G12 ECZ ADDITIONAL MATHEMATICS 2015 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2015 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/2

PAPER 2

Monday 26 OCTOBER 2015

Additional materials:
Answer Booklet
Mathematical tables/Electronic calculators
Graph paper (1 sheet)

Time: 2 hours 30 minutes

Instructions to candidates

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 SERIES

Arithmetic $\quad S_n = \frac{1}{2}n\{2a + (n-1)d\}$

Geometric $\quad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\text{cosec}^2 A = 1 + \cot^2 A.$

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc \cos A.$

$\Delta = \frac{1}{2} bc \sin A$

4 STATISTICS

Mean and standard deviation

Ungrouped data

Mean $(\bar{x}) = \frac{\sum x}{n}$, SD $= \sqrt{\left\{\frac{\sum(x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$

Grouped data

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$, SD $= \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$

1 Solve the simultaneous equations

$4x - y + z = 10,$

$x + 2y - 3z = -3,$

$2x + y + 2z = 5.$ [6]

2 (a) Solve the inequality $2x^2 + x < 3 - 4x$. [3]

(b) Given that $h(x) = -2x^2 + 16x - 24$, express $h(x)$ in the form $a(x - b)^2 + k$ where a, b and k are constants. Hence state the maximum value of $h(x)$. [4]

3 Solve the equations

(a) $7^{x-1} = 0.432$, [3]

(b) $\lg(x + 6) - \lg(x - 3) = 1$. [4]

4 (a) Given that the expression $2x^3 + px^2 - 8x + q$ is exactly divisible by $2x^2 - 7x + 6$, find the values of p and q. [4]

(b) Given that $3x^3 - 13x^2 + 18x - 10 = (Ax + B)(x - 1)(x - 2) + C$ for all values of x. Find the values of A, B and C. [4]

5 (a) In how many ways can seven Additional Mathematics books and two English books be arranged on a shelf if

(i) there are no restrictions, [2]

(ii) one English book is put at one end and the other English book at the other end. [3]

(b) In how many ways can a president, a vice president, a secretary, a treasurer and 2 committee members be appointed from a group of 10 candidates? [3]

6 Find all the angles between 0° and 360° which satisfy the equations

(a) $\cos x = -0.4713$, [2]

(b) $5 \sin \theta - 13 \cos \theta = 0$, [2]

(c) $4 \tan^2 y + 4 \sec y + 1 = 0$. [4]

7 The first term of an arithmetic progression is 8 and the common difference is d, where $d \neq 0$. The first term, the fifth term and the eighth term of this arithmetic progression are the first term, the second term and the third term respectively, of a geometric progression whose common ratio is r.

(a) **(i)** Write two equations connecting d and r. [1]

(ii) Hence find the value of r and the value of d. [3]

(b) Find the sum to infinity of the geometric progression. [2]

(c) Find the sum of the first 8 terms of the arithmetic progression. [3]

8 The masses of 40 packets of tea of a certain brand, measured correct to the nearest gram, are shown in the table below.

Mass (grams)	140 – 144	145 – 149	150 – 154	155 – 159	160 – 164
No of packets	5	10	9	8	8

(a) State the median class. [1]

(b) Find

(i) an estimate of the mean, [3]

(ii) the standard deviation. [5]

9 A particle moves in a straight line so that its velocity V ms^{-1} at time t seconds from the start is given by $V = 4t - \frac{t^2}{2}$.

(a) Find

(i) an expression for acceleration, a, in terms of t, [1]

(ii) the acceleration when $t = 2$. [1]

(iii) the maximum velocity. [3]

(b) If the particle started from a fixed point O on the line, how far is it from O after 8 seconds? [4]

10 **(a)** The equation of a curve is $y = \frac{6}{5-2x}$. Calculate the gradient of the curve at the point where $x = \frac{1}{2}$. [3]

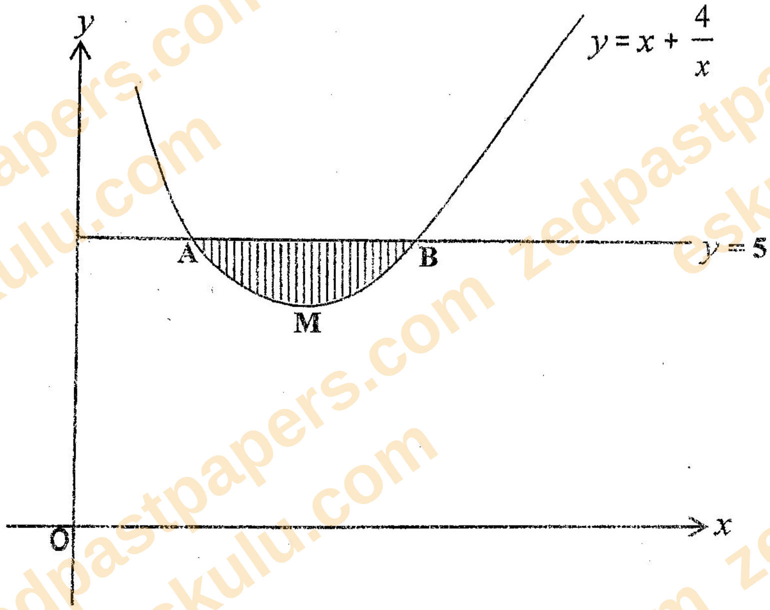
(b) A curve has the equation $p = 2t^3 - 3t^2 - 12t + 18$.

(i) Find the stationary points. [3]

(ii) Determine the nature of the stationary points. [3]

11 The diagram below shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at **M**.

The line $y = 5$ intersects the curve at the points **A** and B.



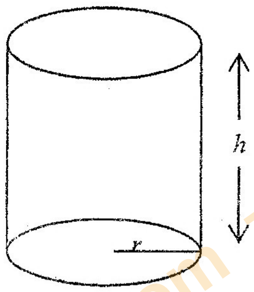
(a) Find the coordinates of **A**, **B** and **M**. [4]

(b) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

12 **Answer only one of the following alternatives:**

EITHER

A closed cylinder shown below is made from thin sheet metal. Its radius is r cm and height h cm. The volume of the cylinder is 600cm^3 and its external surface area is Acm^2.



(a) Show that $\mathbf{A} = 2\pi r^2 + \frac{1200}{r}$. [3]

(b) Find the value, to two significant figures, of r and h for which **A** has a stationary value. [3]

(c) Calculate the stationary value of **A** and determine whether it is a maximum or a minimum. [4]

OR

(a) The third and sixth terms of a geometric progression are $2\frac{2}{3}$ and $\frac{8}{81}$ respectively. Find the sum to infinity. [4]

(b) An arithmetic progression has third term 90 and fifth term 80.

(i) Find the first term and the common difference. [3]

(ii) Find the value of p, given that the sum of the first p terms is equal to the sum of the first $(p + 1)$ terms. [3]

Question 1. Solve the simultaneous equations: $4u-v+z=10$, $u+2v-3z=-3$, $2u+v+2z=5$.

From E1+E3: $6u+3z=15 \Rightarrow 2u+z=5$ ...(A)

From E1+E3 with v: E1+E3 => $6u+3z=15$. Also E1-E3: $2u-2v-z=5$ => (A) and (B) as below.

E3: $2u+v+2z=5$. E1: $4u-v+z=10$. Add: $6u+3z=15 \Rightarrow 2u+z=5$ ...(A)

E3-E1: $-2u+2v+z=-5$. Also E2: $u+2v-3z=-3$. Subtract: E3-E1-E2: $-3u+4z=-2$ ...(B)

From (A): $z=5-2u$. Into (B): $-3u+4(5-2u)=-2 \Rightarrow -3u+20-8u=-2 \Rightarrow -11u=-22 \Rightarrow u=2$.

$z=5-4=1$.

From E3: $4+v+2=5 \Rightarrow v=-1$.

Check E1: $8+1+1=10$ ✓ E2: $2-2-3=-3$ ✓ E3: $4-1+2=5$ ✓

Answer: $u = 2$, $v = -1$, $z = 1$

Question 2. Inequality and completing the square.

(a) Solve $2x^2 + x \le 3 - 4x \Rightarrow 2x^2 + 5x - 3 \le 0$.

Discriminant $= 25+24 = 49$. Roots: $x=(-5+7)/4=1/2$ or $x=(-5-7)/4=-3$.

$(2x-1)(x+3) \le 0$. Since leading coeff > 0: solution between roots.

Answer: (a) $-3 \le x \le 1/2$

(b) $h(x) = -2x^2+16x-24$. Express as $a(x+b)^2 + k$.

$h(x) = -2(x^2-8x)-24 = -2(x-4)^2 + 2*16 - 24 = -2(x-4)^2 + 8$.

$a=-2$, $b=-4$, $k=8$. This is a downward parabola so $k=8$ is the MAXIMUM.

Answer: (b) $h(x) = -2(x-4)^2 + 8$; maximum value of $h(x) = 8$

Question 3. Solve the equations.

(a) $7^{x+5} = 0.432$. Take logarithm of both sides:

$(x+5) \log 7 = \log 0.432 \Rightarrow x+5 = \log(0.432)/\log(7) = -0.3645/0.8451 = -0.4313$.

$x = -0.4313 - 5 = -5.4313$.

Answer: (a) $x \approx -5.43$

(b) $\lg(x+5) - \lg(x-1) = 1$. Combine: $\lg((x+5)/(x-1)) = 1$.

$(x+5)/(x-1) = 10 \Rightarrow x+5 = 10x-10 \Rightarrow 9x = 15 \Rightarrow x = 5/3$.

Check: $x-1 = 2/3 > 0$ and $x+5 = 20/3 > 0$. $\lg(20/3) - \lg(2/3) = \lg(10) = 1$. ✓

Answer: (b) $x = 5/3$

Question 4. Polynomial division and remainder.

(a) $2x^3 + px^2 - 8x + q$ is exactly divisible by $2x^2-7x+6=(2x-3)(x-2)$.

Roots of divisor: $x=2$ and $x=3/2$. So these are roots of the cubic.

At $x=2$: $16+4p-16+q=0 \Rightarrow 4p+q=0$ ...(A)

At $x=3/2$: $2(27/8)+p(9/4)-8(3/2)+q=0 \Rightarrow 27/4+9p/4-12+q=0$.

Multiply by 4: $27+9p-48+4q=0 \Rightarrow 9p+4q=21$ ...(B)

From A: $q=-4p$. Into B: $9p-16p=21 \Rightarrow -7p=21 \Rightarrow p=-3$. $q=12$.

Third factor: $(2x^3-3x^2-8x+12)/(2x^2-7x+6) = x+2$ (by long division).

Answer: (a) p = -3, q = 12; third factor is (x+2)

(b) $3x^3 - 13x^2 + 18x - 2 = (x-1)(x-2)(Ax+C) + B$.

Expand $(x-1)(x-2)(Ax+C)$: first $(x-1)(x-2)=x^2-3x+2$.

$(x^2-3x+2)(Ax+C)=Ax^3+(C-3A)x^2+(2A-3C)x+2C$. Add B.

Match with $3x^3-13x^2+18x-2$: $A=3$, $C-9=-13 \Rightarrow C=-4$, $2A-3C=6+12=18$ ✓, $2(-4)+B=-2 \Rightarrow B=6$.

Answer: (b) A = 3, B = 6, C = -4

Question 5. Arrangements and selections.

5 Additional Mathematics books and 2 English books: 7 books total on a shelf.

(a)(i) No restrictions: $7! = 5040$ arrangements.

Answer: (a)(i) 5040 ways

(a)(ii) One English book at each end of the shelf:

Choose which English book goes at each end: $2! = 2$ ways. Arrange 5 remaining books in middle: $5!=120$ ways.

Total = $2 * 120 = 240$.

Answer: (a)(ii) 240 ways

(b) Appoint a president, VP, secretary, treasurer (4 distinct roles) and 2 committee members from 10 candidates:

Choose and order 4 for the offices: $P(10,4)=10*9*8*7=5040$.

Choose 2 committee from remaining 6: $C(6,2)=15$.

Total = $5040 * 15 = 75600$.

Answer: (b) 75 600 ways

Question 6. Find all angles between 0 and 360 degrees satisfying:

(a) $\cos x = -0.6713$. Basic angle = $\arccos(0.6713) \approx 47.8°$. cos is negative in Q2 and Q3.

$x = 180-47.8 = 132.2°$ or $x = 180+47.8 = 227.8°$.

Answer: (a) x ≈ 132.2° and 227.8°

(b) $5 \sin \theta - 13 \cos \theta = 0 \Rightarrow \tan \theta = 13/5 = 2.6$.

$\theta = \arctan(2.6) \approx 69.0°$ or $69.0°+180° = 249.0°$.

Answer: (b) theta ≈ 69.0° and 249.0°

(c) 4 tan^2(x) + 4 sec(x) - 1 = 0. Use tan^2 x = sec^2 x - 1:

4(sec^2 x-1) + 4 sec x - 1 = 0 => 4 sec^2 x + 4 sec x - 5 = 0.

Discriminant = 16+80 = 96. sec x = (-4 ± sqrt(96))/8 = (-1 ± sqrt(6))/2.

sec x = (-1+sqrt(6))/2 ≈ 0.72 => cos x ≈ 1.38 > 1. No solution.

sec x = (-1-sqrt(6))/2 ≈ -1.72 => cos x ≈ -0.580. arccos(0.580) ≈ 54.6°.

cos negative: Q2: 180-54.6=125.4° or Q3: 180+54.6=234.6°.

Answer: (c) x ≈ 125.4° and 234.6°

Question 7. AP and GP linked by shared terms.

AP first term = 4, common difference = d (d!=0). GP has ratio r.

AP T1=4, T2=4+d, T8=4+7d are the GP T1, T2, T3 respectively.

(a)(i) Two equations: r = (4+d)/4 ....(i) and r = (4+7d)/(4+d) ....(ii)

Answer: (a)(i) r = (4+d)/4 and r = (4+7d)/(4+d)

(a)(ii) Set equations equal: (4+d)/4 = (4+7d)/(4+d) => (4+d)^2 = 4(4+7d).

16+8d+d^2 = 16+28d => d^2-20d=0 => d(d-20)=0 => d=20 (d!=0).

r = (4+20)/4 = 6.

Answer: (a)(ii) d = 20, r = 6

(b) GP: first term=4, common ratio=6. Since |r|=6>1, the series diverges.

Answer: (b) The sum to infinity does not exist because |r| = 6 > 1 (series diverges)

(c) AP: a=4, d=20, n=8. S8=8/2*(2*4+7*20)=4*(8+140)=4*148=592.

Answer: (c) Sum of first 8 terms = 592

Question 8. Grouped frequency table — masses of tea packets.

Mass (grams): 140-144 | 145-149 | 150-154 | 155-159 | 160-164

Frequency: 5 | 10 | 3 | 8 | 4

Total n = 30. Midpoints: 142, 147, 152, 157, 162.

(a) Modal class (highest frequency): 145-149. Median: n/2=15th value.

Cumulative freq: 5, 15, 18, 26, 30. 15th value falls in the 145-149 class.

Answer: (a) Median class = 145-149

(b)(i) Mean: Sigma(fx) = 5(142)+10(147)+3(152)+8(157)+4(162)

= 710+1470+456+1256+648 = 4540.

Mean = 4540/30 = 151.3 grams (to 4 sig. fig.).

Answer: (b)(i) Estimated mean ≈ 151.3 grams

(b)(ii) SD: Sigma(fx^2) = 5(142^2)+10(147^2)+3(152^2)+8(157^2)+4(162^2)

= 100820+216090+69312+197192+104976 = 688390.

Variance = 688390/30 - (151.33)^2 = 22946.3 - 22901.8 = 44.6.

SD = sqrt(44.6) ≈ 6.67 grams.

Answer: (b)(ii) Standard deviation ≈ 6.67 grams

Question 9. Particle motion: V = 4t - t^2/2.

(a)(i) Acceleration a = dV/dt = 4 - t.

Answer: (a)(i) a = 4 - t m/s^2

(a)(ii) Acceleration when t=2: a(2) = 4-2 = 2 m/s^2.

Answer: (a)(ii) Acceleration = 2 m/s^2

(a)(iii) Maximum velocity when a=0: 4-t=0 => t=4. V_max=4(4)-(16)/2=16-8=8 m/s.

Answer: (a)(iii) Maximum velocity = 8 m/s (at t = 4 s)

(b) Distance from O after 8 s: s = integral from 0 to 8 of V dt.

Note: V=4t-t^2/2=0 at t=0 and t=8. V is positive for 0 < t < 8, so distance=displacement.

s = [2t^2 - t^3/6] from 0 to 8 = 2(64)-512/6 = 128-256/3 = (384-256)/3 = 128/3 m.

Answer: (b) Distance from O = 128/3 m ≈ 42.7 m

Question 10. Gradient and stationary points.

(a) y = 4/(5-2x) = 4(5-2x)^{-1}. Differentiate using chain rule:

dy/dx = 4*(-1)*(5-2x)^{-2}*(-2) = 8/(5-2x)^2.

At x=1/2: 5-2(1/2)=4. dy/dx = 8/16 = 1/2.

Answer: (a) Gradient at x = 1/2 is 1/2

(b) Curve p = 2t^3 - 3t^2 - 12t + 18.

(b)(i) Stationary points: dp/dt = 6t^2-6t-12 = 6(t^2-t-2) = 6(t-2)(t+1) = 0.

t=2: p=16-12-24+18=-2. Point (2,-2).

t=-1: p=-2-3+12+18=25. Point (-1,25).

Answer: (b)(i) Stationary points: (2, -2) and (-1, 25)

(b)(ii) d^2p/dt^2 = 12t-6. At t=2: 18>0 => MINIMUM. At t=-1: -18<0 => MAXIMUM.

Answer: (b)(ii) (-1, 25) is a maximum; (2, -2) is a minimum

Question 11. Curve y = x + 4/x with minimum M, intersected by line y=5.

y = x + 4/x. Minimum M: dy/dx = 1 - 4/x^2 = 0 => x^2=4 => x=2.

y(2) = 2+2 = 4. M=(2,4). (d^2y/dx^2 = 8/x^3 > 0 at x=2 => minimum confirmed.)

Line y=5 meets curve: x+4/x=5 => x^2-5x+4=0 => (x-1)(x-4)=0.

x=1: y=5. A=(1,5). x=4: y=5. B=(4,5).

Answer: (a) A = (1, 5), B = (4, 5), M = (2, 4)

(b) Rotate shaded region (between y=5 and y=x+4/x, from x=1 to x=4) about x-axis.

V = pi * integral from 1 to 4 of [5^2 - (x+4/x)^2] dx.

(x+4/x)^2 = x^2+8+16/x^2.

V = pi * integral of [25 - x^2 - 8 - 16/x^2] dx = pi * integral of [17 - x^2 - 16x^{-2}] dx.

= pi * [17x - x^3/3 + 16/x] from 1 to 4.

At x=4: 68-64/3+4 = 72-64/3 = 152/3.

At x=1: 17-1/3+16 = 33-1/3 = 98/3.

V = pi*(152/3 - 98/3) = pi*54/3 = 18*pi.

Answer: (b) Volume = 18*pi cubic units

Question 12. EITHER: Cylinder surface area optimisation.

Closed cylinder: radius r cm, height h cm. Volume=pi*r^2*h=600cm^3 => h=600/(pi*r^2).

(a) Surface area A = 2*pi*r^2 + 2*pi*r*h = 2*pi*r^2 + 2*pi*r*(600/(pi*r^2))

= 2*pi*r^2 + 1200/r. Q.E.D.

Answer: (a) A = 2*pi*r^2 + 1200/r (proved)

(b) dA/dr = 4*pi*r - 1200/r^2 = 0 => 4*pi*r^3 = 1200 => r^3 = 300/pi.

r = (300/pi)^{1/3} ≈ (95.49)^{1/3} ≈ 4.57 ≈ 4.6 cm (to 2 sig. figs.).

h = 600/(pi*r^2). Note: from r^3=300/pi, 300=pi*r^3, so h=2*pi*r^3/(pi*r^2)=2r.

h = 2*4.57 ≈ 9.1 cm (to 2 sig. figs.).

Answer: (b) r ≈ 4.6 cm, h ≈ 9.1 cm

(c) A_stationary = 2*pi*r^2+1200/r = 2*pi*r^2+4*pi*r^2 = 6*pi*r^2.

= 6*pi*(4.57)^2 ≈ 6*pi*20.88 ≈ 393.9 cm^2.

d^2A/dr^2 = 4*pi + 2400/r^3 > 0. => MINIMUM surface area.

Answer: (c) Minimum surface area ≈ 394 cm^2 (minimum confirmed since d^2A/dr^2 > 0)

Question 12or. OR: Geometric and arithmetic progressions.

(a) GP: T3 = 2*(2/3) = 8/3, T6 = 8/81.

T6/T3 = r^3 = (8/81)/(8/3) = (8/81)*(3/8) = 1/27 => r = 1/3.

T3 = a*r^2 = a/9 = 8/3 => a = 24.

S_inf = a/(1-r) = 24/(1-1/3) = 24/(2/3) = 36.

Answer: (a) Sum to infinity = 36

(b) AP: T3=90, T5=80.

$T_3=a+2d=90$ ...(1). $T_5=a+4d=80$ ...(2). Subtract: $2d=-10 \Rightarrow d=-5$.

$a = 90-2(-5) = 100$.

Answer: (b)(i) First term a = 100, common difference d = -5

(b)(ii) $S_p = S_{p+1}$: adding the (p+1)th term leaves sum unchanged => $T_{p+1}=0$.

$T_{p+1} = 100+(p+1-1)(-5) = 100-5p = 0 \Rightarrow p=20$.

Verify: $S_{20}=20/2*(2*100+19*(-5))=10*(200-95)=10*105=1050$. $S_{21}=S_{20}+T_{21}=1050+0=1050$. ✓

Answer: (b)(ii) p = 20

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