

G12 ECZ MATHEMATICS 2016 SPECIMEN PAPER 2 SOLUTIONS

Grade 12 ECZ Mathematics 2016 Specimen Paper 2 (4024/2)

Worked Solutions with Step-by-step Explanations

The full question paper appears first — worked answers are at the end.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Mathematics

4024/2

Paper 2

day

date OCTOBER 2016

Additional materials:

- Answer Booklet
- Silent Electronic Calculator (non programmable)
- Geometrical instruments
- Graph paper (3 sheets)
- Mathematical tables (optional)
- Plain paper (1 sheet)

Time: 2 hours 30 Minutes

Instructions to Candidates

Write your name, centre number and candidate number in the spaces provided on the Answer Booklet.

Write your answers and working in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Omission of essential working will result in loss of marks.

There are twelve (12) questions in this paper.

Section A

Answer **all** questions.

Section B

Answer **any four** questions.

Silent non programmable Calculators or Mathematical tables may be used.

Cell phones should not be brought into the examination room.

Information for Candidates

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 100.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Geometric Progression

$$S_n = \frac{a(1-r^n)}{1-r}, (r < 1)$$

$$S_n = \frac{a(1-r^n)}{1-r}, (r > 1)$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \text{SD} = \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{\sum x^2}{n} - (\bar{x})^2 \right\}}$$

Grouped data

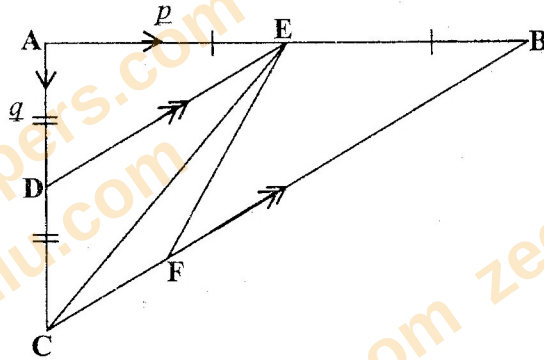
$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \text{SD} = \sqrt{\left\{ \frac{\sum f(x - \bar{x})^2}{\sum f} \right\}} = \sqrt{\left\{ \frac{\sum fx^2}{\sum f} - (\bar{x})^2 \right\}}$$

Section A

Answer all questions in this section

- 1 (a) If matrix $P = \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 12 & 4 \\ -9 & m \end{pmatrix}$, find
- (i) the value of m for which the determinants of P and Q are equal. [2]
 - (ii) the inverse of Q . [2]
- (b) At Sankani School, there are 60 learners in Grade 12. A survey was carried out to determine how many of these learners liked Netball, Football or Volleyball and the following were the findings:-
- 5 liked all the three
 - 8 liked Football only
 - 4 liked volleyball only
 - 3 liked Netball only
 - 12 liked Football and Netball
 - 21 liked Football and Volleyball
 - 18 liked Netball and Volleyball
- (i) Illustrate this information in a Venn diagram. [2]
 - (ii) How many learners did not like any of the three games? [1]
 - (iii) How many learners did not like volleyball? [1]
 - (iv) How many learners liked two games only? [1]
-
- 2 (a) There are 6 girls and 4 boys in a drama club. Two members are chosen at random to represent the club at a meeting. What is the probability that
- (i) both members are boys, [2]
 - (ii) one member is a girl? [3]

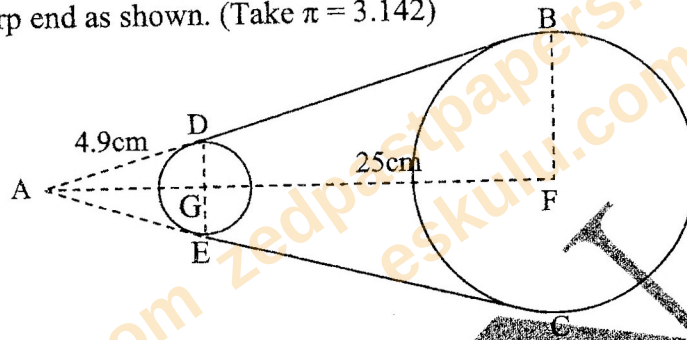
- (b) In the diagram below, $\vec{AE} = p$, $\vec{AD} = q$, $AE = EB$, $AD = DC$, DE is parallel to CB and $CF : FB = 1 : 3$.



- (i) Express in terms of p and/or q
- $\vec{DE}$, [1]
 - $\vec{EC}$, [1]
 - $\vec{CB}$. [1]
- (ii) Show that $\vec{EF} = \frac{1}{2}(3q - p)$. [2]

- 3
- Construct a triangle ABC in which $AB = 6\text{cm}$, $\angle ABC = 120^\circ$ and $AC = 11\text{cm}$. [1]
 - Measure and write the length BC . [2]
 - Within the triangle, draw the locus of points which are
 - equidistant from CA and CB , [1]
 - 4cm from C . [1]
 - A point P , within triangle ABC , is such that it is 4cm from C and equidistant from CA and CB . Label the point P . [2]
 - Another point R , within triangle ABC , is such that it is nearer to BC than to AC and is less than 4cm from C . Indicate, by shading, the region in which R must lie.

- 4 (a) Solve the equation $3x^2 + 5x + 2 = 0$, giving your answers correct to 2 decimal places. [5]
- (b) The figure below is that of a cone ABC from which a Vuvuzela DBCE was made by cutting off its sharp end as shown. (Take $\pi = 3.142$)

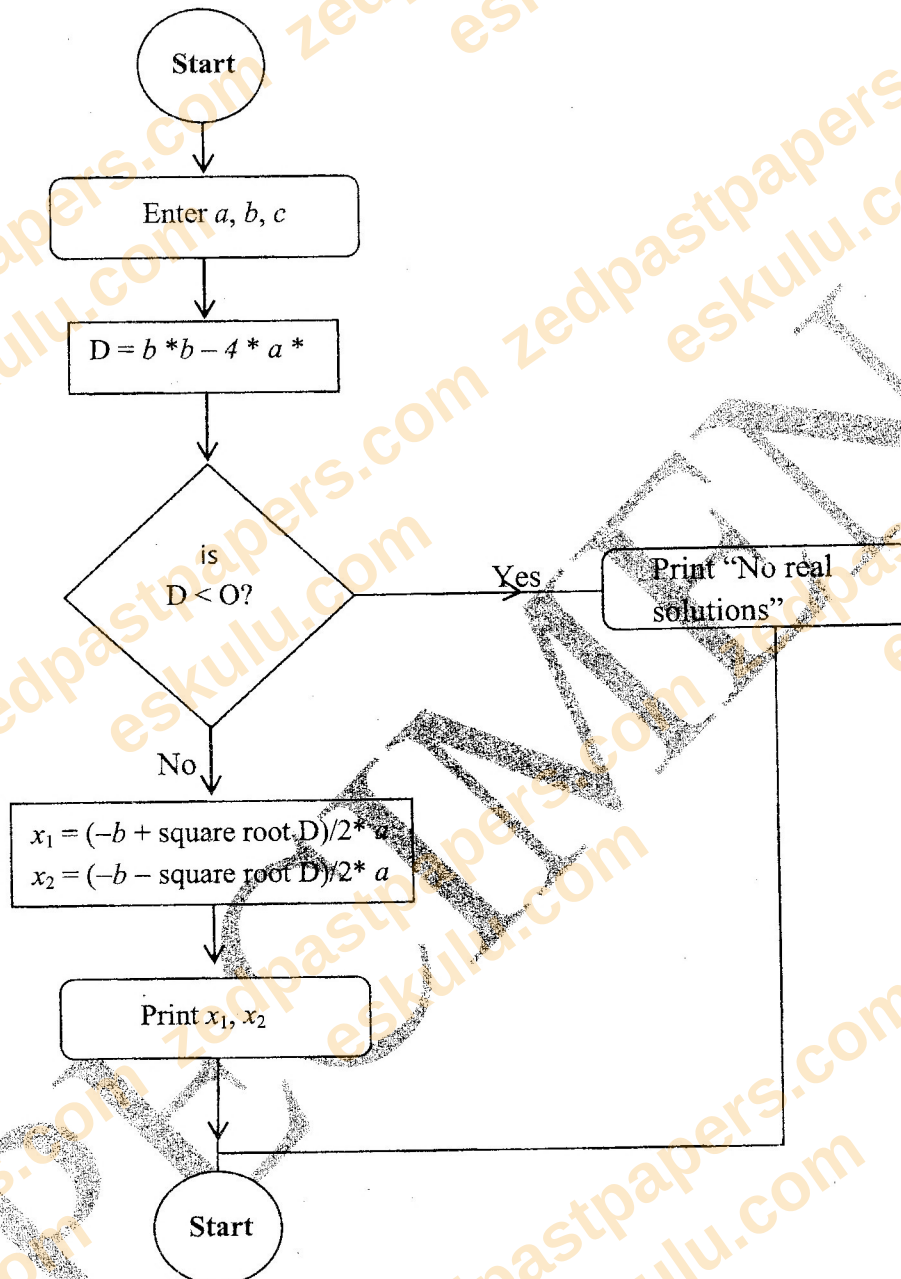


Given that $AF = 30\text{cm}$, $GF = 25\text{cm}$, $DG = 1\text{cm}$ and $AD = 4.9\text{cm}$, calculate

- (i) the curved surface area of the piece that was cut off, [2]
- (ii) the volume of the Vuvuzela. [4]

- 5 (a) The 3rd and 4th terms of a geometric progression are 4 and 8 respectively. Find
- (i) the first term and the common ratio, [3]
- (ii) the sum of the first 10 terms, [2]
- (iii) the sum to infinity of this geometric progression. [2]
- (b) Simplify $\frac{13k}{20a^2} \times \frac{5}{39k}$ [2]

6 Study the flowchart below.



Write a pseudo code corresponding to the flowchart program above.

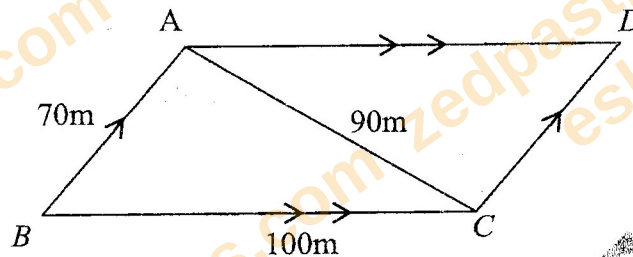
[5]

Section B [48 marks]

Answer any four questions in this section.

Each question in this section carries 12 marks

- 7 (a) A farm is in the shape of a parallelogram ABCD. To accommodate two different crops, the farm is divided into two parts as shown in the diagram below.



Given that $AB = 70\text{m}$, $AC = 90\text{m}$ and $BC = 100\text{m}$, calculate

- (i) angle BAC, [5]
 (ii) the area of triangle ABC, [3]
 (iii) the shortest distance from D to AC. [2]
 (b) Solve the equation $\sin \theta = 0.766$ for $0^\circ \leq \theta \leq 180^\circ$. [2]

- 8 The table below shows the number of days that 52 learners in a Grade 12 class at Masambililo School were absent in one school term.

Days absent	0	1	2	3	4	5	6	7	9
Frequency	2	3	6	7	6	9	11	6	2

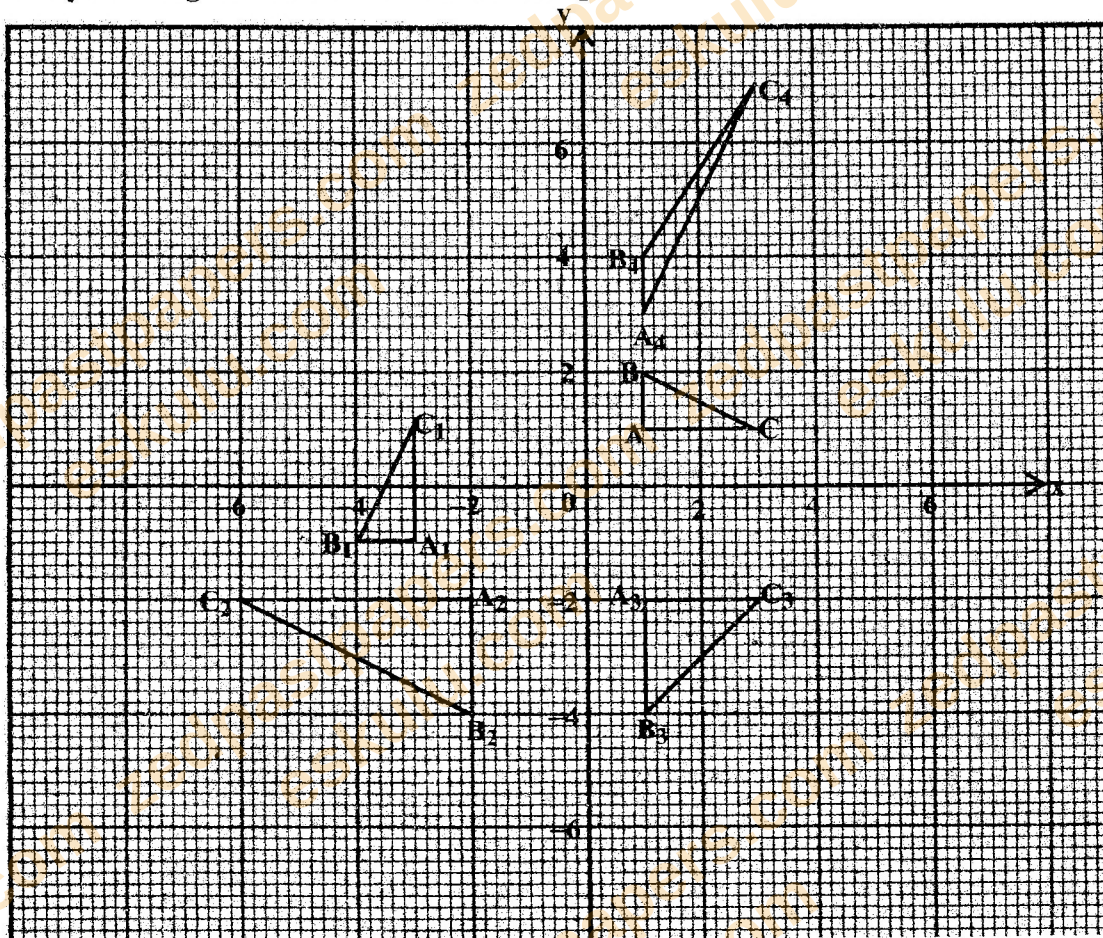
- (a) Calculate the standard deviation [6]
 (b) Answer this part of the question on a sheet of graph paper.

- (i) Using the table above, copy and complete the relative cumulative frequency table below.

Days absent	0	1	2	3	4	5	6	7	9
Cumulative Frequency	2	5	11	18	24	33	44	50	52
Relative cumulative frequency	0.04	0.10	0.21	0.35	0.46				

- (ii) Using a scale of 1cm to represent 1 unit on each axis, draw a smooth relative cumulative frequency curve. [3]
 (iii) Showing your method clearly, use your graph to estimate the 50th percentile. [2]

9 Study the diagram below and answer the questions that follow.



- (a) Triangle ABC is mapped onto triangle $A_1B_1C_1$ by a single transformation. Describe fully this transformation. [3]
- (b) Triangle ABC is mapped onto triangle $A_2B_2C_2$ by an enlargement. Find the
 (i) centre of enlargement, [1]
 (ii) scale factor of an enlargement. [1]
- (c) Triangle ABC is mapped onto triangle $A_3B_3C_3$ by a single transformation find
 (i) matrix representing this transformation, [2]
 (ii) the area scale factor of the transformation. [2]
- (d) Triangle ABC is mapped onto triangle $A_4B_4C_4$ by a single transformation. Describe fully this transformation. [3]

- 10 (a) The values of x and y are connected by the equation $y = x^3 - 6x + 2$ as shown in the table below.

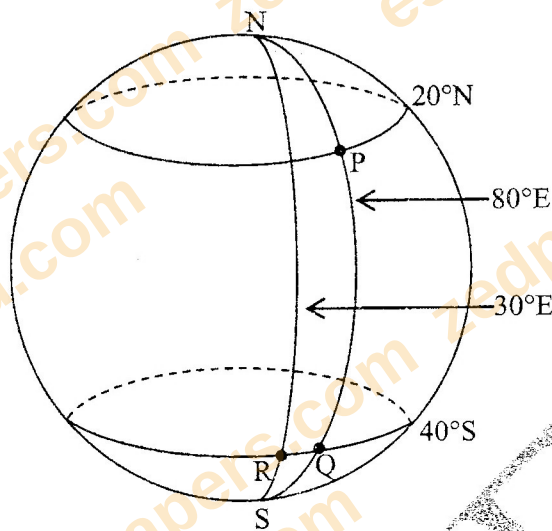
x	-3	-2	-1	0	1	2	3
y	-7	6	7	2	-3	-2	k

- (i) Calculate the value of k . [1]
- (ii) Using a scale of 2cm to represent 1 unit on the x -axis and 1 cm to represent 2 units on the y -axis for $-3 \leq x \leq 3$ and $-8 \leq y \leq 12$, draw the graph of $y = x^3 - 6x + 2$. [3]
- (iii) Use your graph to solve the equation $x^3 - 6x + 2 = 0$. [2]
- (iv) (a) On the same axes, draw the graph of $y = 2x - 2$. [1]
- (b) Hence or otherwise, solve the equation $x^3 - 6x + 2 = 2x - 2$. [2]
- (b) Express $\frac{3}{x+1} - \frac{2}{x-3}$ as a single fraction in its lowest terms. [3]

- 11 Makeke makes two types of cakes for sale namely: Xlent and Yammy. She wants to make not more than 30 cakes in total. She decides that the number of Xlent cakes should not exceed 15 while Yammy cakes should be less than or equal to Xlent cakes. She has at least K300.00 to spend on this project. Each Xlent cake costs K60.00 to make and each Yammy cake costs K30.00 to make.

- (a) Given that x represents Xlent cakes and y represents Yammy cakes, write four inequalities that satisfy these conditions. [4]
- (b) Using a scale of 2cm to represent 5 units on both axes, present this information in graphical form by shading the unwanted regions. [4]
- (c) Given that Makeke makes K20.00 profit on each Xlent cake and K12.00 profit on each Yammy cake, how many of each type should she make to get maximum profit? [2]
- (d) Estimate the maximum profit. [2]

- 12 (a) The diagram below is a sketch of the earth and on it are the points $P(20^\circ\text{N}, 80^\circ\text{E})$, $Q(40^\circ\text{S}, 80^\circ\text{E})$ and $R(40^\circ\text{S}, 30^\circ\text{E})$. [Use $\pi = 3.142$ and $R = 6370\text{km}$]



- (i) Calculate the distance QR in kilometres. [2]
- (ii) An aeroplane starts from P and flies due west on the same latitude covering a distance of 1 232km to point T .
 - (a) Calculate the difference in angles between P and T , [2]
 - (b) Find the position of T . [2]
- (b) (i) Find the equation of the tangent to the curve $y = x^2 - 2x - 3$ at the point $(3, 0)$. [3]
- (ii) A curve is such that $\frac{dy}{dx} = 15x^2 - 12x$. Given that it passes through $(1, 3)$, find its equation. [3]

Question 1. (a) $P = \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 12 & 4 \\ -9 & m \end{pmatrix}$. Find (i) m so $\det P = \det Q$, (ii) the inverse of Q . (b) Sankani School, 60 learners (Netball N, Football F, Volleyball V): all three 5; F only 8; V only 4; N only 3; F and N 12; F and V 21; N and V 18. (i) Venn diagram. (ii) Liked none. (iii) Did not like volleyball. (iv) Liked two games only.

(a)(i) value of m

$$\det P = 4 \times 3 - 2 \times 0 = 12. \det Q = 12m - 4(-9) = 12m + 36. \text{ Set } 12m + 36 = 12.$$

Answer: $m = -2$

(a)(ii) inverse of Q

$$Q = \begin{pmatrix} 12 & 4 \\ -9 & -2 \end{pmatrix}, \det Q = 12. Q^{-1} = \frac{1}{12} \begin{pmatrix} -2 & -4 \\ 9 & 12 \end{pmatrix}.$$

Answer: $Q^{-1} = \begin{pmatrix} -1/6 & -1/3 \\ 3/4 & 1 \end{pmatrix}$

(b) Venn region counts

$$\text{All three} = 5. F \cap N \text{ only} = 12 - 5 = 7; F \cap V \text{ only} = 21 - 5 = 16; N \cap V \text{ only} = 18 - 5 = 13.$$

$$\text{With N only 3, F only 8, V only 4: total inside} = 3 + 8 + 4 + 5 + 7 + 16 + 13 = 56.$$

(b)(ii) liked none

$$60 - 56.$$

Answer: 4 learners

(b)(iii) did not like volleyball

$$\text{Liked volleyball} = 4 + 16 + 13 + 5 = 38. \text{ Not volleyball} = 60 - 38.$$

Answer: 22 learners

(b)(iv) liked two games only

$$F \cap N \text{ only} + F \cap V \text{ only} + N \cap V \text{ only} = 7 + 16 + 13.$$

Answer: 36 learners

Question 2. (a) 6 girls and 4 boys; two chosen at random. P that (i) both are boys, (ii) one member is a girl. (b) $AE = p$, $AD = q$, $AE = EB$, $AD = DC$, $DE \parallel CB$, $CF : FB = 1 : 3$. (i) Express in terms of p and/or q : (a) DE , (b) EC , (c) CB . (ii) Show that $EF = \frac{1}{2}(3q - p)$.

(a)(i) both boys

$$P = \frac{(4/10)(3/9)}{1} = 12/90.$$

Answer: $2/15$

(a)(ii) exactly one girl

$$\text{girl-boy or boy-girl} = 2 \times \frac{(6/10)(4/9)}{1} = 48/90.$$

Answer: $8/15$

(b)(i)(a) DE

$$DE = AE - AD = p - q.$$

Answer: $DE = p - q$

(b)(i)(b) EC

$AD = DC$ so $AC = 2q$. $EC = AC - AE = 2q - p$.

Answer: $EC = 2q - p$

(b)(i)(c) CB

$AE = EB$ so $AB = 2p$. $CB = AB - AC = 2p - 2q$.

Answer: $CB = 2p - 2q$

(b)(ii) show $EF = \frac{1}{2}(3q - p)$

$CF : FB = 1 : 3$ so $CF = \frac{1}{4}CB = \frac{1}{4}(2p - 2q) = \frac{1}{2}(p - q)$.

$EF = EC + CF = (2q - p) + \frac{1}{2}(p - q) = 2q - p + \frac{1}{2}p - \frac{1}{2}q = \frac{3}{2}q - \frac{1}{2}p$.

Answer: $EF = \frac{1}{2}(3q - p)$ ✓

Question 3. (a) Construct triangle ABC with $AB = 6$ cm, $\angle ABC = 120^\circ$, $AC = 11$ cm. (b) Measure BC. (c) Draw the locus (i) equidistant from CA and CB, (ii) 4 cm from C. (d) Mark P: 4 cm from C and equidistant from CA and CB. (e) Shade the region R: nearer BC than AC and less than 4 cm from C.

(a) construction

Draw $AB = 6$ cm; construct $\angle ABC = 120^\circ$ at B; the ray from B and an arc of radius 11 cm from A meet at C.

(b) length BC

Cosine rule check: $11^2 = 6^2 + BC^2 - 2(6)(BC)\cos 120^\circ \Rightarrow BC^2 + 6BC - 85 = 0 \Rightarrow BC = (-6 + \sqrt{376})/2$.

Answer: $BC \approx 6.7$ cm

(c)(i) equidistant from CA and CB

Answer: Bisector of $\angle ACB$ (angle at C)

(c)(ii) 4 cm from C

Answer: Arc / circle of radius 4 cm centred at C

(d) point P

Answer: P = intersection of the $\angle C$ bisector with the 4 cm arc about C

(e) region R

Nearer BC than AC $\rightarrow$ BC-side of the $\angle C$ bisector; less than 4 cm from C $\rightarrow$ inside the arc. Shade the overlap.

Answer: Shade the overlap: BC-side of $\angle C$ bisector AND inside the 4 cm arc about C

Question 4. (a) Solve $3x^2 + 5x + 2 = 0$, correct to 2 decimal places. (b) Cone ABC (apex A) with the small end cut off to make a vuvuzela DBCE. $AF = 30$ cm, $GF = 25$ cm, $DG = 1$ cm, $AD = 4.9$ cm ($\pi = 3.142$). Find (i) the curved surface area of the small piece cut off, (ii) the volume of the vuvuzela.

(a) solve $3x^2 + 5x + 2 = 0$

Factorise: $(3x + 2)(x + 1) = 0$.

Answer: $x = -0.67$ or $x = -1.00$

(b) dimensions

$AG = AF - GF = 30 - 25 = 5$ cm. Small cone ADE: base radius $DG = 1$, height $AG = 5$, slant $AD = 4.9$.

Similar cones: big-cone base radius $BF / AF = DG / AG \Rightarrow BF = 30 \times (1/5) = 6$ cm; big height = $AF = 30$.

(b)(i) CSA of the piece cut off (small cone)

$CSA = \pi \times r \times l = 3.142 \times 1 \times 4.9$.

Answer: $\approx 15.4 \text{ cm}^2$

(b)(ii) volume of the vuvuzela

Big cone = $\frac{1}{3}\pi(6^2)(30) = \frac{1}{3} \times 3.142 \times 36 \times 30 = 1131.1 \text{ cm}^3$.

Small cone = $\frac{1}{3}\pi(1^2)(5) = 5.24 \text{ cm}^3$. Vuvuzela = $1131.1 - 5.24$.

Answer: $\approx 1130 \text{ cm}^3$ (1126 cm^3)

Question 5. (a) The 3rd and 4th terms of a GP are 4 and 8. Find (i) the first term and common ratio, (ii) the sum of the first 10 terms, (iii) the sum to infinity. (b) Simplify $(13k/20a^2) \times (5/39k)$.

(a)(i) first term and common ratio

$r = 8/4 = 2$. $ar^2 = 4 \Rightarrow a(4) = 4 \Rightarrow a = 1$.

Answer: first term $a = 1$, common ratio $r = 2$

(a)(ii) sum of first 10 terms

$S_{10} = a(r^{10} - 1)/(r - 1) = 1(2^{10} - 1)/(2 - 1) = 1023/1$.

Answer: $S_{10} = 1023$

(a)(iii) sum to infinity

Sum to infinity exists only when $|r| < 1$. Here $r = 2$, so the series diverges.

Answer: No sum to infinity (it does not converge)

(b) simplify

$(13k \times 5) / (20a^2 \times 39k) = 65k / (780 a^2 k)$. Cancel k and $65/780 = 1/12$.

Answer: $1/(12a^2)$

Question 6. Study the flowchart (quadratic-formula program): Start; Enter a, b, c; $D = b^2 - 4ac$; is $D < 0$? Yes $\rightarrow$ Print 'No real solutions'; No $\rightarrow x_1 = \frac{-b + \sqrt{D}}{2a}$, $x_2 = \frac{-b - \sqrt{D}}{2a}$; Print x_1 , x_2 ; Stop. Write a pseudocode corresponding to the flowchart.

pseudocode

Start

Enter a, b, c

$D = b^2 - 4ac$

If $D < 0$ then

Print "No real solutions"

Else

$x_1 = \frac{-b + \sqrt{D}}{2a}$

$x_2 = \frac{-b - \sqrt{D}}{2a}$

Print x_1 , x_2

End if

Stop

Answer: Input a,b,c $\rightarrow$ compute $D = b^2 - 4ac \rightarrow$ if $D < 0$ print "No real solutions", else compute and print x_1 , $x_2 \rightarrow$ Stop

Question 7. (a) Parallelogram farm ABCD with $AB = 70$ m, $AC = 90$ m, $BC = 100$ m. Calculate (i) angle BAC, (ii) area of triangle ABC, (iii) the shortest distance from D to AC. (b) Solve $\sin \theta = 0.766$ for $0^\circ \leq \theta \leq 180^\circ$.

(a)(i) angle BAC

Cosine rule: $\cos(BAC) = \frac{(70^2 + 90^2 - 100^2)}{(2 \cdot 70 \cdot 90)} = \frac{3000}{12600} = 0.2381$.

Answer: $\hat{BAC} \approx 76.2^\circ$

(a)(ii) area of triangle ABC

Area = $\frac{1}{2} \times 70 \times 90 \times \sin 76.2^\circ = 3150 \times 0.9712$.

Answer: $\approx 3060 \text{ m}^2$

(a)(iii) shortest distance from D to AC

Diagonal AC splits the parallelogram into equal triangles, so area ACD = area ABC = 3059 m^2 .

Distance = $2 \times \text{area} \div AC = 2 \times 3059 \div 90$.

Answer: $\approx 68.0 \text{ m}$

(b) solve $\sin \theta = 0.766$

$\theta = \sin^{-1}(0.766) = 50.0^\circ$; second solution $180^\circ - 50^\circ = 130^\circ$.

Answer: $\theta = 50.0^\circ$ or 130.0°

Question 8. Days absent for 52 learners — Days: 0,1,2,3,4,5,6,7,9; Frequency: 2,3,6,7,6,9,11,6,2. (a) Calculate the standard deviation. (b)(i) Complete the relative cumulative frequency table. (ii) Draw the relative cumulative frequency curve. (iii) Estimate the 50th percentile.

(a) standard deviation

$$\Sigma f = 52. \Sigma fx = 0+3+12+21+24+45+66+42+18 = 231 \Rightarrow \text{mean} = 231/52 = 4.44.$$

$$\Sigma fx^2 = 0+3+24+63+96+225+396+294+162 = 1263.$$

$$SD = \sqrt{(1263/52 - 4.44^2)} = \sqrt{(24.29 - 19.74)} = \sqrt{4.55}.$$

Answer: Standard deviation $\approx$ 2.13 days

(b)(i) relative cumulative frequency

Cumulative: 2,5,11,18,24,33,44,50,52. Divide each by 52:

Answer: 0.04, 0.10, 0.21, 0.35, 0.46, 0.63, 0.85, 0.96, 1.00

(b)(ii) curve

Answer: Smooth rising curve from (0,0.04) to (9,1.00)

(b)(iii) 50th percentile (relative cf = 0.50)

0.50 lies between days 4 (0.46) and 5 (0.63): $4 + (0.50 - 0.46)/(0.63 - 0.46)$.

Answer: $\approx$ 4.2 days

Question 9. Triangle ABC and its images (coordinates read from the embedded grid). (a) $ABC \rightarrow A'B'C'$ by a single transformation: describe it fully. (b) $ABC \rightarrow A'B'C'$ by an enlargement: find (i) the centre, (ii) the scale factor. (c) $ABC \rightarrow A'B'C'$ by a single transformation: find (i) the matrix, (ii) the area scale factor. (d) $ABC \rightarrow A'B'C'$ by a single transformation: describe it fully.

(a) $A'B'C'$ (from the grid)

$A'B'C'$ is congruent to ABC. Compare orientation: unchanged $\Rightarrow$ translation (give vector); reversed $\Rightarrow$ reflection (give mirror line) or rotation (give centre and angle).

Answer: Congruent single transformation — state type, vector/line/centre as read from the grid

(b)(i) centre of enlargement

Join each vertex to its image and produce; the lines meet at the centre.

Answer: Centre = intersection of the vertex-image lines (read from grid)

(b)(ii) scale factor

Scale factor = image length $\div$ object length (sign shows direction).

Answer: k = image/object (e.g. -2 or 2 — read from grid)

(c)(i) matrix for $A'B'C'$

Find the images of the unit/base vectors (or solve $M \cdot (\text{object}) = \text{image}$ for two vertices) to build the 2×2 matrix.

Answer: 2×2 matrix from the mapped vertices (read from grid)

(c)(ii) area scale factor

Area scale factor = |determinant of the matrix| = |image area ÷ object area|.

Answer: Area factor = |det M|

(d) A■B■C■ (from the grid)

Compare size and orientation with ABC; describe (enlargement / rotation / reflection) with its full parameters.

Answer: State the single transformation with full parameters (read from the grid)

Question 10. $y = x^3 - 6x + 2$. Table $x: -3, -2, -1, 0, 1, 2, 3 \rightarrow y: -7, 6, 7, 2, -3, -2, k$. (i) Find k. (ii) Draw the graph (2 cm to 1 unit on x, 1 cm to 2 units on y). (iii) Solve $x^3 - 6x + 2 = 0$. (iv)(a) Draw $y = 2x - 2$. (b) Solve $x^3 - 6x + 2 = 2x - 2$. (b) Express $3/(x + 1) - 2/(x - 3)$ as a single fraction.

(a)(i) value of k

At $x = 3$: $y = 27 - 18 + 2$.

Answer: $k = 11$

(a)(ii) graph

Answer: Smooth cubic through $(-3, -7), (-2, 6), (-1, 7), (0, 2), (1, -3), (2, -2), (3, 11)$

(a)(iii) solve $x^3 - 6x + 2 = 0$

Read the x-values where the curve cuts the x-axis.

Answer: $x \approx -2.6, 0.3, 2.3$

(a)(iv)(a) line $y = 2x - 2$

Answer: Straight line through $(0, -2)$ and $(3, 4)$

(a)(iv)(b) solve $x^3 - 6x + 2 = 2x - 2$

Intersections of the curve and the line; algebraically $x^3 - 8x + 4 = 0$.

Answer: $x \approx 0.5$ and $x \approx 2.5$ (and ≈ -3.1 near the edge)

(b) single fraction

$[3(x - 3) - 2(x + 1)] / [(x + 1)(x - 3)] = (3x - 9 - 2x - 2) / [(x + 1)(x - 3)]$.

Answer: $(x - 11) / [(x + 1)(x - 3)]$

Question 11. Makeke makes Xlent (x) and Yammy (y) cakes: total not more than 30; Xlent not more than 15; Yammy $\leq$ Xlent; she spends at least K300 (Xlent K60 each, Yammy K30 each). (a) Write four inequalities. (b) Graph and shade the unwanted regions. (c) Profit K20 per Xlent, K12 per Yammy: how many of each for maximum profit? (d) Estimate the maximum profit.

(a) four inequalities

Total: $x + y \leq 30$. Xlent limit: $x \leq 15$. Yammy $\leq$ Xlent: $y \leq x$.

Spending at least K300: $60x + 30y \geq 300 \Rightarrow 2x + y \geq 10$.

Answer: $x + y \leq 30$, $x \leq 15$, $y \leq x$, $2x + y \geq 10$ (with $x, y \geq 0$)

(b) graph

Draw the four boundary lines; shade out the unwanted side of each so the feasible region remains.

Answer: Feasible region bounded by $x = 15$, $y = x$, $x + y = 30$, $2x + y = 10$

(c) numbers for maximum profit

$P = 20x + 12y$ rises with x and y ; the top corner is $x = 15$, $y = 15$ (on $x \leq 15$ and $y = x$, with $x + y = 30$).

Answer: 15 Xlent and 15 Yammy cakes

(d) maximum profit

$P = 20(15) + 12(15) = 300 + 180$.

Answer: Maximum profit = K480.00

Question 12. (a) $P(20^\circ\text{N}, 80^\circ\text{E})$, $Q(40^\circ\text{S}, 80^\circ\text{E})$, $R(40^\circ\text{S}, 30^\circ\text{E})$, $\pi = 3.142$, $R = 6370$ km. (i) Distance QR (km). (ii) Aeroplane flies due west from P along the same latitude 1 232 km to T: (a) the change in longitude $P \rightarrow T$, (b) the position of T. (b)(i) Equation of the tangent to $y = x^2 - 2x - 3$ at $(3, 0)$. (ii) $dy/dx = 15x^2 - 12x$ and the curve passes through $(1, 3)$; find its equation.

(a)(i) distance QR

Q and R share latitude 40°S ; longitude difference = $80^\circ - 30^\circ = 50^\circ$.

$QR = (50/360) \times 2\pi R \cos 40^\circ = (50/360) \times 2 \times 3.142 \times 6370 \times 0.766$.

Answer: $\approx 4\,260$ km

(a)(ii)(a) change in longitude $P \rightarrow T$

$1232 = (\theta/360) \times 2\pi R \cos 20^\circ = (\theta/360) \times 40029 \times 0.9397 = (\theta/360) \times 37615$.

$\theta = 1232 \times 360 / 37615$.

Answer: $\theta \approx 11.8^\circ$

(a)(ii)(b) position of T

Flying west from 80°E : longitude = $80^\circ - 11.8^\circ = 68.2^\circ\text{E}$; latitude unchanged.

Answer: $T(20^\circ\text{N}, 68.2^\circ\text{E})$

(b)(i) tangent at $(3, 0)$

$dy/dx = 2x - 2$; at $x = 3$ gradient = 4. $y - 0 = 4(x - 3)$.

Answer: $y = 4x - 12$

(b)(ii) equation of the curve

$y = \int (15x^2 - 12x) dx = 5x^3 - 6x^2 + C$. At $(1, 3)$: $3 = 5 - 6 + C \Rightarrow C = 4$.

Answer: $y = 5x^3 - 6x^2 + 4$

Disclaimer

Created by eskulu Past Papers Tutor

These solutions were created using artificial intelligence and checked against ECZ marking guidelines. If you find an error, report it at eskulu.com/feedback.

eskulu.com | zedpastpapers.com