G12 ECZ ADDITIONAL MATHEMATICS 2021 GCE PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2021 GCE Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

4030-2-000339

Additional Mathematics

Paper 2

4030/2

2021

Additional Materials:

Answer Booklet;
Silent electronic calculator (non programmable)

Time: 2 hours 30 Minutes **Marks: 100**

Instructions to Candidates

1 Write the **centre number** and your **examination number** on **every page** of the separate **answer booklet** provided.

2 There are **twelve** questions in this paper. Answer **all** questions.

3 Write your answers in the separate Answer Booklet provided.

4 If you use more than one Answer Booklet, fasten the Answer Booklets together.

5 Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for Candidates

1 The number of marks is shown in brackets [ ] at the end of each question or part question.

2 **The use of a non programmable electronic calculator is expected, where appropriate.**

3 You are reminded of the need for clear presentation in your answers.

4 Cell phones and other electronic devices are **not allowed** in the examination room.

5 Check the formulae overleaf.

Mathematical Formulae

1 **ALGEBRA**

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 **SERIES**

Arithmetic $S_n = \frac{1}{2} n\,[2a + (n-1)\,d]$

Geometric $S_n = \frac{a(1-r^n)}{1-r}\ (r \neq 1)$

$S\infty = \frac{a}{1-r}$ for $|r| < 1$

3 **TRIGONOMETRY**

Identities

$\text{Sin}\,(A \pm B) = \sin A \cos B \pm \cos A \sin B.$

$\text{Cos}\,(A \pm B) = \cos A \cos B \mp \sin A \sin B.$

$\text{Tan}\,(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\text{Sin}\,2A = 2 \sin A \cos B$

$\text{Cos}\,2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\text{Tan}\,2A = \frac{2 \tan A}{1 - \tan^2 A}$

4 **STATISTICS**

Mean and standard deviation

Ungrouped data

$\text{Mean}\,(\bar{x}) = \frac{\Sigma x}{n}$, $\text{SD} = \sqrt{\left\{\frac{\Sigma(x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\Sigma x^2}{n} - (\bar{x})^2\right\}}$

Grouped data

$\text{Mean}\,(\bar{x}) = \frac{\Sigma fx}{\Sigma f}$, $\text{SD} = \sqrt{\left\{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}\right\}} = \sqrt{\left\{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2\right\}}$

1 Solve the following systems of equations

$2x + 3y + z = 17,$

$x - 3y + 2z = -8,$

$5x - 2y + 3z = 5.$ [6]

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2 **(a)** Find the range of values of u for which $3u^2 - 8u < 3$. [3]

(b) Express $-3x^2 + 24x - 38$ in the form $a(x + b)^2 + c$, where a, b and c are constants. Hence, find the coordinates of the turning point. [4]

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3 Solve the equations

(a) $13^{3x-4} = \left(\frac{1}{4}\right)^{-3}$, [3]

(b) $\log_3 (x + 25) = 2 + \log_3 (2x - 1)$. [4]

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4 **(a)** Find the value of q, given that the expression $x^3 + 3x^2 + qx - 24$ is divisible by $(x - 3)$. [3]

(b) Solve the equation $x^3 - 7x + 6 = 0$. [4]

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5 **(a)** A group of 5 children consists of 2 girls whose names are Mary and Sarah, and 3 boys whose names are John, Joseph and James.
If these children sit in a straight line on 5 chairs, find the number of different possible seating arrangements if

(i) Joseph must sit in the middle, [2]

(ii) Mary and John must sit together. [3]

(b) A group of 6 designers is to be chosen from 8 men and 5 women. Find the number of ways of choosing at least 5 men. [3]

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6 **(a)** Solve the equation $3\sin(60° - x) = \sin x$ for values of x in the range $0° \leq x \leq 360°$. [4]

(b) **(i)** Express $4\cos x - \frac{5}{3}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence, find the maximum value of $12\cos x - 5\sin x$. [1]

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7 (a) In an arithmetic progression, the sum of the first 5 terms is 50 and the sum of the next 10 terms is 325. Find

(i) the first term and the common difference, [3]

(ii) the sum of the first 13 terms of the progression. [1]

(b) A doctor earns an annual salary of K4 000.00 during the first year of practice. In each succeeding year, the annual salary increases by 10%.

(i) Find the sum of the doctor's annual salaries over the first 6 years. [2]

(ii) Find the number of years the doctor must work, if the sum of his annual salaries is to exceed a million kwacha. [3]

8 The table below shows the heights of 80 plants in a particular forest.

Height (cm)	164 - 168	169 - 173	174 - 178	179 - 183	184 - 188
Frequency	8	18	28	21	5

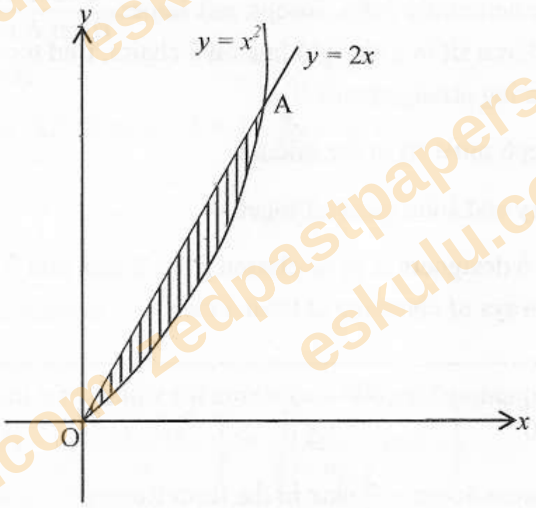
(a) Find the median class. [1]

(b) Calculate

(i) an estimate of the mean, [2]

(ii) the standard deviation. [6]

9 The following diagram shows part of the curve $y = x^2$ and the line $y = 2x$ meeting at A.



Find

(a) the coordinates of A, [3]

(b) the volume obtained by rotating the shaded region through 360° about the x-axis. [6]

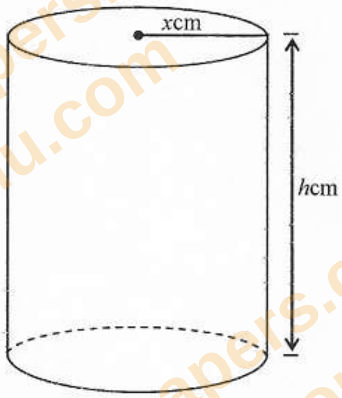
10 The velocity, Vm/s, of a particle moving in a straight line, at time t seconds is given by $V = 3t^2 - 2t + 2$. When $t = 0$, the particle is 3m from a fixed point O on the line.

(a) Find the acceleration when $t = 1.5$. [2]

(b) Find an expression in terms of t, for its distance from the fixed point. [4]

(c) Find the distance between $t = 1$ and $t = 2$. [4]

11 The following diagram shows a cylindrical metal plate of radius $x$cm and height, $h$cm.



(a) Given that its volume is 500cm^3, express h in terms of x. [2]

(b) Hence show that the total surface area (A) of the cylinder is given by

$A = 2\pi x^2 + \dfrac{1000}{x}$ cm^2. [3]

(c) Given that x varies, find the stationary value of A and show that this value is a minimum. [5]

12 The curve $y = e^{-2x} - 3$ meets the x-axis at P and the y-axis at Q.

(a) Find the coordinates of P and of Q. [3]

(b) Sketch the graph of $y = e^{-2x} - 3$ for the domain $-1 \le x \le 1$. [3]

(c) Find the equation of the straight line which must be drawn on the graph of $y = e^{-2x} - 3$ to obtain a solution of the equation $x = \ln\left(\dfrac{1}{\sqrt{x+5}}\right)$. [4]

Question 1. Solve the system of equations: $2x+3y+z=17$, $x-3y+2z=-8$, $5x-2y+3z=5$.

E1: $2x+3y+z=17$. E2: $x-3y+2z=-8$. E3: $5x-2y+3z=5$.

E1+E2: $3x+3z=9 \Rightarrow x+z=3 \Rightarrow z=3-x$ ...(A).

From E1: $3y=17-2x-z=17-2x-(3-x)=14-x \Rightarrow y=(14-x)/3$ ...(B).

Into E3: $5x-2(14-x)/3+3(3-x)=5$. Multiply by 3: $15x-2(14-x)+9(3-x)=15$.

$15x-28+2x+27-9x=15 \Rightarrow 8x-1=15 \Rightarrow x=2$.

$z=3-2=1$. $y=(14-2)/3=4$.

Check E2: $2-12+2=-8$ ✓ E3: $10-8+3=5$ ✓

Answer: $x = 2$, $y = 4$, $z = 1$

Question 2. Range of values and completing the square.

(a) $3u^2 - 8u < 3 \Rightarrow 3u^2-8u-3 < 0$. Factorise: $(3u+1)(u-3) < 0$.

Roots $u=-1/3$ and $u=3$; parabola opens up => negative between the roots.

Answer: (a) $-1/3 < u < 3$

(b) Express $-3x^2+24x-38$ in the form $a(x+b)^2+c$. Factor -3 from the x-terms:

$= -3(x^2-8x) - 38 = -3(x-4)^2 + 48 - 38 = -3(x-4)^2 + 10$.

So $a=-3$, $b=-4$, $c=10$. The coefficient of the square is negative => MAXIMUM.

Answer: (b) $-3(x-4)^2 + 10$; turning point = $(4, 10)$ [maximum]

Question 3. Solve the equations.

(a) $13^{3x-4} = (1/4)^{-3} = 4^3 = 64$.

Take lg: $(3x-4)\lg 13 = \lg 64 \Rightarrow 3x-4 = \lg 64/\lg 13 = 1.8062/1.1139 = 1.621$.

$3x = 5.621 \Rightarrow x = 1.874$.

Answer: (a) $x \approx 1.87$

(b) $\log_3(x+25) = 2 + \log_3(2x-1)$. Rearrange: $\log_3(x+25) - \log_3(2x-1) = 2$.

$\log_3((x+25)/(2x-1)) = 2 \Rightarrow (x+25)/(2x-1) = 9 \Rightarrow x+25 = 18x-9 \Rightarrow 17x = 34 \Rightarrow x = 2$.

(Check: $x+25=27>0$ and $2x-1=3>0$, valid.)

Answer: (b) $x = 2$

Question 4. Polynomial divisibility and a cubic equation.

(a) $x^3+3x^2+qx-24$ is divisible by $(x-3)$, so $f(3)=0$.

$27 + 27 + 3q - 24 = 0 \Rightarrow 30 + 3q = 0 \Rightarrow q = -10$.

Answer: (a) $q = -10$

(b) Solve x^3 - 7x + 6 = 0. Test x=1: 1-7+6=0 ✓ so (x-1) is a factor.

x^3-7x+6 = (x-1)(x^2+x-6) = (x-1)(x+3)(x-2).

Answer: (b) x = 1, x = 2, x = -3

Question 5. Arrangements and combinations.

(a) 5 children (Mary, Sarah, John, Joseph, James) on 5 chairs in a line.

(a)(i) Joseph must sit in the middle (chair 3). Joseph is fixed; the other 4 fill the remaining 4 chairs in 4! = 24 ways.

Answer: (a)(i) 24 arrangements

(a)(ii) Mary and John must sit together. Treat them as one block => 4 units to arrange: 4! ways, and the block can be internally ordered in 2! ways. Total = 4! * 2! = 24*2 = 48.

Answer: (a)(ii) 48 arrangements

(b) 6 designers from 8 men and 5 women with at least 5 men:

5 men, 1 woman: C(8,5)*C(5,1) = 56*5 = 280.

6 men, 0 women: C(8,6)*C(5,0) = 28*1 = 28.

Total = 280 + 28 = 308.

Answer: (b) 308 ways

Question 6. Trigonometry.

(a) 3 sin(60-x) = sin x, 0 <= x <= 360. Expand the left side:

3(sin60 cosx - cos60 sinx) = sinx => 3((sqrt3/2)cosx - (1/2)sinx) = sinx.

(3sqrt3/2)cosx - (3/2)sinx = sinx => (3sqrt3/2)cosx = (5/2)sinx => tan x = 3sqrt3/5 = 1.039.

x = arctan(1.039) = 46.1° or x = 46.1+180 = 226.1°.

Answer: (a) x ≈ 46.1° and 226.1°

(b)(i) Express 4cosx - (5/3)sinx as R cos(x+alpha) = R(cosx cosα - sinx sinα).

R cosα = 4, R sinα = 5/3. R = sqrt(16 + 25/9) = sqrt(169/9) = 13/3.

alpha = arctan((5/3)/4) = arctan(5/12) ≈ 22.6°.

Answer: (b)(i) 4cosx - (5/3)sinx = (13/3) cos(x + 22.6°)

(b)(ii) 12cosx - 5sinx = 3(4cosx - (5/3)sinx) = 3*(13/3)cos(x+22.6°) = 13 cos(x+22.6°).

Maximum when cos(x+22.6°)=1, giving 13.

Answer: (b)(ii) Maximum value of 12cosx - 5sinx = 13

Question 7. Arithmetic and geometric progressions (salary problem).

(a) AP: S5 = 50 and the sum of the NEXT 10 terms is 325 (so S15 - S5 = 325 => S15 = 375).

S5 = 5/2*(2a+4d) = 5(a+2d) = 50 => a+2d = 10 ...(1).

S15 = 15/2*(2a+14d) = 15(a+7d) = 375 => a+7d = 25 ...(2).

(2)-(1): 5d = 15 => d = 3. a = 10-6 = 4.

Answer: (a)(i) First term a = 4, common difference d = 3

S13 = 13/2*(2(4)+12(3)) = 13/2*(8+36) = 13/2*44 = 286.

Answer: (a)(ii) Sum of first 13 terms = 286

(b) GP salary: first year a = K4000, ratio r = 1.1 (10% annual increase).

(b)(i) S6 = a(r^6-1)/(r-1) = 4000(1.1^6-1)/0.1 = 4000(1.771561-1)/0.1 = 4000*7.71561.

Answer: (b)(i) Sum over first 6 years ≈ K30 862.44

(b)(ii) Need Sn > 1 000 000: 4000(1.1^n-1)/0.1 > 1000000 => 40000(1.1^n-1) > 1000000.

1.1^n - 1 > 25 => 1.1^n > 26 => n > lg26/lg1.1 = 1.41497/0.041393 = 34.18.

So the smallest whole number of years is n = 35.

Answer: (b)(ii) The doctor must work 35 years for the total to exceed K1 000 000

Question 8. Heights of 80 plants — frequency table.

Height(cm): 164-168 | 169-173 | 174-178 | 179-183 | 184-188

Frequency: 8 | 18 | 28 | 21 | 5 (n=80)

Midpoints: 166, 171, 176, 181, 186.

(a) Median: n/2 = 40. Cumulative: 8, 26, 54, ... the 40th value is in class 174-178.

Answer: (a) Median class = 174-178

(b)(i) Sigma(fx) = 8(166)+18(171)+28(176)+21(181)+5(186)

= 1328+3078+4928+3801+930 = 14065. Mean = 14065/80 ≈ 175.8.

Answer: (b)(i) Mean ≈ 175.8 cm

(b)(ii) Sigma(fx^2) = 8(27556)+18(29241)+28(30976)+21(32761)+5(34596)

= 220448+526338+867328+687981+172980 = 2475075.

Variance = 2475075/80 - (175.8125)^2 = 30938.44 - 30910.04 = 28.40. SD = sqrt(28.40) ≈ 5.33.

Answer: (b)(ii) Standard deviation ≈ 5.33 cm

Question 9. Curve y = x^2 and line y = 2x meeting at A.

(a) Intersections: x^2 = 2x => x^2-2x = 0 => x(x-2) = 0 => x=0 (origin) or x=2.

At x=2: y=4. So A = (2, 4).

Answer: (a) A = (2, 4)

(b) Shaded region between the line y=2x (outer) and curve y=x^2 (inner), x from 0 to 2.

(At x=1: line y=2 is above curve y=1, confirming the line is the outer boundary.)

V = pi * integral from 0 to 2 of [(2x)^2 - (x^2)^2] dx = pi * integral of (4x^2 - x^4) dx.

= pi*[4x^3/3 - x^5/5]_0^2 = pi*(32/3 - 32/5) = pi*(160/15 - 96/15) = pi*(64/15).

Answer: (b) Volume = 64*pi/15 cubic units

Question 10. Particle velocity V = 3t^2 - 2t + 2; at t=0 it is 3 m from O.

(a) Acceleration: a = dV/dt = 6t - 2. At t=1.5: a = 6(1.5)-2 = 9-2 = 7.

Answer: (a) Acceleration at t=1.5 s is 7 m/s^2

(b) Distance from O: s = integral V dt = t^3 - t^2 + 2t + C. At t=0, s=3 => C=3.

Answer: (b) s = t^3 - t^2 + 2t + 3

(c) V = 3t^2-2t+2 has discriminant 4-24 < 0, so V > 0 for all t (no reversal).

Distance from t=1 to t=2 = s(2) - s(1). s(2)=8-4+4+3=11. s(1)=1-1+2+3=5.

Distance = 11 - 5 = 6.

Answer: (c) Distance between t=1 and t=2 is 6 m

Question 11. Cylindrical metal plate: radius x cm, height h cm, volume 500 cm^3.

(a) Volume: pi*x^2*h = 500 => h = 500/(pi*x^2).

Answer: (a) h = 500 / (pi*x^2)

(b) Total surface area: A = 2*pi*x^2 + 2*pi*x*h = 2*pi*x^2 + 2*pi*x * 500/(pi*x^2).

= 2*pi*x^2 + 1000/x. Q.E.D.

Answer: (b) A = 2*pi*x^2 + 1000/x (proved)

(c) dA/dx = 4*pi*x - 1000/x^2 = 0 => 4*pi*x^3 = 1000 => x^3 = 250/pi => x ≈ 4.30 cm.

d^2A/dx^2 = 4*pi + 2000/x^3 > 0, so the stationary value is a MINIMUM.

A_min = 2*pi*(4.30)^2 + 1000/4.30 ≈ 116.4 + 232.4 ≈ 348.7 cm^2.

Answer: (c) Stationary (minimum) value of A ≈ 348.7 cm^2 (d^2A/dx^2 > 0 confirms minimum)

Question 12. Curve y = e^{-2x} - 3 meeting the x-axis at P and the y-axis at Q.

(a) P is on the x-axis (y=0): e^{-2x}-3 = 0 => e^{-2x} = 3 => -2x = ln3 => x = -(ln3)/2 ≈ -0.549.

Q is on the y-axis (x=0): y = e^0 - 3 = 1 - 3 = -2.

Answer: (a) P = (-(ln3)/2, 0) ≈ (-0.549, 0); Q = (0, -2)

(b) Sketch: y = e^{-2x}-3 is a decreasing exponential shifted down by 3.

At x=-1: y = e^2-3 ≈ 4.39. At x=0: y = -2. At x=1: y = e^{-2}-3 ≈ -2.86.

Curve falls steeply from (-1, 4.39), crosses the x-axis at x ≈ -0.549, and approaches y=-3.

Answer: (b) Decreasing curve through (-1, 4.39), (-0.549, 0) and (0, -2), tending to y = -3

(c) Solve x = ln(1/sqrt(x+5)) = -(1/2) ln(x+5). Multiply by -2: -2x = ln(x+5).

Exponentiate: e^{-2x} = x+5. Subtract 3: e^{-2x} - 3 = x + 2.

Since $y = e^{-2x}-3$, the required straight line is $y = x + 2$; its intersection with the curve gives the solution.

Answer: (c) Draw the straight line $y = x + 2$

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