G12 ECZ ADDITIONAL MATHEMATICS 2017 GCE PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2017 GCE Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

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EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

Additional Mathematics 4030/2

Paper 2

Wednesday **16 AUGUST 2017**

Additional Materials:
Answer Booklet
Silent electronic calculator (non programmable)

Time: 2 hours 30 Minutes

Instructions to Candidates

Write your **name, centre number** and **candidate number** in the spaces on the separate Answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the **Answer Booklet provided.**

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 SERIES

Arithmetic $\quad Sn = \frac{1}{2}n\{2a + (n-1)d\}$

Geometric $\quad Sn = \frac{a(1-r^n)}{1-r}\ (r \neq 1)$

$$S\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \mp \tan B}{1A \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos B$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \textbf{ SD} = \sqrt{\left\{\frac{\sum(x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \textbf{ SD} = \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

1 Solve the following systems of equations

$x + 2y + 2z = 4,$

$3x - y + 4z = 25,$

$3x + 2y - z = -4.$ [6]

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2 (a) Find the range of values of x for which $x(9 - x) \geq 18$. [3]

(b) Express $7 - 5x - 2x^2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. Hence, find the coordinates of the turning point. [4]

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3 Solve the equations

(a) $5^{x-1} = 9$, [3]

(b) $\log_4 (x + 2) = \log_4 (x^2 + 2) - 1$. [4]

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4 (a) Find the value of k, given that the expression $x^3 - kx^2 + 7x + 10$ is divisible by $(x + 2)$. [3]

(b) Solve the equation $x^3 + x^2 - 4x - 4 = 0$. [4]

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5 (a) A committee consists of a patron, a matron and 8 prefects. Find the number of ways of arranging the committee members in a straight line if

(i) there are no restrictions, [2]

(ii) there is **no** prefect at the beginning and at the end of the line. [3]

(b) A delegation of 3 boys and 2 girls is to be chosen from 10 boys and 5 girls. In how many ways can this be done? [3]

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6 (a) Solve the equation $\cos(\theta + 30°) = \sin\theta$, for values of θ in the range $0° \leq \theta \leq 360°$. [4]

(b) Given that $\cos A = \frac{3}{5}$ and $\cos B = \frac{-5}{13}$, where angle A is acute and angle B is obtuse, find the value of $\sin(A + B)$. [4]

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7 (a) The sum of the first four terms of an arithmetic progression is 62 and the second term is 17. Find

(i) the first term and the common difference, [3]

(ii) the sum of the first 40 terms. [2]

(b) A geometric progression is given by 81, 54, 36 …
Find

(i) the 20th term, [2]

(ii) the sum to infinity. [2]

8 The table below shows the results in a Science test for 200 students.

Marks	1 – 20	21 – 40	41 – 60	61 – 80	81 – 100
Frequency	13	56	60	48	23

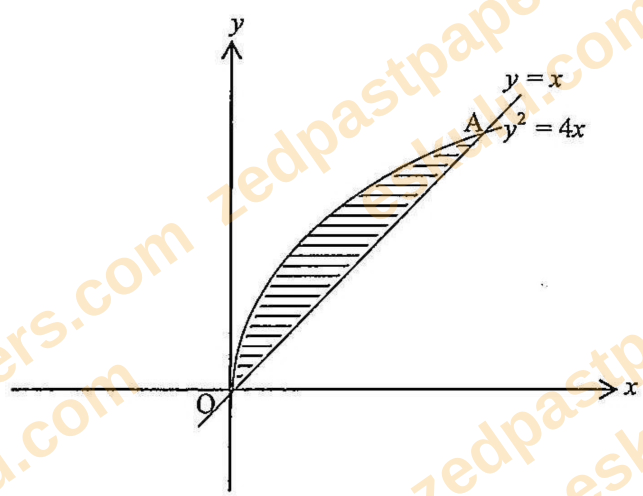
(a) Find the median class. [1]

(b) Calculate an estimate of

(i) the mean, [2]

(ii) the standard deviation. [6]

9 The diagram below shows part of the curve $y^2 = 4x$ and the line $y = x$ meeting at the points O and A.



Find

(a) the coordinates of A, [4]

(b) the volume obtained by rotating the shaded region through 360° about the x-axis. [5]

10 A particle moves in a straight line so that t seconds after passing a fixed point O, its acceleration, a m/s^2 is given by $a = 4t - 12$. Given that the speed at O is 16m/s, find

(a) the values of t at which the particle is at instantaneous rest, [5]

(b) the distance the particle travels in the fifth second. [5]

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11 (a) A curve has the equation $y = 6x - \frac{x^2}{2} - \frac{x^3}{3}$.

(i) Find the coordinates of the stationary points. [4]

(ii) Determine the nature of the stationary points. [2]

(b) Find the coordinates of the point where the curve $y = 3e^{2x-4}$ intersects the line $y = 3$, and the gradient of the curve at that point. [4]

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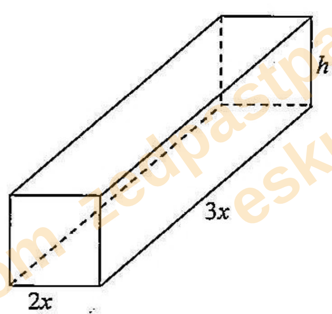
12 **Answer only one of the following alternatives:**

Either

(a) A curve has gradient function given by $\frac{dy}{dx} = 4x + \frac{1}{x^2}$ and passes through the point $(-1, 5)$. Find the equation of the curve. [3]

(b) The diagram below shows a rectangular box without a lid which is made from a thin cardboard.



The base is $3x$ cm long and $2x$ cm wide and the height is h cm. The total surface area of the box is 200cm^2.

(i) Show that $h = \frac{20}{x} - \frac{3x}{5}$. [3]

(ii) Hence, find the dimensions of the box which can give maximum volume. [4]

OR

(a) An agent sponsoring students decides to give a sum of money to a school each year for 12 years. The agent decides to give K75 000.00 in the first year and to increase the sponsorship by K10 000.00 each year.

(i) Find the amount the agent gives out in the twelfth year. [3]

(ii) Find the amount received by the school from the agent altogether. [3]

(b) The first term of a geometric progression is 1 and the sum of the first 3 terms is $\frac{7}{9}$. Find the two possible values of the common ratio of this series.

[4]

Question 1. Solve the system of equations: x+2y+2z=4, 3x-y+4z=25, 3x+2y-z=-4.

E1: x+2y+2z=4. E2: 3x-y+4z=25. E3: 3x+2y-z=-4.

E3-E1: 2x-3z=-8 ...(A).

2*E2+E1: 6x-2y+8z + x+2y+2z = 50+4 => 7x+10z=54 ...(B).

From A: x=(3z-8)/2. Into B: 7(3z-8)/2+10z=54 => 21z-56+20z=108 => 41z=164 => z=4.

x=(12-8)/2=2. From E1: 2+2y+8=4 => 2y=-6 => y=-3.

Check E2: 6-(-3)+16=25 ✓ E3: 6+(-6)-4=-4 ✓

Answer: x = 2, y = -3, z = 4

Question 2. Range of values and completing the square.

(a) x(9-x) >= 18 => 9x-x^2 >= 18 => 0 >= x^2-9x+18 => x^2-9x+18 <= 0.

Factorise: (x-3)(x-6) <= 0. Roots 3 and 6; parabola opens up => <= 0 between roots.

Answer: (a) 3 <= x <= 6

(b) Express 7-5x-2x^2 in the form a(x+b)^2+c. Factor -2 from x-terms:

= -2(x^2 + (5/2)x) + 7 = -2(x + 5/4)^2 + 2*(25/16) + 7 = -2(x+5/4)^2 + 25/8 + 7.

= -2(x+5/4)^2 + 81/8. So a=-2, b=5/4, c=81/8.

Coefficient of the square is negative, so the turning point is a MAXIMUM.

Answer: (b) -2(x+5/4)^2 + 81/8; turning point = (-5/4, 81/8) [maximum]

Question 3. Solve the equations.

(a) 5^{x-1}=9. Take lg: (x-1) lg5 = lg9 => x-1 = lg9/lg5 = 0.9542/0.6990 = 1.365.

x = 2.365.

Answer: (a) x ≈ 2.37

(b) log_4(x+2) = log_4(x^2+2) - 1. Move logs together: log_4(x+2)+1 = log_4(x^2+2).

Write 1 = log_4 4: log_4(4(x+2)) = log_4(x^2+2) => 4(x+2) = x^2+2.

4x+8 = x^2+2 => x^2-4x-6 = 0 => x = (4 ± sqrt(16+24))/2 = (4 ± sqrt(40))/2 = 2 ± sqrt(10).

Both satisfy x+2 > 0 (since 2-sqrt(10) ≈ -1.16 > -2), so both are valid.

Answer: (b) x = 2 + sqrt(10) ≈ 5.16 or x = 2 - sqrt(10) ≈ -1.16

Question 4. Polynomial divisibility and a cubic equation.

(a) x^3 - kx^2 + 7x + 10 is divisible by (x+2), so f(-2)=0.

(-8) - k(4) + 7(-2) + 10 = 0 => -8 - 4k - 14 + 10 = 0 => -12 - 4k = 0 => k = -3.

Answer: (a) k = -3

(b) Solve x^3 + x^2 - 4x - 4 = 0. Factor by grouping:

x^2(x+1) - 4(x+1) = (x+1)(x^2-4) = (x+1)(x-2)(x+2).

Answer: (b) x = -1, x = 2, x = -2

Question 5. Arrangements and combinations.

(a) Committee: a patron, a matron and 8 prefects = 10 members, arranged in a line.

(a)(i) No restrictions: 10! = 3 628 800.

Answer: (a)(i) 3 628 800 ways

(a)(ii) No prefect at the beginning and at the end. Only 2 non-prefects (patron, matron).

So the two ends must be the patron and matron: arrange them in 2! ways.

The 8 prefects fill the 8 middle positions: 8! ways. Total = 2! * 8! = 2 * 40320 = 80640.

Answer: (a)(ii) 80 640 ways

(b) 3 boys from 10 and 2 girls from 5: C(10,3) * C(5,2) = 120 * 10 = 1200.

Answer: (b) 1200 ways

Question 6. Trigonometry.

(a) cos(theta+30) = sin theta, 0 <= theta <= 360. Expand the left side:

cos theta cos30 - sin theta sin30 = sin theta => (sqrt3/2)cos theta - (1/2)sin theta = sin theta.

(sqrt3/2)cos theta = (3/2)sin theta => tan theta = (sqrt3/2)/(3/2) = 1/sqrt3.

theta = 30° or theta = 30+180 = 210°.

Answer: (a) theta = 30° and 210°

(b) cos A = 3/5 (A acute) => sin A = 4/5. cos B = -5/13 (B obtuse) => sin B = 12/13.

sin(A+B) = sinA cosB + cosA sinB = (4/5)(-5/13) + (3/5)(12/13) = -20/65 + 36/65 = 16/65.

Answer: (b) sin(A+B) = 16/65

Question 7. Arithmetic and geometric progressions.

(a) AP: sum of first four terms is 62, and the second term is 17.

S4 = 4/2*(2a+3d) = 4a+6d = 62 => 2a+3d = 31 ...(1).

T2 = a+d = 17 ...(2). From (2): a=17-d. Into (1): 2(17-d)+3d=31 => 34+d=31 => d=-3.

a = 17-(-3) = 20.

Answer: (a)(i) First term a = 20, common difference d = -3

S40 = 40/2*(2(20)+39(-3)) = 20*(40-117) = 20*(-77) = -1540.

Answer: (a)(ii) Sum of first 40 terms = -1540

(b) GP: 81, 54, 36, ... r = 54/81 = 2/3, a = 81.

(b)(i) T20 = a r^19 = 81*(2/3)^19 = 2^19/3^15 = 524288/14348907 ≈ 0.0365.

Answer: (b)(i) 20th term = 2^19/3^15 ≈ 0.0365

S_inf = a/(1-r) = 81/(1-2/3) = 81/(1/3) = 243.

Answer: (b)(ii) Sum to infinity = 243

Question 8. Science test results of 200 students — frequency table.

Marks: 1-20 | 21-40 | 41-60 | 61-80 | 81-100

Frequency: 13 | 56 | 60 | 48 | 23 (n=200)

Midpoints: 10.5, 30.5, 50.5, 70.5, 90.5.

(a) Median: n/2 = 100. Cumulative: 13, 69, 129, ... the 100th value is in class 41-60.

Answer: (a) Median class = 41-60

(b)(i) Sigma(fx) = 13(10.5)+56(30.5)+60(50.5)+48(70.5)+23(90.5)

= 136.5+1708+3030+3384+2081.5 = 10340. Mean = 10340/200 = 51.7.

Answer: (b)(i) Mean = 51.7

(b)(ii) Sigma(fx^2) = 13(110.25)+56(930.25)+60(2550.25)+48(4970.25)+23(8190.25)

= 1433.25+52094+153015+238572+188375.75 = 633490.

Variance = 633490/200 - (51.7)^2 = 3167.45 - 2672.89 = 494.56. SD = sqrt(494.56) ≈ 22.2.

Answer: (b)(ii) Standard deviation ≈ 22.2

Question 9. Curve y^2 = 4x and line y = x meeting at O and A.

(a) Intersections: y=x into y^2=4x gives x^2=4x => x^2-4x=0 => x(x-4)=0.

x=0 gives O=(0,0); x=4 gives y=4, so A=(4,4).

Answer: (a) A = (4, 4)

(b) Shaded region between the parabola y=2sqrt(x) (outer) and line y=x (inner), x from 0 to 4.

(At x=1: parabola y=2 is above line y=1, so parabola is the outer boundary.)

V = pi * integral from 0 to 4 of [(2sqrt(x))^2 - x^2] dx = pi * integral of (4x - x^2) dx.

= pi*[2x^2 - x^3/3]_0^4 = pi*(32 - 64/3) = pi*(96/3 - 64/3) = pi*(32/3).

Answer: (b) Volume = 32*pi/3 cubic units

Question 10. Particle: acceleration a = 4t - 12, speed at O (t=0) is 16 m/s.

Velocity: V = integral a dt = 2t^2 - 12t + C. At t=0, V=16 => C=16.

So V = 2t^2 - 12t + 16.

(a) Instantaneous rest: V=0 => 2t^2-12t+16=0 => t^2-6t+8=0 => (t-2)(t-4)=0.

Answer: (a) t = 2 s and t = 4 s

(b) Distance in the fifth second = displacement from t=4 to t=5.

s = integral V dt = 2t^3/3 - 6t^2 + 16t (taking s=0 at t=0).

s(5) = 250/3 - 150 + 80 = 250/3 - 70 = 40/3. s(4) = 128/3 - 96 + 64 = 128/3 - 32 = 32/3.

V does not change sign on [4,5] (V(5)=6 > 0, V(4)=0), so distance = s(5)-s(4) = 40/3-32/3 = 8/3.

Answer: (b) Distance in the fifth second = 8/3 m ≈ 2.67 m

Question 11. Stationary points and an exponential curve.

(a) y = 6x - x^2/2 - x^3/3. dy/dx = 6 - x - x^2 = -(x^2+x-6) = -(x+3)(x-2).

(a)(i) Stationary at x=-3 and x=2.

y(2) = 12 - 2 - 8/3 = 10 - 8/3 = 22/3.

y(-3) = -18 - 9/2 - (-27/3) = -18 - 4.5 + 9 = -27/2.

Answer: (a)(i) Stationary points: (2, 22/3) and (-3, -27/2)

(a)(ii) d^2y/dx^2 = -1 - 2x. At x=2: -5 < 0 => MAXIMUM. At x=-3: 5 > 0 => MINIMUM.

Answer: (a)(ii) (2, 22/3) is a maximum; (-3, -27/2) is a minimum

(b) Curve y = 3e^{2x-4} meets line y = 3: 3e^{2x-4}=3 => e^{2x-4}=1 => 2x-4=0 => x=2, y=3.

Gradient: dy/dx = 6e^{2x-4}. At x=2: 6e^0 = 6.

Answer: (b) Point = (2, 3); gradient there = 6

Question 12. EITHER: Integration and an open-box optimisation.

(a) dy/dx = 4x + 1/x^2. Integrate: y = 2x^2 - 1/x + C.

Passes through (-1, 5): 2(1) - 1/(-1) + C = 2 + 1 + C = 5 => C = 2.

Answer: (a) y = 2x^2 - 1/x + 2

(b) Open box (no lid): base 3x by 2x, height h. Surface area = base + 4 sides = 200.

SA = 6x^2 + 2(3x*h) + 2(2x*h) = 6x^2 + 10xh = 200.

(b)(i) 10xh = 200 - 6x^2 => h = (200-6x^2)/(10x) = 20/x - 6x/10 = 20/x - 3x/5. Q.E.D.

Answer: (b)(i) h = 20/x - 3x/5 (proved)

(b)(ii) Volume V = 3x*2x*h = 6x^2*h = 6x^2(20/x - 3x/5) = 120x - 18x^3/5.

dV/dx = 120 - 54x^2/5 = 0 => 54x^2 = 600 => x^2 = 100/9 => x = 10/3.

d^2V/dx^2 = -108x/5 < 0, confirming maximum. h = 20/(10/3) - 3(10/3)/5 = 6 - 2 = 4.

Dimensions: length 3x = 10 cm, width 2x = 20/3 cm, height h = 4 cm. (V_max = 2400/9 ≈ 266.7 cm^3.)

Answer: (b)(ii) Box: 10 cm by 20/3 cm by 4 cm (x = 10/3 cm, maximum volume ≈ 266.7 cm^3)

Question 12or. OR: Arithmetic and geometric progressions.

(a) Sponsorship is an AP: first year a=75000, increase d=10000, for n=12 years.

(a)(i) Amount in the 12th year: T12 = a + 11d = 75000 + 11(10000) = 75000 + 110000 = 185000.

Answer: (a)(i) Twelfth-year amount = K185 000

(a)(ii) Total: S12 = 12/2*(2(75000)+11(10000)) = 6*(150000+110000) = 6*260000 = 1 560 000.

Answer: (a)(ii) Total received = K1 560 000

(b) GP: first term a=1, sum of first 3 terms = 7/9.

S3 = 1 + r + r^2 = 7/9 => r^2 + r + 2/9 = 0 => 9r^2 + 9r + 2 = 0.

(3r+1)(3r+2) = 0 => r = -1/3 or r = -2/3.

Check r=-1/3: 1-1/3+1/9 = 7/9 ✓. r=-2/3: 1-2/3+4/9 = 7/9 ✓.

Answer: (b) The two possible common ratios: r = -1/3 or r = -2/3

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