G12 ECZ ADDITIONAL MATHEMATICS 2016 GCE PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2016 GCE Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

Additional Mathematics 4030/2

Paper 2

Wednesday 27 JULY 2016

Additional materials:

Answer Booklet
Mathematical tables/Electronic calculators (non-programmable)

Time: 2 hours 30 minutes

Instructions to candidates

Write your name, centre number and candidate number in the spaces on the separate Answer Booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Arithmetic $Sn = \frac{1}{2} n\,[2a + (n-1)\,d]$

Geometric $Sn = \frac{a(1-r^n)}{1-r}$ $(r \neq 1)$

$S\infty = \frac{a}{1-r}$ for $|r| < 1$

3 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\text{cosec}^2 A = 1 + \cot^2 A.$

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc \cos A.$

$\Delta = \frac{1}{2}\, bc \sin A$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n},\ \text{SD} = \sqrt{\left\{\frac{\sum (x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f},\ \text{SD} = \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

1 Solve the simultaneous equations

$x + 4y + 3z = 10,$
$2x + y - z = -1,$
$3x - y + z = 11.$ [6]

2 (a) Solve the inequality $3x^2 < 10x - 3$. [3]

(b) Given that $y = 7 - 5x - 3x^2$, express y in the form $a - b(x + c)^2$, where a, b and c are constants. Hence or otherwise, write the coordinates of the turning point of y. [4]

3 Solve the equations

(a) $8^{x-1} = 3$, [3]

(b) $\log_3 (2x + 1) = 2 + \log_3 (3x - 1)$. [4]

4 The expression $x^3 + ax^2 + bx + 12$ is divisible by $(x - 1)$ and $(x + 3)$.

(a) Find the values of a and b. [4]

(b) Find the remaining factor of the expression. [1]

(c) Hence or otherwise, solve the equation $x^3 + ax^2 + bx + 12 = 0$. [3]

5 (a) Calculate the number of arrangements of the letters in the word LOGARITHMS. [2]

(b) 12 different books are to be arranged on a shelf. If the shelf has a space for 8 books, how many ways are there of arranging 8 of the books on the shelf? [3]

(c) A committee of 6 members is to be chosen from 5 students and 4 teachers. Find the number of ways of choosing 4 students and 2 teachers. [3]

6 (a) The volume $V\text{cm}^3$ of a liquid in a container when the depth of the liquid is x cm is given by $V = 2x^3 - 4x^2 + 5$.
Given that the depth is increasing at the rate of 1.5cm/s, find the rate of increase in the volume of the liquid when $x = 4$. [2]

(b) The velocity, Vm/s, of a particle moving in a straight line is given by $V = 4 + 3t - t^2$. Find

(i) the value of t when the particle is at instanteous rest, [2]

(ii) the maximum velocity, [1]

(iii) the displacement of the particle from O when $t = 4$. [3]

7 **(a)** The 9^{th} term of an arithmetic progression is 22 and the sum of the first 4 terms is 49. Find the first term and the common difference. [4]

(b) The 4^{th} term and the 10^{th} term of a geometric progression are 256 and 4 respectively. Find

(i) the first term and the common ratio, [2]

(ii) the sum to infinity of the progression. [2]

8 Find all the angles between 0°and 360° which satisfy the equations

(a) $\sin\theta = -0.5921$, [2]

(b) $6 \sin x \cos x = \sin x$, [3]

(c) $2 \sec^2 y = 3 - \tan^2 y$. [4]

9 The table below shows the marks obtained by 35 candidates in an examination.

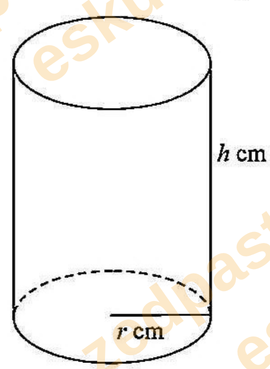
Marks	35	45	55	65	75	85	95
Frequency	3	4	10	5	7	4	2

(a) State the median mark. [1]

(b) Find the mean mark. [4]

(c) Calculate the standard deviation. [4]

10 A closed cylinder has radius r cm and height h cm. The volume of the cylinder is 1 500cm^3.



(a) Find an expression of h in terms of r. [2]

(b) Show that the total surface area, A cm^2, of the cylinder is given by

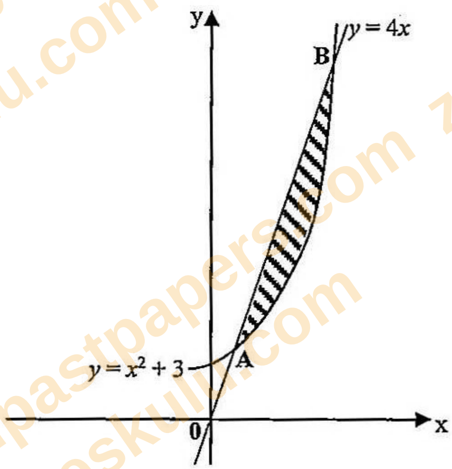
$$A = 2\pi r^2 + \frac{3000}{r}.$$ [2]

(c) Given that r varies, find correct to 2 decimal places, the value of r when A has a stationary value. [4]

(d) Find the stationary value of A and determine its nature. [2]

11 **(a)** Evaluate $\int_1^2 \frac{x+1}{x^3}\,dx.$ [3]

(b) The diagram below shows part of the curve $y = x^2 + 3$ and the line $y = 4x$ intersecting at **A** and **B**.



(i) Find the coordinates of **A** and **B**. [3]

(ii) Calculate the volume generated when the shaded region is rotated through 360° about the x-axis. [4]

12 **Answer only one of the following alternatives:**

EITHER

A curve $y = x + \frac{4}{x}$ passes through a point **P**, where $x = 4$.

(a) Find the equation of the tangent to the curve at the point **P**. [3]

(b) Find the equation of the normal at the point **P**. [3]

(c) Find the co-ordinates of the point **Q**, where the normal meets the curve again. [4]

OR

(a) The first term of an arithmetic progression is 6. Given that the sum of the first 8 terms is 160 and the sum of all the terms is 240, calculate

(i) the common difference, [2]

(ii) the number of terms. [3]

(b) The numbers $x + 3$, $5x + 3$ and $11x + 3$ $(x \neq 0)$ are three consecutive terms of a geometric progression. Find

(i) the value of x, [3]

(ii) the sum to infinity of the progression. [2]

Question 1. Solve the simultaneous equations: x+4y+3z=10, 2x-5y-z=0, 3x-y+z=9.

From E2 substitute into E1 and E3. Let z=2x-5y (from E2: 2x-5y-z=0).

Into E1: x+4y+3(2x-5y)=10 => 7x-11y=10 ...(A).

Into E3: 3x-y+(2x-5y)=9 => 5x-6y=9 ...(B).

5*(A)-7*(B): 35x-55y-35x+42y=50-63 => -13y=-13 => y=1.

From B: 5x-6=9 => 5x=15 => x=3.

z=2(3)-5(1)=6-5=1.

Check E1: 3+4+3=10 ✓ E2: 6-5-1=0 ✓ E3: 9-1+1=9 ✓

Answer: x = 3, y = 1, z = 1

Question 2. Inequality and turning point.

(a) Solve 3x^2 + 10x - 3 >= 0. Factorise: (3x-1)(x+3) = 0 gives x=1/3 or x=-3.

Parabola opens upward, so 3x^2+10x-3 >= 0 OUTSIDE the roots.

Answer: (a) x <= -3 or x >= 1/3

(b) Express y = 7 + 6x - x^2 in the form a - (x+b)^2. Complete the square:

y = -(x^2-6x) + 7 = -(x-3)^2 + 9 + 7 = 16 - (x-3)^2.

Written as a-(x+b)^2: a=16, b=-3 (i.e. 16-(x+(-3))^2 = 16-(x-3)^2).

Since the coefficient of (x-3)^2 is negative, the turning point is a MAXIMUM.

Answer: (b) y = 16 - (x-3)^2; maximum turning point at (3, 16)

Question 3. Solve the equations.

(a) 9^{x-1} = 27^{x+1}. Write both as powers of 3:

3^{2(x-1)} = 3^{3(x+1)} => 2x-2 = 3x+3 => x = -5.

Answer: (a) x = -5

(b) log_2(x+1) - log_2 x = 2. Combine logs:

log_2((x+1)/x) = 2 => (x+1)/x = 4 => x+1 = 4x => 3x = 1.

Answer: (b) x = 1/3

Question 4. Polynomial divisibility.

f(x) = x^3 + ax^2 + bx + 6 is exactly divisible by (x-1) and (x+3), so f(1)=0 and f(-3)=0.

f(1): 1+a+b+6=0 => a+b = -7 ...(A).

f(-3): -27+9a-3b+6=0 => 9a-3b = 21 => 3a-b = 7 ...(B).

(A)+(B): 4a = 0 => a = 0. From A: b = -7.

So f(x) = x^3 - 7x + 6. Divide by (x-1)(x+3) = x^2+2x-3:

x^3-7x+6 = (x^2+2x-3)(x+c). Matching constant: -3c=6 => c=-2.

Check: (x^2+2x-3)(x-2) = x^3-2x^2+2x^2-4x-3x+6 = x^3-7x+6 ✓

Answer: a = 0, b = -7; remaining factor = (x-2)

Answer: x^3 - 7x + 6 = (x-1)(x+3)(x-2)

Question 5. Arrangements and combinations.

(a) Arrange letters of LOGARITHMS (10 letters) with all consonants together.

Vowels: O, A, I (3). Consonants: L, G, R, T, H, M, S (7).

Treat 7 consonants as 1 block. Total units to arrange: 1 block + 3 vowels = 4 units.

Arrangements = 4! * 7! = 24 * 5040 = 120960.

Answer: (a) 120960 arrangements

(b) Committee of 6 from 5 students and 4 teachers with at least 4 teachers.

Since there are only 4 teachers, 'at least 4 teachers' means exactly 4 teachers.

4 teachers and 2 students: C(4,4) * C(5,2) = 1 * 10 = 10.

Answer: (b) 10 ways

Question 6. Particle motion: V = 4 + 3t - t^2.

(a)(i) Instantaneous rest: V = 0. So 4 + 3t - t^2 = 0 => t^2-3t-4=0 => (t-4)(t+1)=0.

t = 4 s (taking positive value).

Answer: (a)(i) The particle is at instantaneous rest at t = 4 s

(a)(ii) Maximum velocity: dV/dt = 3 - 2t = 0 => t = 3/2.

V_max = 4 + 3(3/2) - (3/2)^2 = 4 + 9/2 - 9/4 = 16/4 + 18/4 - 9/4 = 25/4.

Answer: (a)(ii) Maximum velocity = 25/4 = 6.25 m/s (at t = 3/2 s)

(a)(iii) Displacement from O when t=4: s = integral from 0 to 4 of V dt.

s = [4t + 3t^2/2 - t^3/3]_0^4 = 16 + 24 - 64/3 = 40 - 64/3 = 56/3.

Answer: (a)(iii) Displacement = 56/3 ≈ 18.7 m

Question 7. Arithmetic and geometric progressions.

(a) AP: T9 = 49 and sum of first 4 terms S4 = 40.

T9 = a + 8d = 49 ...(A).

S4 = 4/2 * (2a + 3d) = 2(2a+3d) = 4a+6d = 40 ...(B).

4*(A)-(B): 4a+32d-4a-6d = 196-40 => 26d = 156 => d = 6.

From A: a = 49-48 = 1.

Answer: (a) First term a = 1, common difference d = 6

(b) GP: T4 = 256 and T10 = 4.

T10/T4 = r^6 = 4/256 = 1/64. r = (1/64)^{1/6} = 1/2.

T4 = ar^3 = a*(1/8) = 256 => a = 2048.

Answer: (b)(i) Common ratio r = 1/2, first term a = 2048

S_inf = a/(1-r) = 2048 / (1-1/2) = 2048/(1/2) = 4096.

Answer: (b)(ii) Sum to infinity = 4096

Question 8. Find all angles between 0 and 360 degrees satisfying:

(a) sin theta = -0.6921. Basic angle = arcsin(0.6921) ≈ 43.8°. sin negative: Q3 and Q4.

theta = 180+43.8 = 223.8° or theta = 360-43.8 = 316.2°.

Answer: (a) theta ≈ 223.8° and 316.2°

(b) 6 sin x - cos x = 0 => 6 sin x = cos x => tan x = 1/6.

x = arctan(1/6) ≈ 9.5° or 9.5+180 = 189.5°.

Answer: (b) x ≈ 9.5° and 189.5°

(c) 2cos^2 x + sin x = 1. Use cos^2 x = 1-sin^2 x:

2(1-sin^2 x)+sin x = 1 => 2sin^2 x-sin x-1 = 0 => (2sin x+1)(sin x-1) = 0.

sin x = -1/2: x = 210° or x = 330°.

sin x = 1: x = 90°.

Answer: (c) x = 90°, 210°, 330°

Question 9. Marks of 35 candidates — frequency table.

Marks: 11-20 | 21-30 | 31-40 | 41-50 | 51-60 | 61-70 | 71-80

Frequency: 3 | 4 | 10 | 5 | 7 | 4 | 2 (total=35)

Midpoints: 15.5, 25.5, 35.5, 45.5, 55.5, 65.5, 75.5.

(a) Median: n=35, median is the 18th value. Cumulative: 3,7,17,22,...

The 18th value falls in class 41-50. Median = 40.5 + (18-17)/5 * 10 = 41.5.

Answer: (a) Median mark = 41.5

(b)(i) Sigma(fx) = 3(15.5)+4(25.5)+10(35.5)+5(45.5)+7(55.5)+4(65.5)+2(75.5)

= 46.5+102+355+227.5+388.5+262+151 = 1532.5.

Mean = 1532.5/35 ≈ 43.8.

Answer: (b)(i) Mean ≈ 43.8

(b)(ii) Sigma(fx^2) = 3(240.25)+4(650.25)+10(1260.25)+5(2070.25)+7(3080.25)+4(4290.25)+2(5700.25)

= 720.75+2601+12602.5+10351.25+21561.75+17161+11400.5 = 76398.75.

Variance = 76398.75/35 - (43.8)^2 = 2182.82 - 1918.44 = 264.38.

SD = sqrt(264.38) ≈ 16.3.

Answer: (b)(ii) Standard deviation ≈ 16.3

Question 10. Closed cylinder: radius r cm, height h cm, volume 1500 cm^3.

(a) Volume: pi*r^2*h = 1500 => h = 1500/(pi*r^2).

Answer: (a) h = 1500 / (pi*r^2)

(b) Total surface area: A = 2*pi*r^2 + 2*pi*r*h = 2*pi*r^2 + 2*pi*r * 1500/(pi*r^2).

= 2*pi*r^2 + 3000/r. Q.E.D.

Answer: (b) A = 2*pi*r^2 + 3000/r (proved)

(c) dA/dr = 4*pi*r - 3000/r^2 = 0 => 4*pi*r^3 = 3000 => r^3 = 750/pi.

r = (750/pi)^{1/3} = (238.73)^{1/3} ≈ 6.20 cm.

Answer: (c) r ≈ 6.20 cm

(d) d^2A/dr^2 = 4*pi + 6000/r^3 > 0, so the stationary point is a MINIMUM.

A_min = 2*pi*(6.20)^2 + 3000/6.20 ≈ 241.7 + 483.9 ≈ 726 cm^2.

Answer: (d) Minimum surface area ≈ 726 cm^2 (d^2A/dr^2 > 0 confirms minimum)

Question 11. Integration and area.

(a) Evaluate integral from 1 to 2 of (x+1)/x^3 dx = integral of (x^{-2} + x^{-3}) dx.

= [-x^{-1} - x^{-2}/2]_1^2 = [-1/x - 1/(2x^2)]_1^2.

At x=2: -1/2 - 1/8 = -5/8. At x=1: -1 - 1/2 = -3/2.

= -5/8 - (-3/2) = -5/8 + 12/8 = 7/8.

Answer: (a) Integral = 7/8

(b) Curve y = x^2+3 and line y = 4x intersect at A and B.

Set equal: x^2+3 = 4x => x^2-4x+3=0 => (x-1)(x-3)=0.

x=1: y=4. A=(1,4). x=3: y=12. B=(3,12).

Answer: (b)(i) A = (1, 4) and B = (3, 12)

(b)(ii) Volume rotating shaded region (line y=4x above curve y=x^2+3) about x-axis:

V = pi * integral from 1 to 3 of [(4x)^2 - (x^2+3)^2] dx.

(4x)^2 = 16x^2. (x^2+3)^2 = x^4+6x^2+9. Integrand = -x^4+10x^2-9.

= pi * [-x^5/5 + 10x^3/3 - 9x]_1^3.

At x=3: -243/5+90-27 = -243/5+63 = 72/5.

At x=1: -1/5+10/3-9 = (-3+50-135)/15 = -88/15.

V = pi*(72/5 - (-88/15)) = pi*(216/15+88/15) = pi*(304/15).

Answer: (b)(ii) Volume = 304*pi/15 cubic units

Question 12. EITHER: Curve y = x + 4/x through point P at x = 4.

At x=4: y = 4+4/4 = 5. So P = (4, 5).

dy/dx = 1 - 4/x^2. At x=4: 1-4/16 = 1-1/4 = 3/4. Gradient of tangent = 3/4.

(a) Tangent at P: y-5 = 3/4*(x-4) => y = 3x/4-3+5 => y = 3x/4+2.

Answer: (a) Tangent: y = 3x/4 + 2

(b) Normal gradient = -4/3 (negative reciprocal). Normal at P=(4,5):

y-5 = -4/3*(x-4) => y = -4x/3+16/3+5 => 3y = -4x+16+15 => 4x+3y=31.

Answer: (b) Normal: 4x + 3y = 31

(c) Normal meets curve y=x+4/x again. Substitute y=(-4x+31)/3 into curve:

(-4x+31)/3 = x+4/x. Multiply by 3x: -4x^2+31x = 3x^2+12.

7x^2-31x+12=0. x=(31 ± sqrt(961-336))/14 = (31 ± 25)/14.

x=56/14=4 (point P) or x=6/14=3/7.

At x=3/7: y = 3/7 + 4/(3/7) = 3/7+28/3 = 9/21+196/21 = 205/21.

Answer: (c) Q = (3/7, 205/21)

Question 12or. OR: Arithmetic and geometric progressions.

(a) AP: first term a=6, sum of first 8 terms S8=160, sum of all terms=240.

S8 = 8/2*(2(6)+7d) = 4(12+7d) = 48+28d = 160 => 28d=112 => d=4.

Answer: (a)(i) Common difference d = 4

Total sum Sn=240: n/2*(12+(n-1)4) = n(4+2n) = 2n^2+4n = 240.

n^2+2n-120=0 => (n+12)(n-10)=0 => n=10.

Check: S10=10/2*(12+36)=5*48=240 ✓

Answer: (a)(ii) Number of terms = 10

(b) x+3, 5x+3, 11x+3 are consecutive GP terms.

(b)(i) For GP: (5x+3)^2 = (x+3)(11x+3).

25x^2+30x+9 = 11x^2+36x+9 => 14x^2-6x=0 => 2x(7x-3)=0.

x=0 (rejected, x≠0) or x=3/7.

Answer: (b)(i) x = 3/7

(b)(ii) Terms: x+3=24/7, 5x+3=36/7, 11x+3=54/7. Common ratio r=36/24=3/2.

|r| = 3/2 > 1, so the GP is divergent.

Answer: (b)(ii) Sum to infinity does NOT exist (|r| = 3/2 > 1, series diverges)

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