

G12 ECZ ADDITIONAL MATHEMATICS 2017 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2017 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics

4030/2

Paper 2

Monday

20 NOVEMBER 2017

Additional Materials:

Answer Booklet
Silent electronic calculator (non programmable)

Time: 2 hours 30 Minutes

Instructions to Candidates

Write your **name**, **centre number** and **candidate number** in the spaces on the separate Answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the **Answer Booklet** provided.

If you use more than one Answer Booklet, **fasten** the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Arithmetic $S_n = \frac{1}{2} n \{2a + (n-1)d\}$

Geometric $S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

3 TRIGONOMETRY

Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \mp \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \text{SD} = \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{\sum x^2}{n} - (\bar{x})^2 \right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \text{SD} = \sqrt{\left\{ \frac{\sum f(x - \bar{x})^2}{\sum f} \right\}} = \sqrt{\left\{ \frac{\sum fx^2}{\sum f} - (\bar{x})^2 \right\}}$$

- 1 Solve the following systems of equations

$$2a + b - c = -1,$$

$$a + 3b - 2c = -6,$$

$$5a + 2b + c = 5.$$

[6]

- 2 (a) Find the range of values of x for which $2x^2 < 3x + 14$. [3]

- (b) Express $6 + 4x - x^2$ in the form $a(x + b)^2 + c$, where a , b and c are constants.
Hence, find the coordinates of the turning point. [4]

- 3 Solve the equations

(a) $3^{x+1} = 5$, [3]

(b) $\log_7(2x - 3) + 2 = \log_7(17x + 15)$. [4]

- 4 (a) Find the value of k , given that the expression $x^3 + kx^2 + 2x + 12$ is divisible by $(x - 3)$. [3]

(b) Factorise the expression $2x^3 + x^2 - 13x + 6$. [4]

- 5 (a) In how many ways can 5 girls and 4 boys be put in a straight line if
(i) the girls are together, [2]

- (ii) no two boys are together. [3]

- (b) A group of 5 students is to be chosen from 5 girls and 4 boys.
Find the number of ways of choosing 3 girls and 2 boys. [3]

- 6 (a) Solve the equation $\sqrt{2} \cos(x - 45^\circ) = 4 \sin x$ for values of x in the range $0^\circ \leq x \leq 360^\circ$. [4]

- (b) (i) Express $5 \sin \theta - 12 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$
and $0^\circ < \alpha < 90^\circ$. [3]

- (ii) Hence find the maximum value of $R \sin(\theta - \alpha)$. [1]

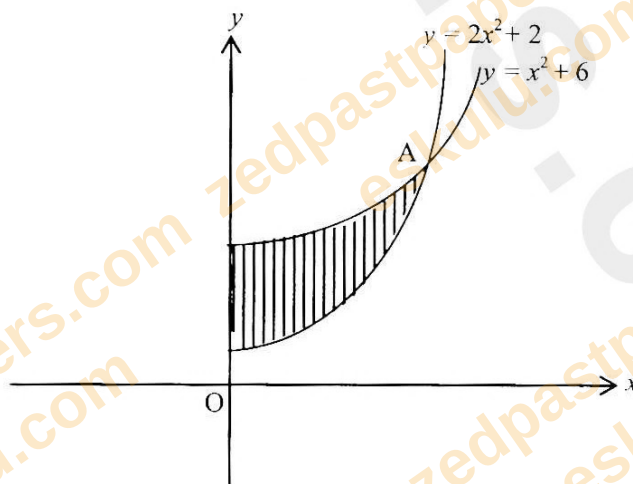
- 7 (a) The sum of the first 6 terms of an arithmetic progression is 57 and the sum of the next 7 terms is 203. Find the
- (i) first term and the common difference, [3]
 - (ii) sum of the first 12 terms of the progression. [2]
- (b) The third term of a geometric progression is 32 and the ninth term is $\frac{1}{2}$. Find the
- (i) first term and the common ratio, [3]
 - (ii) sum to infinity. [1]

- 8 The table below shows the points scored by 110 students in an examination.

Points scored	6 – 8	8 – 10	10 – 12	12 – 14	14 – 16	16 – 18	18 – 20
Frequency	13	16	9	15	22	15	20

- (a) Find the median class. [1]
- (b) Calculate
 - (i) an estimate of the mean, [2]
 - (ii) the standard deviation. [6]

- 9 The diagram below shows part of the curves $y = x^2 + 6$ and $y = 2x^2 + 2$ meeting at the point A.



Find

- (a) the coordinates of A, [3]
- (b) the volume obtained by rotating the shaded region through 360° about the x-axis. [6]

- 10 The velocity, V m/s, of a particle travelling in a straight line, at time t seconds after leaving a fixed point O , is given by $V = qt - 3t^2 + 10$, where $t \geq 0$ and q is a constant. When $t = 0$, the particle is at O and its acceleration is 1 m/s^2 . Find

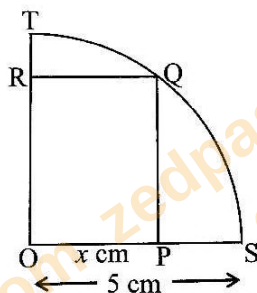
- (a) the value of q , [3]
- (b) the value of t when the particle is at instantaneous rest, [3]
- (c) the distance the particle has travelled when it is at instantaneous rest. [4]

- 11 (a) A curve has the equation $y = -2x^3 + 3x^2 + 12x + 27$.
- (i) Find the stationary points. [4]
 - (ii) Determine the nature of the stationary points. [2]
- (b) A curve has an equation $y = \ln x$. Determine the equation of the straight line which can be drawn to find the solution of the equation $x^2 e^{x-2} = 1$. [4]

- 12 Answer only one of the following alternatives:

EITHER

- (a) A curve has gradient function $\frac{dy}{dx} = 6x^2 + 4x - 5$ and passes through the point $(2, 10)$. Find the equation of the curve. [4]
- (b) The diagram below shows a quarter circle with radius 5 cm. $OPQR$ are vertices of a rectangle on the quarter circle and $OP = x$ cm.



- (i) Given that the area of the rectangle $OPQR$ is $A \text{ cm}^2$, show that the square of the rectangular area, A^2 , is given by $A^2 = (25 - x^2)x^2$. [3]
- (ii) Find the values of x for which A^2 is stationary. [3]

OR

- (a) In an arithmetic progression, the 5th term is three times the second term. The sum of the first ten terms is 50. Find the first term and the sum of the first 30 terms. [6]
- (b) The fifth term of a geometric progression is 8, the third term is 4 and the common ratio is positive. Find the first term, the common ratio and the sum of the first twelve terms. [4]

Question 1. Solve the system of equations: $2x+y-z=-1$, $x+3y-2z=-6$, $5x+2y+z=5$.

Add E1+E3 to eliminate z: $(2x+y-z)+(5x+2y+z)=(-1+5) \Rightarrow 7x+3y=4 \dots(A)$.

From E1: $z=2x+y+1$. Substitute into E2: $x+3y-2(2x+y+1)=-6 \Rightarrow -3x+y=-4 \dots(B)$.

From B: $y=3x-4$. Into A: $7x+3(3x-4)=4 \Rightarrow 16x=16 \Rightarrow x=1$.

$y=3-4=-1$. $z=2+(-1)+1=2$.

Check E3: $5-2+2=5$ ✓ Check E2: $1-3+4=-6$? $1-3-4=-6$ ✓

Answer: $x = 1$, $y = -1$, $z = 2$

Question 2. Quadratic inequality and completing the square.

(a) Solve $2x^2 < 3x+14 \Rightarrow 2x^2-3x-14 < 0$. Factorise: $(2x-7)(x+2) = 0 \Rightarrow x=7/2$ or $x=-2$.

Parabola opens up: $2x^2-3x-14 < 0$ between the roots.

Answer: (a) $-2 < x < 7/2$

(b) Express $6+4x-x^2$ in the form $a-(x+b)^2$. Complete the square on x:

$6+4x-x^2 = -(x^2-4x)+6 = -(x-2)^2+4+6 = 10-(x-2)^2$.

Written as $a-(x+b)^2$: $a=10$, $b=-2$ (i.e. $10-(x+(-2))^2 = 10-(x-2)^2$).

Since coefficient of $(x-2)^2$ is negative, the turning point is a MAXIMUM.

Answer: (b) $y = 10 - (x-2)^2$; maximum turning point = (2, 10)

Question 3. Solve the equations.

(a) $3^{x-5} = 5$. Take lg of both sides: $(x-5) \lg 3 = \lg 5$.

$x-5 = \lg 5 / \lg 3 = 0.6990/0.4771 = 1.465$. $x = 1.465+5 = 6.465$.

Answer: (a) $x \approx 6.47$

(b) $\lg(2x-3)+2=\lg(13x+15)$. Write $2=\lg 100$: $\lg(100(2x-3))=\lg(13x+15)$.

$100(2x-3)=13x+15$. $200x-300=13x+15$. $187x=315$. $x=315/187$.

Check: $2(315/187)-3=315/93.5>0$ ✓ (domain valid).

Answer: (b) $x = 315/187 \approx 1.68$

Question 4. Polynomial divisibility and factorisation.

(a) $x^3+2x^2+kx+12$ is divisible by $(x-3)$. So $f(3)=0$:

$27+18+3k+12=0 \Rightarrow 57+3k=0 \Rightarrow k=-19$.

Answer: (a) $k = -19$

(b) Factorise $2x^3+x^2-13x+6$. Test $x=2$: $16+4-26+6=0$ ✓ So $(x-2)$ is a factor.

Divide: $2x^3+x^2-13x+6 = (x-2)(2x^2+5x-3) = (x-2)(2x-1)(x+3)$.

Check: $(2x-1)(x+3)=2x^2+6x-x-3=2x^2+5x-3$ ✓

Answer: (b) $2x^3+x^2-13x+6 = (x-2)(2x-1)(x+3)$

Question 5. Arrangements and combinations.

(a) 5 girls and 4 boys arranged in a straight line.

(a)(i) Girls together: treat 5 girls as 1 block. Arrange 1 block+4 boys = 5 units: 5! ways.

Girls within block: 5! ways. Total = $5! \times 5! = 120 \times 120 = 14400$.

Answer: (a)(i) 14400 arrangements

(a)(ii) No two boys adjacent: place 5 girls first (5! ways). Gaps created: $_G_G_G_G_G_ = 6$ gaps.

Choose 4 gaps for 4 boys and arrange them: $P(6,4) = 6 \times 5 \times 4 \times 3 = 360$.

Total = $5! \times P(6,4) = 120 \times 360 = 43200$.

Answer: (a)(ii) 43200 arrangements

(b) Choose 3 girls from 5 and 2 boys from 4: $C(5,3) \times C(4,2) = 10 \times 6 = 60$.

Answer: (b) 60 ways

Question 6. Trigonometry — equation and R-form.

(a) $\sqrt{2} \cos(x-45) = 4 \sin x$. Expand: $\sqrt{2}(\cos x \cos 45 + \sin x \sin 45) = 4 \sin x$.

$\sqrt{2}(\cos x / \sqrt{2} + \sin x / \sqrt{2}) = 4 \sin x \Rightarrow \cos x + \sin x = 4 \sin x \Rightarrow \cos x = 3 \sin x$.

$\tan x = 1/3$. $x = \arctan(1/3) \approx 18.4^\circ$. Also $x = 180 + 18.4 = 198.4^\circ$.

Answer: (a) $x \approx 18.4^\circ$ and 198.4°

(b) Express $5 \sin \theta - 12 \cos \theta$ in the form $R \sin(\theta - \alpha)$.

$R \sin(\theta - \alpha) = R(\sin \theta \cos \alpha - \cos \theta \sin \alpha)$. Matching: $R \cos \alpha = 5$, $R \sin \alpha = 12$.

$R = \sqrt{25 + 144} = \sqrt{169} = 13$. $\alpha = \arctan(12/5) \approx 67.4^\circ$.

Maximum value = $R = 13$ (when $\sin(\theta - 67.4^\circ) = 1$).

Answer: (b) $5 \sin \theta - 12 \cos \theta = 13 \sin(\theta - 67.4^\circ)$; maximum value = 13

Question 7. Arithmetic and geometric progressions.

(a) $S_6 = 57$ and sum of next 7 terms is 203 (i.e. $S_{13} - S_6 = 203$, so $S_{13} = 260$).

$S_6 = 3(2a + 5d) = 57 \Rightarrow 2a + 5d = 19 \dots (A)$.

$S_{13} = 13/2(2a + 12d) = 13(a + 6d) = 260 \Rightarrow a + 6d = 20 \dots (B)$.

From A: $a = (19 - 5d)/2$. Into B: $(19 - 5d)/2 + 6d = 20 \Rightarrow 19 + 7d = 40 \Rightarrow d = 3$. $a = 2$.

Answer: (a)(i) First term $a = 2$, common difference $d = 3$

$S_{12} = 12/2(2(2) + 11(3)) = 6(4 + 33) = 6 \times 37 = 222$.

Answer: (a)(ii) Sum of first 12 terms = 222

(b) GP: $T_3 = 4/9$ and $T_9 = 1/2$.

$$T_9/T_3 = r^6 = (1/2)/(4/9) = 9/8. \quad r = (9/8)^{1/6} \approx 1.020.$$

Since $|r| = (9/8)^{1/6} > 1$, the GP is divergent.

$$\text{First term: } a = T_3/r^2 = (4/9)/(9/8)^{1/3} \approx 0.431.$$

Answer: (b)(i) $r = (9/8)^{1/6} \approx 1.020$ ($r > 1$)

Answer: (b)(ii) Sum to infinity does NOT exist ($|r| > 1$, series diverges)

Question 8. Points scored by 35 students — frequency table.

Marks: 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69

Frequency: 3 | 4 | 10 | 5 | 7 | 4 | 2 ($n=35$)

Midpoints: 4.5, 14.5, 24.5, 34.5, 44.5, 54.5, 64.5.

(a) Modal class: highest frequency = 10 (class 20-29).

Answer: (a) Modal class = 20-29

$$(b)(i) \Sigma(fx) = 3(4.5) + 4(14.5) + 10(24.5) + 5(34.5) + 7(44.5) + 4(54.5) + 2(64.5)$$

$$= 13.5 + 58 + 245 + 172.5 + 311.5 + 218 + 129 = 1147.5. \quad \text{Mean} = 1147.5/35 \approx 32.8.$$

Answer: (b)(i) Mean ≈ 32.8

$$(b)(ii) \Sigma(fx^2) = 3(20.25) + 4(210.25) + 10(600.25) + 5(1190.25) + 7(1980.25) + 4(2970.25) + 2(4160.25)$$

$$= 60.75 + 841 + 6002.5 + 5951.25 + 13861.75 + 11881 + 8320.5 = 46918.75.$$

$$\text{Var} = 46918.75/35 - (32.8)^2 = 1340.5 - 1075.8 = 264.7. \quad \text{SD} = \sqrt{264.7} \approx 16.3.$$

Answer: (b)(ii) Standard deviation ≈ 16.3

Question 9. Curves $y = 2x^2 + 2$ and $y = 2x + 2$ intersecting at A and B.

Find intersections: $2x^2 + 2 = 2x + 2 \Rightarrow 2x^2 - 2x = 0 \Rightarrow 2x(x-1) = 0.$

$x=0: y=2. \quad A=(0,2). \quad x=1: y=4. \quad B=(1,4).$

Answer: (a) A = (0, 2) and B = (1, 4)

(b) Volume rotating shaded region (line $y=2x+2$ above, curve $y=2x^2+2$ below, 0 to 1) about x-axis:

$$V = \pi \int_0^1 [(2x+2)^2 - (2x^2+2)^2] dx.$$

$$(2x+2)^2 = 4x^2 + 8x + 4. \quad (2x^2+2)^2 = 4x^4 + 8x^2 + 4. \quad \text{Integrand} = -4x^4 - 4x^2 + 8x.$$

$$= \pi \left[-\frac{4x^5}{5} - \frac{4x^3}{3} + 4x^2 \right]_0^1 = \pi \left(-\frac{4}{5} - \frac{4}{3} + 4 \right).$$

$$= \pi \left(-\frac{12}{15} - \frac{20}{15} + \frac{60}{15} \right) = \pi \left(\frac{28}{15} \right).$$

Answer: (b) Volume = $28\pi/15$ cubic units

Question 10. Particle: $V = qt + t^2 - 3t$, returns to O at $t = 3$.

Position from O: $s = \int V dt = \frac{qt^2}{2} + \frac{t^3}{3} - 3t^2/2.$

Particle returns to O at $t=3: s(3)=0.$

$$9q/2 + 9 - 27/2 = 0 \Rightarrow 9q/2 - 9/2 = 0 \Rightarrow 9(q-1)/2 = 0 \Rightarrow q=1.$$

Answer: (a) $q = 1$

$$V = t + t^2 - 3t = t^2 - 2t = t(t-2). \text{ At rest: } t=0 \text{ or } t=2 \text{ s.}$$

Answer: (b) Particle at instantaneous rest at $t = 2$ s

$$(c) V = t(t-2) < 0 \text{ for } 0 < t < 2 \text{ (particle moves backward).}$$

$$\text{Distance} = \left| \int_0^2 t(t-2) dt \right| = \left| \left[\frac{t^3}{3} - t^2 \right]_0^2 \right|.$$

$$= \left| \frac{8}{3} - 4 \right| = \left| \frac{8}{3} - \frac{12}{3} \right| = \left| -\frac{4}{3} \right| = \frac{4}{3}.$$

Answer: (c) Distance travelled = $\frac{4}{3}$ m

Question 11. Stationary points and graphical solution of equation.

$$(a) y = -2x^3 + 9x^2 - 12x + 27. \quad dy/dx = -6x^2 + 18x - 12 = -6(x^2 - 3x + 2) = -6(x-1)(x-2).$$

$$(a)(i) \text{ Stationary at } x=1 \text{ and } x=2. \quad y(1) = -2 + 9 - 12 + 27 = 22. \quad y(2) = -16 + 36 - 24 + 27 = 23.$$

Answer: (a)(i) Stationary points: (1, 22) and (2, 23)

$$(a)(ii) \quad d^2y/dx^2 = -12x + 18. \text{ At } x=1: 6 > 0 \Rightarrow \text{MINIMUM. At } x=2: -6 < 0 \Rightarrow \text{MAXIMUM.}$$

Answer: (a)(ii) (1, 22) is a local minimum; (2, 23) is a local maximum

$$(b) \text{ Equation: } 2x^3 - 9x^2 + 10x - 3 = 0. \text{ Rearrange to use curve } y = -2x^3 + 9x^2 - 12x + 27:$$

$$\text{Multiply by } -1: -2x^3 + 9x^2 - 10x + 3 = 0.$$

$$\text{Add } 2x - 24 \text{ to both sides: } -2x^3 + 9x^2 - 12x + 27 = 24 - 2x.$$

$$\text{So the solutions are where the curve } y = -2x^3 + 9x^2 - 12x + 27 \text{ meets the line } y = 24 - 2x.$$

Answer: (b) The straight line is $y = 24 - 2x$ (i.e. $y = -2x + 24$)

Question 12. EITHER: Integration and metal-plate optimisation.

$$(a) dy/dx = 4x - 4/x^2. \text{ Integrate: } y = 2x^2 + 4x^{-1} + C = 2x^2 + 4/x + C.$$

$$\text{Passes through } (2, 10): 10 = 8 + 2 + C = 10 + C \Rightarrow C = 0.$$

Answer: (a) Equation of curve: $y = 2x^2 + 4/x$

$$(b) \text{ Rectangle } y \times x \text{ and quarter-circle radius } x, \text{ perimeter} = 60 \text{ cm.}$$

$$\text{Perimeter: } 2y + 2x + (\pi x)/2 = 60 \Rightarrow y = 30 - x - \pi x/4.$$

$$(b)(i) A = x \cdot y = x(30 - x - \pi x/4) = 30x - x^2 - \pi x^2/4. \text{ Q.E.D.}$$

Answer: (b)(i) $A = 30x - x^2 - \pi x^2/4$ (proved)

$$(b)(ii) dA/dx = 30 - 2x - \pi x/2 = 0 \Rightarrow x = 60/(4 + \pi).$$

Answer: (b)(ii) $x = 60/(4 + \pi) \approx 8.40 \text{ cm}$

$$(b)(iii) d^2A/dx^2 = -2 - \pi/2 < 0 \Rightarrow \text{MAXIMUM. } A_{\max} = 900/(4 + \pi) \approx 126 \text{ cm}^2.$$

Answer: (b)(iii) Maximum area = $900/(4 + \pi) \approx 126 \text{ cm}^2$

Question 12or. OR: Arithmetic and geometric progressions.

(a) AP: $T_5 = 3 \cdot T_2$ and $S_{10} = 50$.

$$a + 4d = 3(a + d) = 3a + 3d \Rightarrow d = 2a \dots (A)$$

$$S_{10} = 5(2a + 9d) = 5(2a + 18a) = 100a = 50 \Rightarrow a = \frac{1}{2}.$$

$$d = 2 \cdot \left(\frac{1}{2}\right) = 1.$$

Answer: (a) First term $a = \frac{1}{2}$, common difference $d = 1$

$$S_{30} = 30/2 \cdot (2 \cdot \left(\frac{1}{2}\right) + 29 \cdot 1) = 15 \cdot (1 + 29) = 15 \cdot 30 = 450.$$

Answer: Sum of first 30 terms = 450

(b) GP: $T_5 = 8$, $T_3 = 4$, $r > 0$.

$$T_5/T_3 = r^2 = 8/4 = 2 \Rightarrow r = \sqrt{2} \text{ (positive root).}$$

$$T_3 = ar^2 = 2a = 4 \Rightarrow a = 2.$$

Answer: (b) First term $a = 2$, common ratio $r = \sqrt{2}$

$$S_{12} = a \cdot (r^{12} - 1)/(r - 1) = 2 \cdot ((\sqrt{2})^{12} - 1)/(\sqrt{2} - 1) = 2 \cdot (64 - 1)/(\sqrt{2} - 1).$$

$$= 126/(\sqrt{2} - 1). \text{ Rationalize: } \cdot ((\sqrt{2} + 1)/(\sqrt{2} + 1)) = 126(\sqrt{2} + 1).$$

Answer: (b) Sum of first 12 terms = $126(1 + \sqrt{2}) \approx 304.2$

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