

G12 ECZ MATHEMATICS 2019 PAPER 2 SOLUTIONS

Examinations Council of Zambia — School Certificate Ordinary Level (November)

Worked solutions — Section A (answer all) and Section B (any four)

The full question paper appears first — worked answers are at the end.



EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Mathematics

Paper 2

4024/2

Friday

15 NOVEMBER 2019

Additional materials:

Answer Booklet
Silent Electronic Calculator (non programmable)
Geometrical instruments
Graph paper (3 sheets)
Plain paper (1 sheet)

Time: 2 hours 30 minutes

Instructions to Candidates

Write your **name**, **centre number** and **candidate number** in the spaces provided on the **Answer Booklet**.

Write your answers and working in the **Answer Booklet provided**.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Omission of essential working will result in loss of marks.

There are **twelve (12)** questions in this paper.

Section A

Answer **all** questions.

Section B

Answer any **four** questions.

Silent non programmable Calculators may be used.

Cell phones are not allowed in the examination room.

Information for Candidates

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 100.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Geometric Progression

$$S_n = \frac{a(1-r^n)}{1-r}, (r < 1)$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, (r > 1)$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A$$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \text{SD} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \text{SD} = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$$

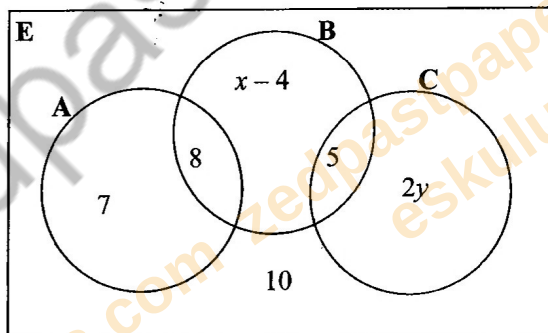
Section A (52 Marks)

Answer all questions in this section

- 1 (a) Given that matrix $A = \begin{pmatrix} 6 & x \\ 2 & 3 \end{pmatrix}$,
- (i) find the value of x for which the determinant of A is 36, [2]
 - (ii) hence, find the inverse of A . [2]
- (b) A box contains 4 red pens and 5 green pens. A pen is picked at random from the box without replacement and a second pen is then picked.
- (i) Draw a tree diagram to illustrate the outcomes. [3]
 - (ii) What is the probability of picking one red pen and one green pen? [2]

- 2 (a) Simplify $\frac{6x^2 - 24}{x - 2}$. [2]
- (b) Given the geometric progression 4, 8, 16, ..., find
- (i) the geometric mean of 256 and 1 024, [2]
 - (ii) the 11th term, [2]
 - (iii) the sum of the first 11 terms. [3]

- 3 (a) Solve the equation $p^2 - p = 4$, giving your answers correct to 2 decimal places. [5]
- (b) The Venn diagram below shows the number of elements in sets A , B and C .



Find

- (i) x , such that $n(B) = n(B \cup C)$, [2]
- (ii) y , such that $n(C) = n(A)$, [1]
- (iii) $n(E)$, [1]
- (iv) $n(B')$. [1]

4 (a) Express $\frac{5}{2x-4} - \frac{4}{3x+5}$ as a single fraction in its simplest form. [3]

(b) The program below is given in the form of a pseudocode.

Begin

Enter length

If length < 0

Then display "error message and re-enter positive length

Else enter height

If height < 0

Then display "error message" and re-enter positive height

Else volume = $\frac{1}{3} * l * l * h$

End if

Display volume

End

Draw the corresponding flowchart for the information given above.

[5]

5 • (a) (i) Construct triangle ABC in which AB = 8cm, $\hat{BAC} = 110^\circ$ and $\hat{ABC} = 35^\circ$. [1]

(ii) Measure and write the length of BC. [1]

(b) Within the triangle ABC, construct the locus of points which are

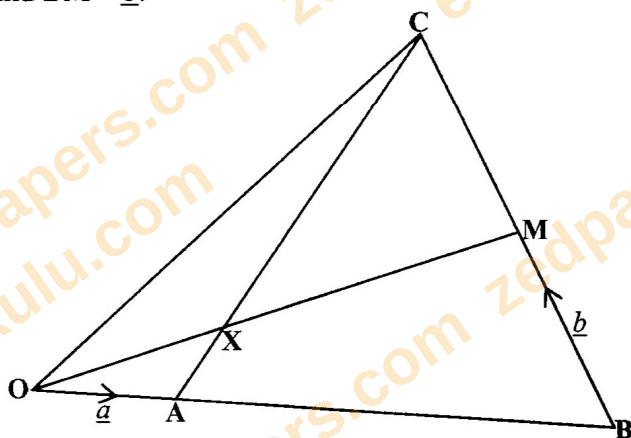
(i) 3cm from AB, [1]

(ii) equidistant from AC and BC, [2]

(iii) 4cm from A. [1]

(c) A point Q inside triangle ABC is greater than or equal to 3cm from AB, less than or equal to 4cm from A and nearer to AC than BC. Indicate clearly, by shading, the region in which Q must lie. [2]

- 6 (a) In the diagram below, $\vec{OB} = 4\vec{OA}$ and $\vec{AC} = 5\vec{AX}$. M is the midpoint of BC, $\vec{OA} = \underline{a}$ and $\vec{BM} = \underline{b}$.



- (i) Express in terms of $\underline{a}$ and/or $\underline{b}$.

(a) $\vec{AB}$,

[1]

(b) $\vec{AC}$,

[1]

(c) $\vec{OM}$.

[1]

(ii) Show that $\vec{OX} = \frac{2}{5}(4\underline{a} + \underline{b})$.

[2]

- (b) Find the equation of the normal to the curve $y = 5x^3 - 6x^2 + 2x + 5$ at the point (1, 2).

[3]

Section B [48 marks]

Answer any four questions in this section.

Each question in this section carries 12 marks.

7 Answer the whole of this question on a sheet of graph paper.

The vertices of triangle ABC are A(1, 1), B(1, 3) and C(3, 3). The vertices of triangle $A_1B_1C_1$ are $A_1(-1, 1)$, $B_1(-3, 1)$ and $C_1(-3, 3)$.

- (a) Using a scale of 1cm to represent 1 unit on each axis, draw the x and y axes for $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$. Draw and label triangles ABC and $A_1B_1C_1$. [2]
 - (b) Describe fully a single transformation that maps triangle ABC onto triangle $A_1B_1C_1$. [2]
 - (c) An enlargement maps triangle ABC onto triangle $A_2B_2C_2$ with vertices $A_2(-2, -2)$, $B_2(-2, -6)$ and $C_2(-6, -6)$.
 - (i) Draw and label triangle $A_2B_2C_2$. [1]
 - (ii) Find the scale factor. [1]
 - (d) The transformation represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ maps triangle ABC onto triangle $A_3B_3C_3$.
 - (i) Find the coordinates of the vertices A_3 , B_3 and C_3 . [2]
 - (ii) Draw and label triangle $A_3B_3C_3$. [1]
 - (e) Triangle ABC is mapped onto triangle $A_4B_4C_4$ with vertices $A_4(1, -2)$, $B_4(1, 0)$ and $C_4(3, -6)$.
 - (i) Draw and label triangle $A_4B_4C_4$. [1]
 - (ii) Find the matrix representing this transformation. [2]
-

8 The table below shows the expenditure of 90 farmers in a particular farming season.

Amount (K)	$0 < x \leq 100$	$100 < x \leq 200$	$200 < x \leq 300$	$300 < x \leq 400$	$400 < x \leq 500$	$500 < x \leq 600$	$600 < x \leq 700$	$700 < x \leq 800$
No. of farmers	5	16	17	17	14	12	7	2

(a) Calculate the standard deviation. [6]

(b) Answer this part of the question on a sheet of graph paper.

(i) Using the table above, copy and complete the cumulative frequency table below.

Amount (K)	≤ 0	≤ 100	≤ 200	≤ 300	≤ 400	≤ 500	≤ 600	≤ 700	≤ 800
Frequency	0	5	21	38	55	69			

[1]

(ii) Using a scale of 2cm to represent 100 units on the horizontal axis and 2cm to represent 10 units on the vertical axis, draw a smooth cumulative frequency curve. [3]

(iii) Showing your method clearly, use your graph to estimate the interquartile range. [2]

9 Kuunika wishes to build a lodge with single and double rooms. He needs to decide the number of each room type he should build to maximize profit.

Let x represent the number of single rooms and y the number of double rooms.

(a) Write the inequalities which represent each of the following conditions:

(i) There must be at least one single room. [1]

(ii) There must be at least 10 rooms altogether. [1]

(iii) The total number of rooms should not exceed 15. [1]

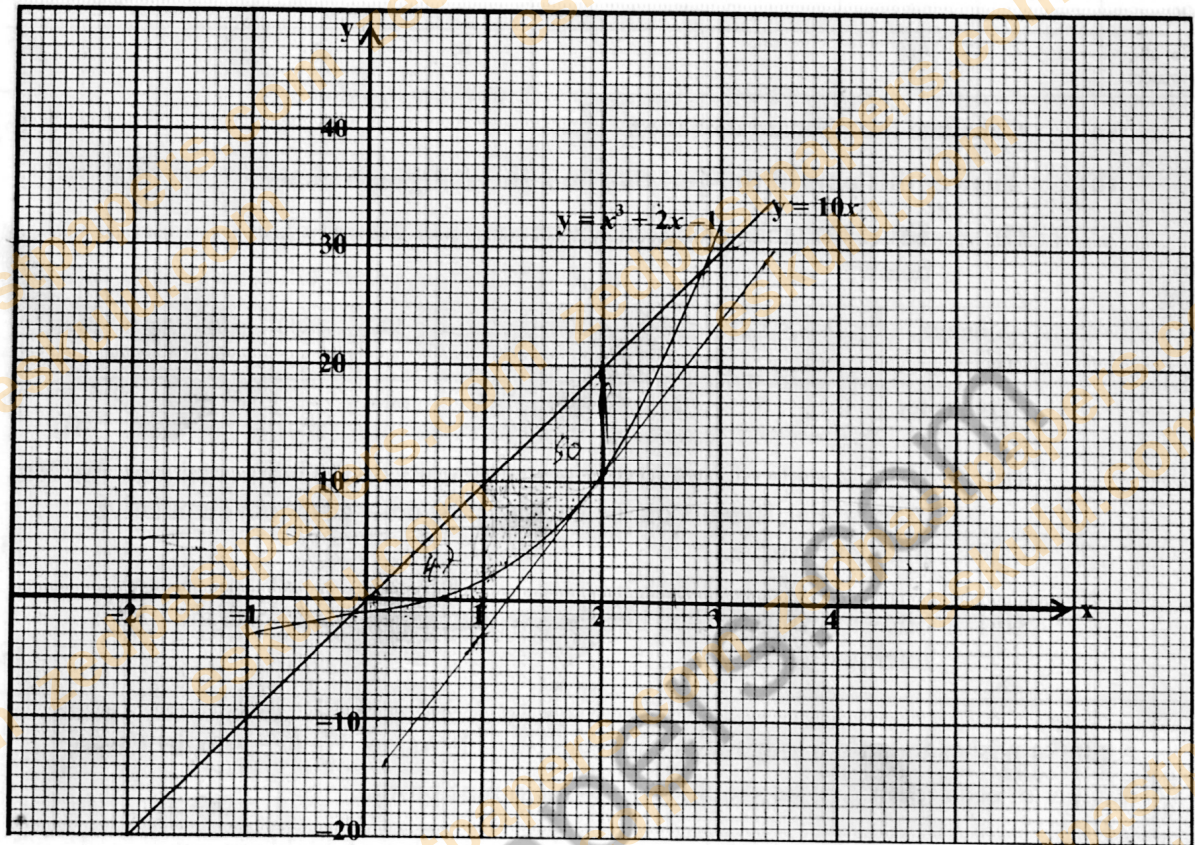
(iv) The number of double rooms must be at least twice the number of single rooms. [1]

(v) The number of double rooms should not be more than 12. [1]

(b) Using a scale of 2cm to 5 units on both axes, draw x and y axes for $0 \leq x \leq 16$ and $0 \leq y \leq 16$ respectively and shade the unwanted region to indicate clearly the region where the solution of the inequalities lie. [5]

(c) The rate for a single room is K600.00 and K900.00 for a double room. How many rooms of each type should Kuunika build to maximize the income? [2]

- 10 (a) The diagram below shows part of the graphs of $y = x^3 + 2x - 1$ and $y = 10x$.



- (i) Use the graphs to solve the equations

(a) $x^3 + 2x = 6$,

[2]

(b) $x^3 + 2x - 1 = 10x$.

[2]

- (ii) Calculate an estimate of

(a) the gradient of the curve at the point (2, 11),

[2]

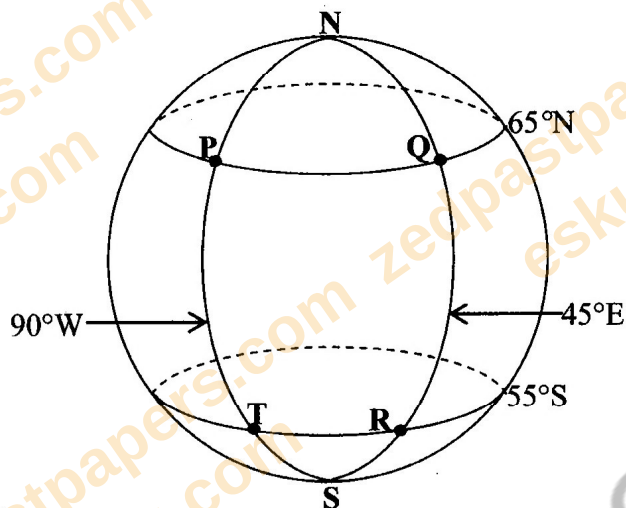
(b) the area bounded by the curve, $y = 10x$, $y = 0$ and $x = 2$.

[3]

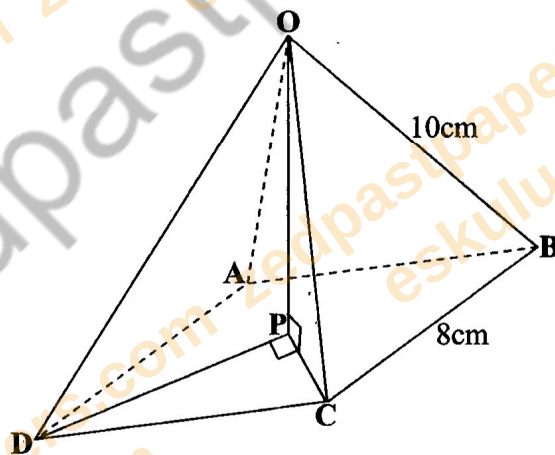
(b) Evaluate $\int_1^3 (3x^2 + 4x) dx$.

[3]

- 11 (a) The points P, Q, R and T are on the surface of the earth as shown in the diagram below. [Take π as 3.142 and $R = 3437\text{nm}$]



- (i) Find the difference in longitude between the points T and R. [2]
 (ii) Find, in nautical miles, the distance between
 (a) P and Q along the latitude 65°N , [2]
 (b) P and T along the longitude 90°W . [2]
 (b) The figure below shows a right pyramid with a vertex O and a square base ABCD of side 8cm. $\angle CPD = \angle OPC = 90^\circ$.



Given that $OA = OB = OC = OD = 10\text{cm}$, calculate

- (i) the height OP, [4]
 (ii) the angle between the edge OC and the base PC. [2]



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Question 1. (a) $A = \begin{pmatrix} 6 & x \\ 2 & 3 \end{pmatrix}$. (i) Find x for which $\det A = 36$. (ii) Hence find A^{-1} . (b) Box: 4 red and 5 green pens; a pen is picked without replacement, then a second. (i) Tree diagram. (ii) $P(\text{one red and one green})$.

(a)(i) value of x

$$\det A = (6)(3) - (x)(2) = 18 - 2x.$$

$$18 - 2x = 36 \Rightarrow 2x = -18 \Rightarrow x = -9.$$

Answer: $x = -9$

(a)(ii) inverse of A

$$A = \begin{pmatrix} 6 & -9 \\ 2 & 3 \end{pmatrix}, \det A = 36.$$

$$A^{-1} = (1/36) \begin{pmatrix} 3 & 9 \\ -2 & 6 \end{pmatrix}.$$

Answer: $A^{-1} = (1/36) \begin{pmatrix} 3 & 9 \\ -2 & 6 \end{pmatrix} = \begin{pmatrix} 1/12 & 1/4 \\ -1/18 & 1/6 \end{pmatrix}$

(b)(i) tree diagram

First pick from 9: $P(R)=4/9$, $P(G)=5/9$. Second pick from 8 (no replacement).

Answer: Two-stage tree, first denominators 9, second denominators 8

(b)(ii) $P(\text{one red and one green})$

$$P(RG) + P(GR) = (4/9)(5/8) + (5/9)(4/8) = 20/72 + 20/72 = 40/72.$$

Answer: $P(\text{one red and one green}) = 5/9$

Question 2. (a) Simplify $(6x^2 - 24)/(x - 2)$. (b) For the GP 4, 8, 16, ..., find (i) the geometric mean of 256 and 1024, (ii) the 11th term, (iii) the sum of the first 11 terms.

(a) simplify $(6x^2 - 24)/(x - 2)$

$$6x^2 - 24 = 6(x^2 - 4) = 6(x - 2)(x + 2).$$

$$= 6(x - 2)(x + 2)/(x - 2) = 6(x + 2).$$

Answer: $6(x + 2) = 6x + 12$

(b) GP 4, 8, 16, ... ($a = 4$, $r = 2$)

(b)(i) geometric mean of 256 and 1024

$$GM = \sqrt{(256 \times 1024)} = \sqrt{262144} = 512.$$

Answer: Geometric mean = 512

(b)(ii) 11th term

$$T_{11} = ar^{10} = 4(2)^{10} = 4(1024).$$

Answer: $T_{11} = 4096$

(b)(iii) sum of first 11 terms

$$S_{11} = a(r^{11} - 1)/(r - 1) = 4(2^{11} - 1)/(2 - 1) = 4(2048 - 1).$$

Answer: $S_{11} = 8188$

Question 3. (a) Solve $p^2 - p = 4$, answers to 2 d.p. (b) Venn diagram for sets A, B, C: A only 7, $A \cap B$ only 8, B only $x-4$, $B \cap C$ only 5, C only $2y$, and 10 outside all (A and C do not overlap). Find (i) x with $n(B) = n(B \cup C)'$, (ii) y with $n(C) = n(A)$, (iii) $n(E)$, (iv) $n(B')$.

(a) solve $p^2 - p = 4$

$$p^2 - p - 4 = 0, \text{ with } a = 1, b = -1, c = -4.$$

$$p = [1 \pm \sqrt{(1 + 16)}] / 2 = [1 \pm \sqrt{17}] / 2 = [1 \pm 4.123] / 2.$$

$$p = 5.123/2 = 2.56 \text{ or } p = -3.123/2 = -1.56.$$

Answer: $p = 2.56$ or $p = -1.56$

(b)(i) value of x

$$n(B) = 8 + (x - 4) + 5 = x + 9.$$

$$(B \cup C)' = \text{regions outside B and C} = (A \text{ only}) + (\text{outside all}) = 7 + 10 = 17.$$

$$x + 9 = 17 \Rightarrow x = 8.$$

Answer: $x = 8$

(b)(ii) value of y

$$n(A) = 7 + 8 = 15; n(C) = 5 + 2y.$$

$$5 + 2y = 15 \Rightarrow y = 5.$$

Answer: $y = 5$

(b)(iii) $n(E)$

$$\text{With B only} = 4, \text{ C only} = 10; \text{ total} = 7 + 8 + 4 + 5 + 10 + 10.$$

Answer: $n(E) = 44$

(b)(iv) $n(B')$

$$n(B') = n(E) - n(B) = 44 - 17.$$

Answer: $n(B') = 27$

Question 4. (a) Express $5/(2x - 4) - 4/(3x + 5)$ as a single fraction in simplest form. (b) Pseudocode (Begin; Enter length; If length < 0 then error and re-enter; Else enter height; If height < 0 then error and re-enter; Else volume = $\text{length} \times \text{width} \times \text{height}$; End if; Display volume; End). Draw the corresponding flowchart.

(a) single fraction

$$= [5(3x + 5) - 4(2x - 4)] / [(2x - 4)(3x + 5)].$$

$$\text{Numerator} = 15x + 25 - 8x + 16 = 7x + 41.$$

Answer: $(7x + 41) / [(2x - 4)(3x + 5)]$

(b) flowchart (description)

Start (oval) $\rightarrow$ Input: Enter length.

Decision: Is length < 0 ? Yes $\rightarrow$ Output error "re-enter positive length" (loop back to Enter length); No $\rightarrow$ Input: Enter height.

Decision: Is height < 0 ? Yes $\rightarrow$ Output error "re-enter positive height" (loop back to Enter height); No $\rightarrow$ Process: volume = $(1/3) \cdot l \cdot l \cdot h$.

$\rightarrow$ Output: Display volume $\rightarrow$ Stop (oval).

Answer: Flowchart with two validation diamonds (length < 0 , height < 0) feeding the process box volume = $\frac{1}{3} \cdot l^2 \cdot h$, then Display volume $\rightarrow$ Stop

Question 5. (a) Construct triangle ABC with AB = 8 cm, $\angle BAC = 110^\circ$, $\angle ABC = 35^\circ$; (ii) measure BC. (b) Within ABC draw the loci (i) 3 cm from AB, (ii) equidistant from AC and BC, (iii) 4 cm from A. (c) Point Q: ≥ 3 cm from AB, ≤ 4 cm from A, nearer AC than BC — shade the region.

(a)(ii) length of BC (sine rule / measured)

$$\angle ACB = 180^\circ - 110^\circ - 35^\circ = 35^\circ.$$

$$BC/\sin \angle BAC = AB/\sin \angle ACB \Rightarrow BC = 8 \cdot \sin 110^\circ / \sin 35^\circ = 8(0.9397)/0.5736.$$

Answer: BC $\approx$ 13.1 cm (confirm by measuring)

(b)(i) locus 3 cm from AB

A line parallel to AB at a distance of 3 cm (inside the triangle).

(b)(ii) locus equidistant from AC and BC

The bisector of angle C (between AC and BC).

(b)(iii) locus 4 cm from A

An arc of a circle, centre A, radius 4 cm.

(c) region for Q

≥ 3 cm from AB $\Rightarrow$ on the far side of the parallel line from AB.

≤ 4 cm from A $\Rightarrow$ inside (or on) the arc centred A.

Nearer AC than BC $\Rightarrow$ the AC side of the bisector of angle C.

Answer: Shade the overlap of all three conditions

Question 6. (a) $OB = 4OA$, $AC = 5AX$, M is the midpoint of BC, $OA = a$, $BM = b$. (i) Express in a and/or b: (a) AB, (b) AC, (c) OM. (ii) Show $OX = (2/5)(4a + b)$. (b) Find the equation of the normal to $y = 5x^3 - 6x^2 + 2x + 5$ at the point (1, 2).

(a)(i)(a) AB

$$AB = OB - OA = 4a - a = 3a.$$

Answer: AB = 3a

(a)(i)(b) AC

M midpoint of BC, $BM = b$, so $BC = 2b$.

$$AC = AB + BC = 3a + 2b.$$

Answer: $AC = 3a + 2b$

(a)(i)(c) OM

$$OM = OB + BM = 4a + b.$$

Answer: $OM = 4a + b$

(a)(ii) show $OX = (2/5)(4a + b)$

$$AC = 5AX \Rightarrow AX = (1/5)AC = (1/5)(3a + 2b).$$

$$OX = OA + AX = a + (3/5)a + (2/5)b = (8/5)a + (2/5)b = (2/5)(4a + b). \text{ (shown)}$$

Answer: $OX = (2/5)(4a + b)$ ✓

(b) normal to $y = 5x^3 - 6x^2 + 2x + 5$ at $(1, 2)$

$$dy/dx = 15x^2 - 12x + 2; \text{ at } x = 1 \text{ the tangent gradient} = 15 - 12 + 2 = 5.$$

$$\text{Normal gradient} = -1/5, \text{ so } y - 2 = -(1/5)(x - 1).$$

$$5y - 10 = -x + 1 \Rightarrow x + 5y = 11.$$

Answer: Normal: $x + 5y = 11$ ($y = -x/5 + 11/5$)

Question 7. Triangle ABC: A(1,1), B(1,3), C(3,3); triangle A'B'C': A'(-1,1), B'(-3,1), C'(-3,3). (b) Describe fully $ABC \rightarrow A'B'C'$. (c) Enlargement maps $ABC \rightarrow A'B'C'$ with A'(-2,-2), B'(-2,-6), C'(-6,-6): find the scale factor. (d) Matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ maps $ABC \rightarrow A'B'C'$: find the vertices. (e) $ABC \rightarrow A'B'C'$ with A'(1,-2), B'(1,0), C'(3,-6): find the matrix.

(b) describe $ABC \rightarrow A'B'C'$

$$(x, y) \rightarrow (-y, x): A(1,1) \rightarrow (-1,1), B(1,3) \rightarrow (-3,1), C(3,3) \rightarrow (-3,3).$$

This is matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Answer: Rotation, 90° anticlockwise (positive quarter turn), centre (0, 0)

(c)(ii) scale factor of the enlargement

$$A(1,1) \rightarrow (-2,-2), B(1,3) \rightarrow (-2,-6), C(3,3) \rightarrow (-6,-6): \text{ each image} = -2 \times \text{original}.$$

Answer: Scale factor = -2 (centre (0, 0))

(d) vertices of A'B'C' under $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$$(x, y) \rightarrow (2x, y).$$

$$A(1,1) \rightarrow (2,1); B(1,3) \rightarrow (2,3); C(3,3) \rightarrow (6,3).$$

Answer: A'(2,1), B'(2,3), C'(6,3)

(e) matrix for $ABC \rightarrow A'B'C'$

$$x \text{ unchanged; } y' = cx + dy \text{ with } A(1,1) \rightarrow -2, B(1,3) \rightarrow 0: c + d = -2, c + 3d = 0 \Rightarrow d = 1, c = -3 \text{ (check C: } -9 + 3 = -6 \checkmark).$$

$$\text{So } (x, y) \rightarrow (x, -3x + y).$$

Answer: Matrix = $\begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$ (a shear, x-axis invariant)

Question 8. Expenditure of 90 farmers: $0 < x \leq 100$ (5), $100 < x \leq 200$ (16), $200 < x \leq 300$ (17), $300 < x \leq 400$ (17), $400 < x \leq 500$ (14), $500 < x \leq 600$ (12), $600 < x \leq 700$ (7), $700 < x \leq 800$ (2). (a) Standard deviation. (b)(i) complete the cumulative frequency table, (ii) draw the ogive, (iii) estimate the interquartile range.

(a) standard deviation

Midpoints: 50, 150, 250, 350, 450, 550, 650, 750; $\Sigma f = 90$.

$\Sigma fx = 250 + 2400 + 4250 + 5950 + 6300 + 6600 + 4550 + 1500 = 31800$, mean = 353.33.

$\Sigma fx^2 = 14\,065\,000$.

Variance = $14065000/90 - 353.33^2 = 156277.8 - 124844.4 = 31433.4$.

SD = $\sqrt{31433.4} = 177.3$.

Answer: Standard deviation $\approx$ K177

(b)(i) cumulative frequency table

$\leq 0:0$, $\leq 100:5$, $\leq 200:21$, $\leq 300:38$, $\leq 400:55$, $\leq 500:69$, $\leq 600:81$, $\leq 700:88$, $\leq 800:90$.

Answer: Cumulative: 0, 5, 21, 38, 55, 69, 81, 88, 90

(b)(ii) cumulative frequency curve

Plot (100,5), (200,21), (300,38), (400,55), (500,69), (600,81), (700,88), (800,90) from (0,0) and join with a smooth ogive.

(b)(iii) interquartile range (from graph)

$Q_{22.5}$ at 22.5th: $Q_{22.5} = 200 + (22.5 - 21)/17 \times 100 \approx 208.8$.

$Q_{67.5}$ at 67.5th: $Q_{67.5} = 400 + (67.5 - 55)/14 \times 100 \approx 489.3$.

IQR = $Q_{67.5} - Q_{22.5} = 489.3 - 208.8 \approx 280$.

Answer: Interquartile range $\approx$ K280 (read your graph to confirm)

Question 9. Kuunika builds single rooms (x) and double rooms (y). (a) Inequalities: (i) at least one single, (ii) at least 10 rooms, (iii) total not exceed 15, (iv) doubles at least twice singles, (v) doubles not more than 12. (b) Draw and shade the unwanted region. (c) Rate K600/single, K900/double — rooms of each type for maximum income.

(a) inequalities

(i) $x \geq 1$. (ii) $x + y \geq 10$. (iii) $x + y \leq 15$.

(iv) $y \geq 2x$. (v) $y \leq 12$.

Answer: $x \geq 1$, $x + y \geq 10$, $x + y \leq 15$, $y \geq 2x$, $y \leq 12$

(b) region (method)

Draw $x = 1$, $x + y = 10$, $x + y = 15$, $y = 2x$ and $y = 12$; shade away the unwanted side so the clear region satisfies all five.

(c) rooms for maximum income

Income $I = 600x + 900y$. Test the corners:

$(5,10) \rightarrow 12000$; $(1,12) \rightarrow 11400$; $(3,12) \rightarrow 12600$.

At $(3,12)$: $x + y = 15$ ✓, $y = 12$ ✓, $y \geq 2x$ ($12 \geq 6$) ✓.

Answer: Build 3 single rooms and 12 double rooms, giving maximum income K12 600.00

Question 10. (a) Graphs of $y = x^3 + 2x - 1$ and $y = 10x$. (i) Use the graphs to solve (a) $x^3 + 2x = 6$, (b) $x^3 + 2x - 1 = 10x$. (ii) Estimate (a) the gradient at $(2, 11)$, (b) the area bounded by the curve, $y = 10x$, $y = 0$ and $x = 2$. (b) Evaluate $\int_{-3}^3 (3x^2 + 4x) dx$.

(a)(i)(a) solve $x^3 + 2x = 6$

$x^3 + 2x = 6$ means $x^3 + 2x - 1 = 5$; read where the curve meets $y = 5$.

Answer: $x \approx 1.46$ (read from the graph)

(a)(i)(b) solve $x^3 + 2x - 1 = 10x$

Read the x -values where the curve meets the line $y = 10x$ (equivalently solve $x^3 - 8x - 1 = 0$).

Answer: $x \approx -2.8$, $x \approx -0.1$, $x \approx 2.9$ (read from the graph)

(a)(ii)(a) gradient at $(2, 11)$

Draw the tangent at $(2, 11)$ and measure rise/run.

Check: $dy/dx = 3x^2 + 2$; at $x = 2$, $= 12 + 2 = 14$.

Answer: Gradient ≈ 14

(a)(ii)(b) area bounded by curve, $y = 10x$, $y = 0$ and $x = 2$

Estimate by counting unit squares inside the boundaries on the graph.

Between the line and the curve over $0 \leq x \leq 2$: $\int_0^2 (10x - (x^3 + 2x - 1)) dx = [-x^4/4 + 4x^2 + x]_0^2 = -4 + 16 + 2 = 14$.

Answer: Area ≈ 14 square units (count squares on your graph to confirm)

(b) evaluate $\int_{-3}^3 (3x^2 + 4x) dx$

$\int (3x^2 + 4x) dx = x^3 + 2x^2$.

$[x^3 + 2x^2]_{-3}^3 = (27 + 18) - (1 + 2) = 45 - 3$.

Answer: $\int_{-3}^3 (3x^2 + 4x) dx = 42$

Question 11. (a) Points on the earth ($\pi = 3.142$, $R = 3437$ nm): $P(65^\circ N, 90^\circ W)$, $Q(65^\circ N, 45^\circ E)$, $T(55^\circ S, 90^\circ W)$, $R(55^\circ S, 45^\circ E)$. (i) Difference in longitude between T and R. (ii) Distances (a) P to Q along latitude $65^\circ N$, (b) P to T along longitude $90^\circ W$. (b) Right pyramid, vertex O, square base ABCD side 8 cm, $OA = OB = OC = OD = 10$ cm, P the centre of the base. Find (i) the height OP, (ii) the angle between edge OC and the base.

(a)(i) difference in longitude T to R

T at $90^\circ W$, R at $45^\circ E$: difference $= 90 + 45 = 135^\circ$.

Answer: 135°

(a)(ii)(a) P to Q along latitude 65°N

Longitude difference = $90 + 45 = 135^\circ$; radius of parallel = $R \cos 65^\circ$.

$$PQ = (135/360) \cdot 2\pi(R \cos 65^\circ) = (0.375)(2)(3.142)(3437)(0.4226) = (0.375)(9127.4).$$

Answer: PQ ≈ 3423 nm

(a)(ii)(b) P to T along longitude 90°W

Latitude difference P(65°N) to T(55°S) = $65 + 55 = 120^\circ$.

$$PT = (120/360) \cdot 2\pi R = (1/3)(2)(3.142)(3437) = (1/3)(21597.0).$$

Answer: PT ≈ 7199 nm (≈ 7200 nm)

(b)(i) height OP

Base diagonal AC = $8\sqrt{2}$; PC = half-diagonal = $4\sqrt{2}$, so $PC^2 = 32$.

$$OP^2 = OC^2 - PC^2 = 100 - 32 = 68, OP = \sqrt{68}.$$

Answer: OP ≈ 8.25 cm

(b)(ii) angle between edge OC and the base

This is $\angle OCP$, where $\cos \angle OCP = PC/OC = 4\sqrt{2}/10 = 0.5657$.

$$\angle OCP = \cos^{-1}(0.5657) \text{ (or } \tan^{-1}(OP/PC) = \tan^{-1}(8.25/5.657)).$$

Answer: Angle ≈ 55.5°

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