G12 ECZ ADDITIONAL MATHEMATICS 2020 SPECIMEN PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2020 Specimen Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/2

Paper 2

Specimen

Additional Materials:

Answer Booklet;
Silent electronic calculator (non programmable)

Time: 2 hours 30 Minutes

Instructions to Candidates

Write your **name, centre number** and **candidate number** in the spaces on the separate Answer Booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the **Answer Booklet** provided.

If you use more than one Answer Booklet, **fasten** the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 SERIES

Arithmetic $S_n = \frac{1}{2} n\,[2a + (n-1)\,d]$

Geometric $S_n = \frac{a(1-r^n)}{1-r}\ (r \neq 1)$

$$S\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

Identities

$\text{Sin}\,(A \pm B) = \sin A \cos B \pm \cos A \sin B.$

$\text{Cos}\,(A \pm B) = \cos A \cos B \mp \sin A \sin B.$

$\text{Tan}\,(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\text{Sin}\,2A = 2 \sin A \cos B$

$\text{Cos}\,2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\text{Tan}\,2A = \frac{2 \tan A}{1 - \tan^2 A}$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean}\,(\bar{x}) = \frac{\sum x}{n},\ \text{SD} = \sqrt{\left\{\frac{\sum (x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean}\,(\bar{x}) = \frac{\sum fx}{\sum f},\ \text{SD} = \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

1 Solve the following systems of equations

$2x + 3y - z = 2,$

$x - 2y - 4z = 8,$

$3x + 5y + 2z = -7.$ [6]

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2 (a) Find the range of values of x for which $3x^2 - 10x - 8 < 0$. [3]

(b) Express $12x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. Hence, find the minimum value of $12x^2 - 6x + 5$. [4]

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3 Solve the equations

(a) $4^{3x-2} = 19$, [3]

(b) $\log_2(2x^2 + 3x + 5) = 3 + \log_2(x + 1)$. [4]

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4 (a) Find the value of k, given that the expression $3x^3 - 14x^2 - 7kx + 10$ is exactly divisible by $(x + 1)$. [3]

(b) Solve the equation $2x^3 - 3x^2 - 30x + 56 = 0$. [4]

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5 (a) In how many ways can the letters of the word 'UNGROUPED' be arranged? [2]

(b) A team of 5 people is to be selected from 7 women and 6 men. Find the number of different teams that could be selected if there must be more women than men in the team. [5]

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6 (a) Solve the equation $\cos(x + 60°) = 2\sin x$ for values of x in the range $0° \le x \le 360°$. [4]

(b) (i) Express $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence, find the maximum value of $R\sin(x + \alpha)$. [1]

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7 (a) The sum of the first 6 terms of an arithmetic progression is 555 and the sum of the next 6 terms is 1 455. Find the first term and the common difference. [4]

(b) In a geometric progression, the third term is 45 and the fifth term is 405. Find the

(i) first term and the common ratio $(r > 0)$, [3]

(ii) sum of the first 8 terms. [2]

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8 The table below shows the points scored by 95 students in an examination.

Points scored	1 – 3	4 – 6	7 – 9	10 – 12	13 – 15	16 – 18	19 – 21	22 – 24
Frequency	12	9	8	11	7	20	13	15

(a) Find the median class. [1]

(b) Calculate

(i) an estimate of the mean, [2]

(ii) the standard deviation. [6]

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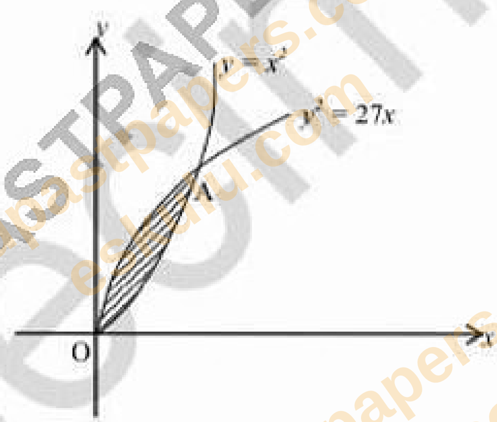
9 **(a)** A curve has equation $y = \dfrac{x+a}{x+2}$. Given that $\dfrac{dy}{dx} = -\dfrac{1}{25}$ when $x = 3$, find the value of a. [3]

(b) The diagram below shows part of the curves $y^2 = 27x$ and $y = x^2$ intersecting at O and A.



Find

(i) the coordinates of A, [3]

(ii) the volume obtained by rotating the shaded region through 360° about the x-axis. [4]

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10 The velocity, Vms^{-1} of a particle moving in a straight line, t seconds after passing through a fixed point O, is given by $V = 36t - 3t^2$.

Find the

(a) value of t when the velocity is constant, [3]

(b) value of t when the velocity is at instantaneous rest, [3]

(c) distance of the particle from O when the particle is at instantaneous rest. [4]

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11 (a) The curves $y = e^{3x}$ and $y = e^{2-x}$ intersect at the point B.

Find the

(i) coordinates of B. [3]

(ii) gradient of each curve in terms of e, at B. [3]

(b) When $\log_{10} y$ is plotted against x, a straight line is obtained passing through the points (0.6, 0.3) and (1.1, 0.2). Find $\log_{10} y$ in terms of x. [4]

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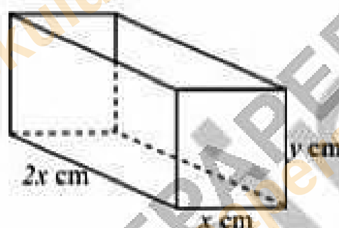
12 (a) A curve has equation $y = 2x - 3\sin x$. find the smallest positive value of x for which the curve has gradient $\frac{1}{2}$. [3]

(b) The diagram shows a cuboid with a rectangular base of sides x cm and $2x$ cm. The height of the cuboid is y cm and its total surface area is 120cm^2.



(i) Show that the volume V cm^3, of the cuboid is given by $V = 40x - \frac{4x^3}{3}$. [3]

(ii) Given that x can vary, find the dimensions of the cuboid when V is a minimum. [4]

Question 1. Solve the system of equations: $2x+3y-z=2$, $x-2y-4z=8$, $3x+5y+2z=-7$.

E1: $2x+3y-z=2$. E2: $x-2y-4z=8$. E3: $3x+5y+2z=-7$.

From E1: $z=2x+3y-2$ ...(A).

Into E2: $x-2y-4(2x+3y-2)=8 \Rightarrow x-2y-8x-12y+8=8 \Rightarrow -7x-14y=0 \Rightarrow x=-2y$ ...(B).

Into E3: $3x+5y+2(2x+3y-2)=-7 \Rightarrow 7x+11y-4=-7 \Rightarrow 7x+11y=-3$ ...(C).

Sub $x=-2y$ into C: $-14y+11y=-3 \Rightarrow -3y=-3 \Rightarrow y=1$. $x=-2$. $z=2(-2)+3(1)-2=-3$.

Check E2: $-2-2-4(-3)=8$ ✓ E3: $-6+5-6=-7$ ✓

Answer: $x = -2$, $y = 1$, $z = -3$

Question 2. Range of values and completing the square.

(a) $3x^2-10x-8 < 0$. Factorise: $(3x+2)(x-4) < 0$. Roots $x=-2/3$ and $x=4$.

Parabola opens up => negative between the roots.

Answer: (a) $-2/3 < x < 4$

(b) Express $12x^2-6x+5$ in the form $a(x+b)^2+c$. Factor 12 from x-terms:

$= 12(x^2 - x/2) + 5 = 12(x-1/4)^2 - 12(1/16) + 5 = 12(x-1/4)^2 - 3/4 + 5$.

$= 12(x-1/4)^2 + 17/4$. So $a=12$, $b=-1/4$, $c=17/4$. Minimum value $= c = 17/4$.

Answer: (b) $12(x-1/4)^2 + 17/4$; minimum value of $12x^2-6x+5 = 17/4$

Question 3. Solve the equations.

(a) $4^{3x-2}=19$. Take lg: $(3x-2)\lg 4 = \lg 19 \Rightarrow 3x-2 = \lg 19/\lg 4 = 1.2788/0.6021 = 2.124$.

$3x = 4.124 \Rightarrow x = 1.375$.

Answer: (a) $x \approx 1.37$

(b) $\log_2(2x^2+3x+5) = 3 + \log_2(x+1)$. Rearrange: $\log_2((2x^2+3x+5)/(x+1)) = 3$.

$(2x^2+3x+5)/(x+1) = 8 \Rightarrow 2x^2+3x+5 = 8x+8 \Rightarrow 2x^2-5x-3 = 0 \Rightarrow (2x+1)(x-3) = 0$.

$x = -1/2$ or $x = 3$. (Both satisfy $x+1>0$ since $-1/2 > -1$, so both valid.)

Answer: (b) $x = -1/2$ or $x = 3$

Question 4. Polynomial divisibility and a cubic equation.

(a) $3x^3-14x^2-7kx+10$ is exactly divisible by $(x+1)$, so $f(-1)=0$.

$3(-1)-14(1)-7k(-1)+10 = -3-14+7k+10 = -7+7k = 0 \Rightarrow k = 1$.

Answer: (a) $k = 1$

(b) Solve $2x^3-3x^2-30x+56=0$. Test $x=2$: $16-12-60+56=0$ ✓ so $(x-2)$ is a factor.

$2x^3-3x^2-30x+56 = (x-2)(2x^2+x-28) = (x-2)(2x-7)(x+4)$.

Answer: (b) $x = 2$, $x = 7/2$, $x = -4$

Question 5. Arrangements and combinations.

(a) Arrange the letters of 'UNGROUPED' (9 letters, with U appearing twice).

Number of arrangements = 9!/2! = 362880/2 = 181440.

Answer: (a) 181 440 arrangements

(b) Team of 5 from 7 women and 6 men with MORE women than men. Possible splits (w,m):

(5,0): C(7,5)*C(6,0) = 21*1 = 21.

(4,1): C(7,4)*C(6,1) = 35*6 = 210.

(3,2): C(7,3)*C(6,2) = 35*15 = 525.

Total = 21+210+525 = 756.

Answer: (b) 756 teams

Question 6. Trigonometry.

(a) cos(x+60) = 2 sin x, 0 <= x <= 360. Expand the left side:

cosx cos60 - sinx sin60 = 2 sinx => (1/2)cosx - (sqrt3/2)sinx = 2 sinx.

(1/2)cosx = (2 + sqrt3/2)sinx = ((4+sqrt3)/2)sinx => tan x = 1/(4+sqrt3) ≈ 0.1745.

x = arctan(0.1745) = 9.9° or x = 9.9+180 = 189.9°.

Answer: (a) x ≈ 9.9° and 189.9°

(b)(i) Express 5 sinx + 12 cosx as R sin(x+alpha) = R(sinx cosα + cosx sinα).

R cosα = 5, R sinα = 12. R = sqrt(25+144) = 13. alpha = arctan(12/5) ≈ 67.4°.

Answer: (b)(i) 5 sinx + 12 cosx = 13 sin(x + 67.4°)

(b)(ii) Maximum of R sin(x+alpha) is R, when sin(x+alpha)=1.

Answer: (b)(ii) Maximum value = 13

Question 7. Arithmetic and geometric progressions.

(a) AP: S6 = 555 and the sum of the NEXT 6 terms is 1455 (so S12 - S6 = 1455 => S12 = 2010).

S6 = 6/2*(2a+5d) = 3(2a+5d) = 555 => 2a+5d = 185 ...(1).

S12 = 12/2*(2a+11d) = 6(2a+11d) = 2010 => 2a+11d = 335 ...(2).

(2)-(1): 6d = 150 => d = 25. 2a = 185-125 = 60 => a = 30.

Answer: (a) First term a = 30, common difference d = 25

(b) GP: T3 = 45, T5 = 405, r > 0.

T5/T3 = r^2 = 405/45 = 9 => r = 3. T3 = ar^2 = 9a = 45 => a = 5.

Answer: (b)(i) First term a = 5, common ratio r = 3

S8 = a(r^8-1)/(r-1) = 5(3^8-1)/(3-1) = 5(6561-1)/2 = 5*6560/2 = 16400.

Answer: (b)(ii) Sum of first 8 terms = 16 400

Question 8. Points scored by 95 students — frequency table.

Points: 1-3 | 4-6 | 7-9 | 10-12 | 13-15 | 16-18 | 19-21 | 22-24

Frequency: 12 | 6 | 8 | 11 | 7 | 20 | 13 | 18 (n=95)

Midpoints: 2, 5, 8, 11, 14, 17, 20, 23.

(a) Median: n/2 = 47.5. Cumulative: 12,18,26,37,44,64,77,95. The 48th value is in class 16-18.

Answer: (a) Median class = 16-18

(b)(i) Sigma(fx) = 12(2)+6(5)+8(8)+11(11)+7(14)+20(17)+13(20)+18(23)

= 24+30+64+121+98+340+260+414 = 1351. Mean = 1351/95 ≈ 14.2.

Answer: (b)(i) Mean ≈ 14.2

(b)(ii) Sigma(fx^2) = 12(4)+6(25)+8(64)+11(121)+7(196)+20(289)+13(400)+18(529)

= 48+150+512+1331+1372+5780+5200+9522 = 23915.

Variance = 23915/95 - (14.2211)^2 = 251.74 - 202.24 = 49.50. SD = sqrt(49.50) ≈ 7.04.

Answer: (b)(ii) Standard deviation ≈ 7.04

Question 9. Differentiation of a quotient; area between curves.

(a) y = (x+a)/(x+2). Quotient rule: dy/dx = [(x+2) - (x+a)]/(x+2)^2 = (2-a)/(x+2)^2.

At x=3: (2-a)/25 = -1/25 => 2-a = -1 => a = 3.

Answer: (a) a = 3

(b) Curves y^2 = 27x and y = x^2 intersect at O and A.

Find A: substitute y=x^2 into y^2=27x: (x^2)^2 = 27x => x^4 - 27x = 0 => x(x^3-27)=0.

x=0 gives O; x=3 gives y=9. So A = (3, 9).

Answer: (b)(i) A = (3, 9)

(b)(ii) Shaded region between y=sqrt(27x) (outer) and y=x^2 (inner), x from 0 to 3.

(At x=1: sqrt(27) ≈ 5.2 is above x^2 = 1, so y^2=27x is the outer boundary.)

V = pi * integral from 0 to 3 of [27x - (x^2)^2] dx = pi * integral of (27x - x^4) dx.

= pi*[27x^2/2 - x^5/5]_0^3 = pi*(243/2 - 243/5) = pi*(1215/10 - 486/10) = pi*(729/10).

Answer: (b)(ii) Volume = 729*pi/10 cubic units

Question 10. Particle velocity V = 36t - 3t^2.

(a) Velocity is constant (maximum) when acceleration = 0. a = dV/dt = 36 - 6t = 0 => t = 6.

Answer: (a) t = 6 s

(b) Instantaneous rest: V = 0 => 36t - 3t^2 = 0 => 3t(12-t) = 0 => t=0 or t=12.

Taking the non-zero value, the particle is at instantaneous rest at t = 12 s.

Answer: (b) t = 12 s

(c) Distance from O at t=12: s = integral from 0 to 12 of (36t-3t^2) dt = [18t^2 - t^3]_0^12.

= 18(144) - 1728 = 2592 - 1728 = 864.

Answer: (c) Distance from O = 864 m

Question 11. Exponential curves and a log-linear graph.

(a) Curves y = e^{3x} and y = e^{2-x} intersect at B.

(a)(i) e^{3x} = e^{2-x} => 3x = 2-x => 4x = 2 => x = 1/2. y = e^{3(1/2)} = e^{3/2}.

Answer: (a)(i) B = (1/2, e^{3/2})

(a)(ii) For y=e^{3x}: dy/dx = 3e^{3x}; at x=1/2 this is 3e^{3/2}.

For y=e^{2-x}: dy/dx = -e^{2-x}; at x=1/2 this is -e^{3/2}.

Answer: (a)(ii) Gradients at B: 3e^{3/2} and -e^{3/2}

(b) lg y plotted against x gives a straight line through (0.6, 0.3) and (1.1, 0.2).

Gradient = (0.2-0.3)/(1.1-0.6) = -0.1/0.5 = -0.2.

lg y = -0.2x + c. Using (0.6, 0.3): 0.3 = -0.2(0.6) + c = -0.12 + c => c = 0.42.

Answer: (b) lg y = -0.2x + 0.42

Question 12. Curve gradient and cuboid optimisation.

(a) y = 2x - 3 sin x. dy/dx = 2 - 3 cos x. Set equal to 1/2:

2 - 3 cos x = 1/2 => 3 cos x = 3/2 => cos x = 1/2 => x = pi/3 (smallest positive value).

Answer: (a) x = pi/3 (= 60°)

(b) Cuboid: rectangular base x by 2x, height y, total surface area 120 cm^2.

SA = 2(x*2x) + perimeter*y = 4x^2 + 2(x+2x)y = 4x^2 + 6xy = 120.

(b)(i) 6xy = 120 - 4x^2 => y = 20/x - 2x/3. Volume V = (2x^2)y = 2x^2(20/x - 2x/3).

V = 40x - 4x^3/3. Q.E.D.

Answer: (b)(i) V = 40x - 4x^3/3 (proved)

(b)(ii) dV/dx = 40 - 4x^2 = 0 => x^2 = 10 => x = sqrt(10).

d^2V/dx^2 = -8x < 0, confirming a maximum. y = 20/sqrt(10) - 2sqrt(10)/3 = 2sqrt(10) - 2sqrt(10)/3 = 4sqrt(10)/3.

Dimensions: base sqrt(10) cm by 2sqrt(10) cm, height 4sqrt(10)/3 cm.

Answer: (b)(ii) x = sqrt(10) ≈ 3.16 cm; base sqrt(10) by 2sqrt(10) cm (≈3.16 by 6.32 cm), height 4sqrt(10)/3 ≈ 4.22 cm

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