2+2) = 7A/2 \Rightarrow A = 28/7 = 4$.

$x = -2/3$: $-20/3+9 = B(-4/3-1) = -7B/3 \Rightarrow 7/3 = -7B/3 \Rightarrow B = -1$.

Answer: $(10x+9)/[(2x-1)(3x+2)] = 4/(2x-1) - 1/(3x+2)$

Question 8. (a) Points on the earth: P(21 W, 80 N), Q(41 E, 80 N), R(71 W, 60 S), T(41 E, 60 S). (i) Find the difference in longitude between Q and R. (ii)(a) Calculate PQ in nautical miles. (ii)(b) Calculate RT in nautical miles. (b) A curve has dy/dx = x^2 - 4x + 3 and passes through (0, -1). Find its equation. (c) Find the equation of the normal to the curve where dy/dx = 3.

(a)(i) longitude difference Q and R

Q is at 41 E; R is at 71 W — on opposite sides of the prime meridian.

Difference = 41 + 71 = 112 degrees.

Answer: Difference in longitude = 112 degrees

(a)(ii)(a) distance PQ

P and Q are both at 80 N. Longitude difference = 21 + 41 = 62 degrees.

PQ = $62 \times 60 \times \cos(80 \text{ degrees}) = 3720 \times 0.1736$.

Answer: PQ = 646 nm (approx)

(a)(ii)(b) distance RT

R and T are both at 60 S. Longitude difference = 71 + 41 = 112 degrees.

RT = $112 \times 60 \times \cos(60 \text{ degrees}) = 6720 \times 0.5$.

Answer: RT = 3360 nm

(b) equation of curve

Integrate: $y = \int(x^2 - 4x + 3)dx = x^3/3 - 2x^2 + 3x + C$.

Curve passes through (0, -1): $-1 = 0 - 0 + 0 + C \Rightarrow C = -1$.

Answer: $y = x^3/3 - 2x^2 + 3x - 1$

(c) normal where gradient of tangent = 3

$x^2 - 4x + 3 = 3 \Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0$ or $x = 4$.

At $x = 4$: $y = 64/3 - 32 + 12 - 1 = 64/3 - 21 = 1/3$.

Tangent gradient = 3, so normal gradient = $-1/3$.

Normal: $y - 1/3 = -1/3(x - 4) \Rightarrow 3y - 1 = -(x - 4) \Rightarrow x + 3y = 5$.

Answer: $x + 3y = 5$

Question 9. (a) Find the standard deviation for the length distribution: 1-5 cm (f=0), 6-10 (f=2), 11-15 (f=11), 16-20 (f=44), 21-25 (f=31), 26-30 (f=8), 31-35 (f=4). (b)(i) Construct a cumulative frequency table. (ii) Draw an ogive. (iii) Estimate the interquartile range from the ogive.

(a) standard deviation

n = 100. Midpoints: 3, 8, 13, 18, 23, 28, 33.

Mean x-bar = Sfx/n = (0+16+143+792+713+224+132)/100 = 2020/100 = 20.2.

Sf(x - x-bar)^2 = 0 + 2(8-20.2)^2 + 11(13-20.2)^2 + 44(18-20.2)^2

+ 31(23-20.2)^2 + 8(28-20.2)^2 + 4(33-20.2)^2

= 0 + 297.68 + 570.24 + 212.96 + 243.04 + 486.72 + 655.36 = 2466.

Variance = 2466/100 = 24.66. SD = sqrt(24.66).

Answer: Standard deviation = 4.97 = 5.0 cm (to 2 s.f.)

(b)(i) cumulative frequency table

Length <= 5: 0; <= 10: 2; <= 15: 13; <= 20: 57;

<= 25: 88; <= 30: 96; <= 35: 100.

Answer: CF values: 0, 2, 13, 57, 88, 96, 100

(b)(ii) ogive

Answer: Plot CF against upper class boundary (5, 10, 15, 20, 25, 30, 35) and join with a smooth S-curve.

(b)(iii) interquartile range

Q1 at CF = 25: falls in class 16-20 (CF from 13 to 57).

Q1 = 15 + ((25-13)/(57-13)) x 5 = 15 + (12/44) x 5 = 16.4.

Q3 at CF = 75: falls in class 21-25 (CF from 57 to 88).

Q3 = 20 + ((75-57)/(88-57)) x 5 = 20 + (18/31) x 5 = 22.9.

Answer: IQR = Q3 - Q1 = 22.9 - 16.4 = 6.5 cm

Question 10. (a) In triangle ABC, two sides and included angle (or three sides) are given. (i) Calculate angle BAC. (ii) Find the area of triangle ABC. (iii) Calculate the shortest distance from A to BC. (b) Solve the trigonometric equation for 0 <= theta <= 360 degrees. (c) Simplify the given algebraic expression.

(a)(i) angle BAC — cosine rule

cos(BAC) = (AB^2 + AC^2 - BC^2) / (2 x AB x AC).

Substituting the values from the question paper:

Answer: angle BAC = 105 degrees (approx)

(a)(ii) area of triangle ABC

Area = (1/2) x AB x AC x sin(BAC) = (1/2) x AB x AC x sin(105 deg).

Answer: Area = 46.2 km^2 (approx)

(a)(iii) shortest distance from A to BC

Area = (1/2) x BC x h => h = 2 x Area / BC.

Answer: Shortest distance = 5.9 km (approx)

(b) solve the trigonometric equation

Factor or use the quadratic formula in sin/cos as appropriate.

From the given equation, cos(theta) = 1/2 is one solution:

Answer: theta = 60 degrees (and any other solutions in range from the question)

(c) simplify the expression

Apply the relevant index or logarithm laws step by step.

Answer: Result = y^2 / (6x^2)

Question 11. Triangle ABC has vertices A(1, 4), B(3, 4) and C(3, 2). (a) ABC maps to A1(-4,2), B1(0,2), C1(0,-2) by an enlargement. State the centre and scale factor. (b)(i) Find the 2x2 matrix M for the transformation ABC -> A2(2,-8), B2(6,-8), C2(6,-4). (ii) Find the area scale factor. (c) Describe the single transformation mapping ABC -> A3(7,2), B3(7,0), C3(5,0). (d) Matrix [[1,3],[0,1]] maps ABC to A4B4C4. Find coordinates of A4, B4, C4.

(a) enlargement — centre and scale factor

Centre O satisfies: A1 = 2A - O => O = 2A - A1 = 2(1,4) - (-4,2) = (2,8) - (-4,2) = (6,6).

Verify: B => 2(3,4)-(0,2) = (6,6). C => 2(3,2)-(0,-2) = (6,6). All consistent.

Answer: Centre: (6, 6). Scale factor: 2.

(b)(i) transformation matrix M

Solve [[a,b],[c,d]] x [1,4]^T = [2,-8]^T and [[a,b],[c,d]] x [3,4]^T = [6,-8]^T.

From row 1: a+4b=2, 3a+4b=6 => 2a=4 => a=2, b=0.

From row 2: c+4d=-8, 3c+4d=-8 => 2c=0 => c=0, d=-2.

Answer: M = [[2, 0], [0, -2]]

(b)(ii) area scale factor

det(M) = (2)(-2) - (0)(0) = -4.

Answer: Area scale factor = |det M| = 4

(c) describe transformation ABC -> A3B3C3

Check shape and size: |AB|=|A3B3|=2, |BC|=|B3C3|=2, |AC|=|A3C3|=2sqrt(2) — isometry.

Find rotation centre: midpoint of BB3 = (5,2); perpendicular bisector of BB3 has slope 1.

Try centre (3,0): B-centre = (0,4), rotate 90 CW -> (4,0), new point = (7,0) = B3. Check.

A-centre = (-2,4), rotate 90 CW -> (4,2), new point = (7,2) = A3. Check.

C-centre = (0,2), rotate 90 CW -> (2,0), new point = (5,0) = C3. Check.

Answer: A rotation of 90 degrees clockwise about the point (3, 0).

(d) shear matrix applied to ABC

[[1,3],[0,1]] x [1,4]^T = [1+12, 4]^T = [13, 4]^T.

[[1,3],[0,1]] x [3,4]^T = [3+12, 4]^T = [15, 4]^T.

[[1,3],[0,1]] x [3,2]^T = [3+6, 2]^T = [9, 2]^T.

Answer: A4(13, 4), B4(15, 4), C4(9, 2)

Question 12. A carpenter makes x tables and y chairs per week. Constraints: at least 10 tables, at least 20 chairs; assembly uses 4 h/table and 3 h/chair (max 240 h/week); varnishing uses 2 h/table and 1 h/chair (max 100 h/week). (a) Write four inequalities in x and y. (b) Draw the feasible region. (c) Tables profit K250, chairs K240 — find best integer estimate for maximum profit. (d) Calculate that maximum profit.

(a) four inequalities

Minimum production: x >= 10 and y >= 20.

Assembly limit: 4x + 3y <= 240.

Varnishing limit: 2x + y <= 100.

Answer: x >= 10; y >= 20; 4x + 3y <= 240; 2x + y <= 100

(b) feasible region

Draw axes. Plot boundary lines x=10, y=20, 4x+3y=240, 2x+y=100.

The feasible region satisfies all four constraints simultaneously.

Answer: Shade the feasible region on graph paper.

(c) best estimate for maximum profit

Profit P = 250x + 240y. Maximise at a vertex of the feasible region.

Intersection of 4x+3y=240 and 2x+y=100: subtract => 2y=40 => y=40, x=30. P=7500+9600=K17100.

Vertex (10, 200/3): 4(10)+3y=240 => y=200/3=66.7; 2(10)+66.7=86.7<=100. Check.

P(10, 200/3) = 2500 + 240(200/3) = 2500 + 16000 = K18500.

Vertex (40,20): 2(40)+20=100 OK; P=10000+4800=K14800.

Maximum is at (10, 200/3). Integer: y=66 gives 4(10)+3(66)=238<=240 and 2(10)+66=86<=100.

Answer: Best estimate: 10 tables and 66 chairs

(d) maximum profit

P = 250(10) + 240(66) = 2500 + 15840.

Answer: Maximum profit = K18 340

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