G12 ECZ ADDITIONAL MATHEMATICS 2016 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2016 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/2

Paper 2

Tuesday **15 NOVEMBER 2016**

Additional Materials:

Answer Booklet;
Electronic calculators

Time: 2 hours 30 Minutes

Instructions to Candidates

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 SERIES

Arithmetic $\quad Sn = \frac{1}{2} n \{2a + (n - 1) d\}$

Geometric $\quad Sn = \frac{a(1 - r^n)}{1 - r} \ (r \neq 1)$

$$S\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\operatorname{cosec}^2 A = 1 + \cot^2 A.$

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc \cos A.$

$$\Delta = \frac{1}{2} bc \sin A$$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \ \textbf{SD} = \sqrt{\left\{\frac{\sum (x - \bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \ \textbf{SD} = \sqrt{\left\{\frac{\sum f(x - \bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

1 Solve the following systems of equations

$x + 3y - 2z = 5,$

$3x - y + z = 9,$

$x - 2y - 3z = -7.$ [6]

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2 (a) Find the range of values of x for which $2x^2 - 7x + 3 \geq 0$. [3]

(b) Express $3x^2 - 24x + 50$ in the form $a(x + b)^2 + c$, where a, b and c are constants. Hence, find the coordinates of the turning point. [4]

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3 Solve the equations

(a) $4^{x-1} - 2^x = 8,$ [4]

(b) $\log_3 (2x + 1) - 2 = \log_3 (3x - 11).$ [3]

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4 (a) Find the value of q, given that the expression $2x^3 - 3x^2 - qx + 6$ is divisible by $2x - 1$. [3]

(b) The expression $7x^2 + 35$ and $44x - 2x^3$ leaves the same remainder when divided by $x - a$. find the three possible values of a. [4]

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5 (a) In how man ways can 7 red marbles and 3 green marbles be put in a straight line if

(i) there are no restrictions, [2]

(ii) green marbles should not be next to each other? [3]

(b) A group of 6 pupils is to be chosen from 8 boys and 6 girls. Find the number of ways of choosing at least 4 girls. [3]

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6 (a) Solve the equation $2\sin 2x + 1 = 0$ for values of x in the range $0° \leq x \leq 360°$. [3]

(b) (i) Express $5\sin \theta - 12\cos \theta$ in the form $R \sin(\theta - \propto)$ where $R > 0$ and $0 < \propto < 90°$. [3]

(ii) Hence solve the equation $5\sin \theta - 12\cos \theta = 6.5$, giving all solutions between $0°$ and $360°$. [2]

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7 **(a)** The third and sixth terms of a geometric progression are $2\frac{2}{3}$ and $\frac{8}{81}$ respectively.

Find

(i) the common ratio and the first term, [3]

(ii) the sum to infinity of the geometric progression. [2]

(b) The nth term of an arithmetic progression is $4n + 1$. Find, in terms of n, the sum of the first n terms of the progression. [4]

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8 A new drug for a certain disease was administered to 60 patients of different age groups. The table below shows the results obtained.

Age (x)	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35
Frequency	5	8	12	18	9	6	2

(a) State the median class. [1]

(b) Calculate

(i) an estimate of the mean, [2]

(ii) the standard deviation. [6]

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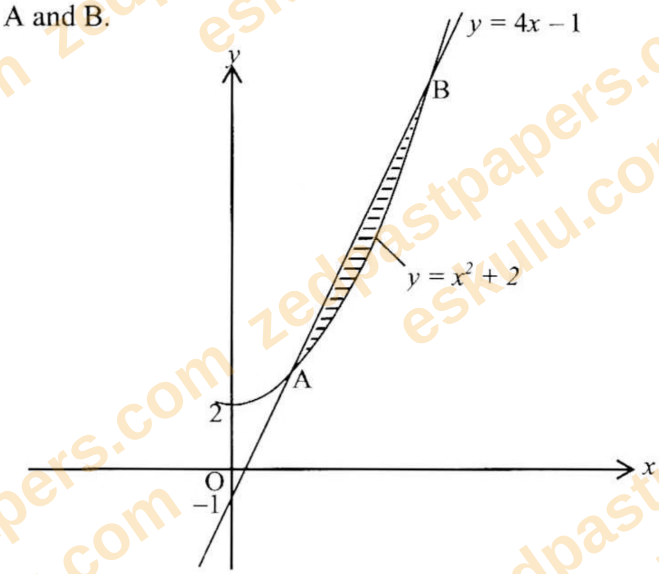
9 The diagram below shows parts of the curve $y = x^2 + 2$ and the line $y = 4x - 1$ intersecting at the points A and B.



Find

(a) the coordinates of A and B, [4]

(b) the volume obtained by rotating the shaded region through 360° about the x-axis. [5]

10 A small body moves in a straight line so that its velocity v m/s at time t seconds from the starting point O, is given by $v = 6t^2 - 18t + 12$.

(a) Find an expression in terms of t for

(i) its acceleration, [2]

(ii) its distance from O. [2]

(b) Calculate the maximum velocity and the distance covered when the velocity is maximum. [6]

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11 **(a)** A curve has the equation $y = 2x^3 - 9x^2 + 12x + 8$.

(i) Find the coordinates of the stationary points of the curve. [3]

(ii) Determine the nature of the stationary points. [3]

(b) Find the coordinates of the point where the curves $y = e^{\frac{x}{2} - 3}$ and $y = e^{7+3x}$ meet and the gradient of each curve at the point. [4]

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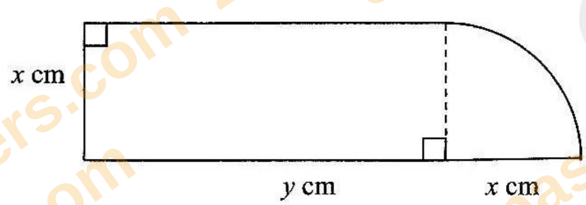
12 **Answer only one of the following alternatives:**

Either

(a) Given that $y = x + \sin 2x$, find the values of x for which $\frac{dy}{dx} = 0$, for the range $0 \leq x \leq \pi$ [3]

(b) The diagram below shows a metal plate consisting of a rectangle of length y cm and width x cm, and a quarter-circle of radius x cm. The perimeter of the metal plate is 60 cm.



(i) Show that the area of the plate, A cm^2, is given by $A = 30x - x^2$. [3]

(ii) Given that x can vary, find the value of x at which A is stationary, [1]

(iii) Hence, find the stationary value of A and determine whether it is a maximum or a minimum. [3]

Or

(a) The eighth term of an arithmetic progression is four times the fifth term and the sum of the first eight terms is 20. Calculate the sum of the first ten terms. [5]

(b) The second and the fifth terms of a geometric progression are $\frac{1}{18}$ and $\frac{4}{243}$ respectively.

Find

(i) the common ratio and the first term, [3]

(ii) the sum to infinity of the progression. [2]

Question 1. Solve the simultaneous equations: x-3y+2z=5, 3x-y+z=8, 2x-2y-3z=-7.

From E1+E3: 3x-5y-z=-2 ...(A). Wait, use systematic elimination.

E1: x-3y+2z=5. From E3: 2x-2y-3z=-7. Eliminate x:

E3-2*E1: 2x-2y-3z-2x+6y-4z=-7-10 => 4y-7z=-17 ...(A)

E2-3*E1: 3x-y+z-3x+9y-6z=8-15 => 8y-5z=-7 ...(B)

From (A)*5-(B)*7: 20y-35z-56y+35z=-85+49 => -36y=-36 => y=1.

From A: 4(1)-7z=-17 => -7z=-21 => z=3.

From E1: x-3+6=5 => x=2.

Check E2: 6-1+3=8 ✓ E3: 4-2-9=-7 ✓

Answer: x = 2, y = 1, z = 3

Question 2. Inequality and completing the square.

(a) Solve 2x^2 - 7x + 3 >= 0. Factorise: (2x-1)(x-3) >= 0.

Roots: x=1/2 and x=3. Parabola opens up, so >= 0 outside the roots.

Answer: (a) x <= 1/2 or x >= 3

(b) Express 3x^2 + 24x + 50 in the form a(x+b)^2 + c. Find turning point.

3x^2+24x+50 = 3(x^2+8x)+50 = 3(x+4)^2-48+50 = 3(x+4)^2+2.

a=3, b=4, c=2. Since a>0, this is a MINIMUM.

Answer: (b) 3(x+4)^2 + 2; minimum turning point = (-4, 2)

Question 3. Solve the equations.

(a) 7^{x-1} = 10. Take log of both sides:

(x-1) log 7 = log 10 = 1 => x-1 = 1/log 7 = 1/0.8451 = 1.183.

x = 1+1.183 = 2.183.

Answer: (a) x ≈ 2.18

(b) lg(2x+1) - 2 lg x = 1. Rewrite: lg(2x+1) - lg(x^2) = 1.

lg((2x+1)/x^2) = 1 => (2x+1)/x^2 = 10 => 2x+1 = 10x^2.

10x^2 - 2x - 1 = 0. Quadratic formula: x = (2 ± sqrt(4+40))/20 = (1 ± sqrt(11))/10.

x = (1+sqrt(11))/10 ≈ 0.432. (Reject negative root since x>0.)

Answer: (b) x = (1+sqrt(11))/10 ≈ 0.432

Question 4. Polynomial divisibility and remainder.

(a) 2x^3 - x^2 - qx + 6 is exactly divisible by (2x-1). So f(1/2)=0.

f(1/2) = 2(1/8)-(1/4)-q(1/2)+6 = 1/4-1/4-q/2+6 = -q/2+6 = 0 => q=12.

Polynomial becomes 2x^3-x^2-12x+6. Divide by (2x-1):

2x^3-x^2-12x+6 = (2x-1)(x^2-6). [Check: (2x-1)(x^2-6)=2x^3-12x-x^2+6 ✓]

x^2-6 = (x-sqrt(6))(x+sqrt(6)).

Answer: (a) q = 12; factors: (2x-1)(x - sqrt(6))(x + sqrt(6))

(b) 3x^3 - 13x^2 + 18x - 2 = (x-1)(x-2)(Ax+C) + B. Expand (x-1)(x-2)=x^2-3x+2.

(x^2-3x+2)(Ax+C)+B = Ax^3+(C-3A)x^2+(2A-3C)x+2C+B.

Match with 3x^3-13x^2+18x-2: A=3; C-9=-13 => C=-4; 2(3)-3(-4)=18 ✓; -8+B=-2 => B=6.

Answer: (b) A = 3, B = 6, C = -4

Question 5. Arrangements and combinations.

(a) 3 distinct red marbles and 3 distinct green marbles arranged in a straight line.

(a)(i) No restrictions: 6! = 720 ways.

Answer: (a)(i) 720 ways

(a)(ii) No two marbles of the same colour adjacent: colours must alternate.

Pattern RGRGRG or GRGRG: 2 colour patterns. Red arranged in 3! ways, Green in 3! ways.

Total = 2 * 3! * 3! = 2*6*6 = 72.

Answer: (a)(ii) 72 ways

(b) Group of 6 from 8 boys and 9 girls with at least 4 girls:

4 girls, 2 boys: C(9,4)*C(8,2) = 126*28 = 3528.

5 girls, 1 boy: C(9,5)*C(8,1) = 126*8 = 1008.

6 girls, 0 boys: C(9,6)*C(8,0) = 84*1 = 84.

Total = 3528+1008+84 = 4620.

Answer: (b) 4620 ways

Question 6. Find all angles between 0 and 360 degrees satisfying:

(a) cos x = -0.4713. Basic angle = arccos(0.4713) ≈ 61.9°. cos negative: Q2 and Q3.

x = 180-61.9 = 118.1° or x = 180+61.9 = 241.9°.

Answer: (a) x ≈ 118.1° and 241.9°

(b) 5 sin theta - 13 cos theta = 0 => tan theta = 13/5 = 2.6.

theta = arctan(2.6) ≈ 69.0° or 69.0+180 = 249.0°.

Answer: (b) theta ≈ 69.0° and 249.0°

(c) 4 tan^2 x + 4 sec x - 1 = 0. Use tan^2 x = sec^2 x - 1:

4 sec^2 x + 4 sec x - 5 = 0. sec x = (-1 ± sqrt(6))/2.

sec x = (-1+sqrt(6))/2 ≈ 0.72 => cos x > 1. No solution.

sec x = (-1-sqrt(6))/2 ≈ -1.72 => cos x ≈ -0.580. arccos(0.580) ≈ 54.6°.

x = 180-54.6 = 125.4° or x = 180+54.6 = 234.6°.

Answer: (c) x ≈ 125.4° and 234.6°

Question 7. Geometric and arithmetic progressions.

(a) GP: T3 = 8/3 (= 2 2/3), T6 = 8/81.

T6/T3 = r^3 = (8/81)/(8/3) = 1/27 => r = 1/3.

T3 = a*r^2 = a/9 = 8/3 => a = 24.

Answer: (a)(i) Common ratio r = 1/3, first term a = 24

S_inf = a/(1-r) = 24/(2/3) = 36.

Answer: (a)(ii) Sum to infinity = 36

(b) The nth term of an AP is 4n+1. T1=5, T2=9, so d=4 and a=5.

Sn = n/2*(T1+Tn) = n/2*(5+4n+1) = n/2*(4n+6) = n(2n+3).

Alternatively: Sn = Sum from k=1 to n of (4k+1) = 4*n(n+1)/2+n = 2n^2+2n+n = 2n^2+3n.

Answer: (b) Sum of first n terms = 2n^2 + 3n = n(2n+3)

Question 8. Drug trial frequency table — 60 patients of different ages.

Age: 1-5 | 6-10 | 11-15 | 16-20 | 21-25 | 26-30 | 31-35

Freq: 5 | 8 | 12 | 18 | 9 | 6 | 2 Total=60.

Midpoints: 3, 8, 13, 18, 23, 28, 33.

(a) Median: n=60, n/2=30. Cumulative: 5,13,25,43,... 30th is in 16-20 class.

Answer: (a) Median class = 16-20

(b)(i) Mean: Sigma(fx) = 5(3)+8(8)+12(13)+18(18)+9(23)+6(28)+2(33)

= 15+64+156+324+207+168+66 = 1000. Mean = 1000/60 = 50/3 ≈ 16.7.

Answer: (b)(i) Estimated mean = 50/3 ≈ 16.7

(b)(ii) Sigma(fx^2) = 5(9)+8(64)+12(169)+18(324)+9(529)+6(784)+2(1089)

= 45+512+2028+5832+4761+4704+2178 = 20060.

Variance = 20060/60 - (50/3)^2 = 334.33 - 277.78 = 56.55.

SD = sqrt(56.55) ≈ 7.52.

Answer: (b)(ii) Standard deviation ≈ 7.52

Question 9. Curve y=x^2+2 intersected by line y=4x-1 at points A and B.

Find A and B: x^2+2=4x-1 => x^2-4x+3=0 => (x-1)(x-3)=0.

x=1: y=3. A=(1,3). x=3: y=11. B=(3,11).

Answer: (a) A = (1, 3) and B = (3, 11)

(b) Volume of shaded region (line above, curve below, x from 1 to 3) rotated about x-axis:

V = pi * integral from 1 to 3 of [(4x-1)^2 - (x^2+2)^2] dx.

(4x-1)^2 = 16x^2-8x+1. (x^2+2)^2 = x^4+4x^2+4.

Integrand: -x^4+12x^2-8x-3.

= pi * [-x^5/5 + 4x^3 - 4x^2 - 3x] from 1 to 3.

At x=3: -243/5+108-36-9=-243/5+63=72/5.

At x=1: -1/5+4-4-3=-1/5-3=-16/5.

V = pi*(72/5-(-16/5)) = pi*(88/5).

Answer: (b) Volume = 88*pi/5 cubic units

Question 10. Particle velocity V = 6t + 18/t - 12.

(a)(i) Acceleration a = dV/dt = 6 - 18*t^{-2} = 6 - 18/t^2.

Answer: (a)(i) a = 6 - 18/t^2 m/s^2

(a)(ii) Distance from O: s = integral of V dt = 3t^2 + 18 ln(t) - 12t + C.

Answer: (a)(ii) s = 3t^2 + 18 ln(t) - 12t + C

(b) Stationary velocity when a=0: 6-18/t^2=0 => t^2=3 => t=sqrt(3) s.

V_min = 6*sqrt(3) + 18/sqrt(3) - 12 = 6*sqrt(3) + 6*sqrt(3) - 12 = 12*sqrt(3)-12 = 12(sqrt(3)-1) ≈ 8.78 m/s.

d^2V/dt^2 = 36/t^3 > 0, confirming this is a minimum.

Answer: (b) Minimum velocity = 12(sqrt(3)-1) ≈ 8.78 m/s at t = sqrt(3) s

Distance at t=sqrt(3) (measuring from t=1, starting point):

s = 3(3) + 18 ln(sqrt(3)) - 12*sqrt(3) - (3+0-12) = 18 + 9 ln 3 - 12*sqrt(3) ≈ 7.1 m.

Answer: Distance covered ≈ 7.1 m

Question 11. Stationary points and exponential curves.

(a) y = 2x^3 - 9x^2 + 12x + 8. dy/dx = 6x^2-18x+12 = 6(x^2-3x+2) = 6(x-1)(x-2).

(a)(i) Stationary points at x=1 and x=2.

y(1) = 2-9+12+8 = 13. y(2) = 16-36+24+8 = 12.

Answer: (a)(i) Stationary points: (1, 13) and (2, 12)

(a)(ii) d^2y/dx^2 = 12x-18. At x=1: -6 < 0 => MAXIMUM. At x=2: 6 > 0 => MINIMUM.

Answer: (a)(ii) (1, 13) is a local maximum; (2, 12) is a local minimum

(b) Curves y = e^{x-1} and y = e^{2-x}. Set equal: e^{x-1} = e^{2-x}.

x-1 = 2-x => 2x = 3 => x = 3/2. y = e^{1/2} = sqrt(e).

Gradient of y=e^{x-1}: dy/dx=e^{x-1}. At x=3/2: e^{1/2} = sqrt(e) ≈ 1.65.

Gradient of y=e^{2-x}: dy/dx=-e^{2-x}. At x=3/2: -e^{1/2} = -sqrt(e) ≈ -1.65.

Answer: (b) Point of meeting: (3/2, sqrt(e)). Gradients: sqrt(e) and -sqrt(e).

Question 12. EITHER: Trig derivative and metal-plate optimisation.

(a) y = x + sin(2x), 0 <= x <= pi. dy/dx = 1 + 2cos(2x) = 0.

cos(2x) = -1/2. 2x = 2*pi/3 or 4*pi/3. x = pi/3 or x = 2*pi/3.

Answer: (a) x = pi/3 and x = 2*pi/3

(b) Rectangle (y cm by x cm) + quarter-circle radius x cm. Perimeter=60cm.

Perimeter: 2y + 2x + (pi*x)/2 = 60 => y = 30 - x - pi*x/4.

(b)(i) Area A = x*y = x(30-x-pi*x/4) = 30x - x^2 - pi*x^2/4. Q.E.D.

Answer: (b)(i) A = 30x - x^2 - pi*x^2/4 (proved)

(b)(ii) dA/dx = 30 - 2x - pi*x/2 = 0 => 30 = x(2+pi/2) = x*(4+pi)/2.

x = 60/(4+pi).

Answer: (b)(ii) x = 60/(4+pi) ≈ 8.40 cm

(b)(iii) d^2A/dx^2 = -2-pi/2 < 0. MAXIMUM.

A_max = 900/(4+pi) ≈ 126 cm^2.

Answer: (b)(iii) Maximum area ≈ 126 cm^2 (d^2A/dx^2 < 0 => maximum confirmed)

Question 12or. OR: Arithmetic and geometric progressions.

(a) AP: T8 = 4*T5 and S8 = 20.

a+7d = 4(a+4d) => a+7d=4a+16d => -3a=9d => a=-3d ...(1)

S8 = 8/2*(2a+7d) = 4(2(-3d)+7d) = 4(d) = 4d = 20 => d=5.

a = -3*5 = -15.

S10 = 10/2*(2(-15)+9*5) = 5*(-30+45) = 5*15 = 75.

Answer: (a) Sum of first ten terms = 75

(b) GP: T2 = 1/18, T5 = 4/243.

T5/T2 = r^3 = (4/243)/(1/18) = (4*18)/243 = 72/243 = 8/27.

r = 2/3.

T2 = a*r = a*(2/3) = 1/18 => a = 1/18 * 3/2 = 1/12.

Answer: (b)(i) Common ratio r = 2/3, first term a = 1/12

S_inf = (1/12)/(1-2/3) = (1/12)/(1/3) = 3/12 = 1/4.

Answer: (b)(ii) Sum to infinity = 1/4

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