G12 ECZ ADDITIONAL MATHEMATICS 2014 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2014 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate
and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4030/2

PAPER 2

Tuesday 28 OCTOBER 2014

Additional materials:
Answer Booklet
Mathematical tables/Electronic calculators
Graph paper (1 sheet)

TIME: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones should not be brought in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Arithmetic $S_n = \frac{1}{2} n\, [2a + (n - 1)\, d]$

Geometric $S_n = \frac{a(1 - r^n)}{1 - r}$ $(r \neq 1)$

$S_\infty = \frac{a}{1 - r}$ for $|r| < 1$

3 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\operatorname{cosec}^2 A = 1 + \cot^2 A.$

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc \cos A.$

$\Delta = \frac{1}{2} bc \sin A$

4 STATISTICS

Mean and standard deviation

Ungrouped data

Mean $(\bar{x}) = \frac{\sum x}{n}$, SD $= \sqrt{\left\{\frac{\sum (x - \bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$

Grouped data

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$, SD $= \sqrt{\left\{\frac{\sum f(x - \bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$

1 Solve the simultaneous equations

$p - 3q + 2r = 2,$

$2p - q + r = 5,$

$2p + q + 4r = -3.$ [6]

2 **(a)** Find the range of values of x for which $4x^2 - 8x - 5 \geq 0$. [3]

(b) Express $7 - 5x - 2x^2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. Hence, find the co-ordinates of the maximum turning point. [4]

3 **(a)** Find the value of p, given that the expression $2x^3 + px^2 - x - 2$ is divisible by $x + 3$. [3]

(b) Factorise the expression $2x^3 - 11x^2 + 17x - 6$. [4]

4 Solve the equations

(a) $3^{2x+1} - 3^x = 4$, [4]

(b) $\lg(3x + 2) + 6\lg 2 = 2 + \lg(2x + 1)$. [3]

5 **(a)** A family consists of a father, mother and six children. Find the number of ways of arranging them in a straight line if

(i) there are no restrictions, [2]

(ii) the arrangements start and end with a parent. [2]

(b) A chess team consisting of 8 boys and 5 girls is to be chosen from 10 boys and 7 girls. In how many ways can this be done? [3]

6 Find all the angles between 0° and 360° which satisfy the equations

(a) $\sin 2x = -0.645$, [3]

(b) $5\sin x - 3\cos x = 0$, [2]

(c) $3\tan^2 x + 5 = 7\sec x$. [4]

7 The table below shows a record of performance of pupils at school A and school B.

School A	1	1	2	3	3	3	4	6	6	6
School B	1	2	2	4	5	5	5	5	6	6

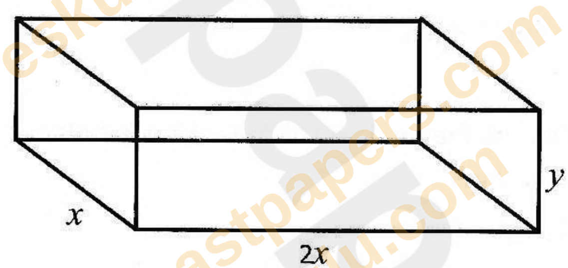
(a) Find the mean performance of each school. [2]

(b) Calculate the standard deviation for each school. [6]

(c) Which of the two schools has better results? Give a reason for your answer. [1]

8 **(a)** Find $\int\left(x^3 + \frac{1}{x^3}\right)dx$. [2]

(b) The diagram below shows a rectangular box made up of a thin sheet of metal with a lid. The length of the box is $2x$ cm, the breadth is x cm and the height is y cm.



(i) Given that the volume of the box is 72cm^3, show that the surface area, Acm^3, of the thin sheet of metal used is given by $A = 4x^2 + \frac{216}{x}$. [3]

(ii) Given that x can vary, find the minimum area of the box. Show that this is a minimum value. [4]

9 **(a)** The eighth term of a geometric progression is 32 and the fourth term is 2. Find the first term and the positive value of the common ratio. [3]

(b) Find the sum to infinity of a geometric progression whose first term is 5 and the second term is –4. [2]

(c) The nth term of an arithmetic progression is $3n + 7$. Find the sum of the first 40 terms. [4]

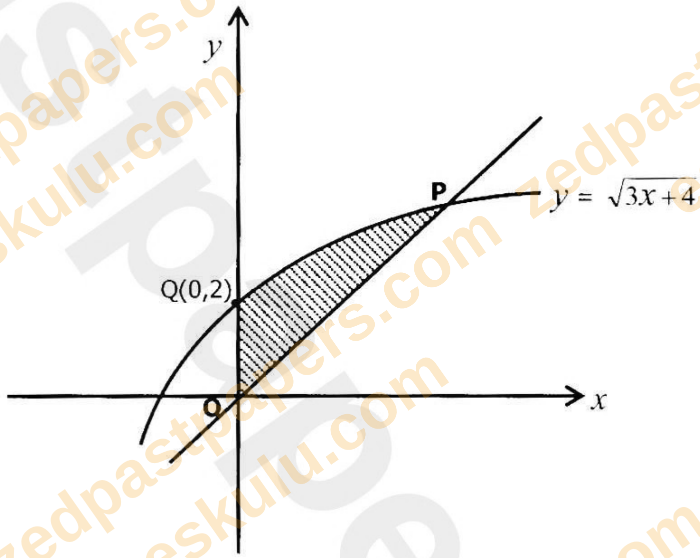
10 The velocity in m/s of a particle, travelling in a straight line at time t seconds after leaving a fixed point O, is given by $V = 10 + kt - 3t^2$, where $t \geq 0$ and k is a constant. When $t = 0$ the particle is at O and its acceleration is 1m/s^2. Find the

(a) value of k, [3]

(b) value of t when the particle is instantaneously at rest, [3]

(c) distance the particle has travelled when it is again at O. [4]

11 The diagram shows part of the curve $y = \sqrt{3x+4}$ and a straight line. The curve meets the straight line at P. The gradient of the straight line is 1.



(a) Find

(i) the equation of the straight line OP, [1]

(ii) the co-ordinates of the point P. [3]

(b) The shaded region is rotated through 360° about the x-axis to form a solid of revolution. Calculate the volume generated in terms of π. [6]

[Turn over

12 Answer only one of the following alternatives:

EITHER

(a) A tangent touches the curve $y = 3x^2 + 2x + 1$ at the point where it is parallel to the line $4x + y = 5$. Find the co-ordinates of the point where the tangent touches the curve. [3]

(b) (i) Draw the graph of $y = e^{2x} - 4$ for values of x at intervals of 0.5, for $-2 \le x \le 1\frac{1}{2}$. [4]

(ii) By drawing a suitable straight line on your graph, obtain an approximate solution to the equation $x = \ln\sqrt{1-4x}$. [3]

OR

(a) The sixth term of an arithmetic progression with 12 terms is 14 and the sum of the last six terms is 126. Calculate the sum of the first six terms. [5]

(b) The fifth term of a geometric progression is $\frac{2}{81}$ and the sum of the third and fourth terms is $\frac{8}{27}$. Find the

(i) positive value of the common ratio, r, [3]

(ii) sum to infinity of the progression. [2]

Question 1. Solve the simultaneous equations: $p-3q+2r=2$, $2p-q+r=5$, $2p+q+4r=-3$.

E2-E1: $p+2q-r=3$ ...(A)

From 2*E1-E2: $-5q+3r=-1$ ...(B) [$2(p-3q+2r)-(2p-q+r)$: $-5q+3r=2*2-5=-1$]

From E3-E2: $2q+3r=-8$ ...(C)

B-C: $-7q=7 \Rightarrow q=-1$.

From C: $-2+3r=-8 \Rightarrow r=-2$.

From E2: $2p+1-2=5 \Rightarrow 2p=6 \Rightarrow p=3$.

Verify E1: $3+3-4=2$ ✓ E3: $6-1-8=-3$ ✓

Answer: $p = 3$, $q = -1$, $r = -2$

Question 2. Inequalities and completing the square.

(a) Find range of values of a for which $4x^2 - 8x + a > 0$ for all real x.

For quadratic > 0 for ALL x: discriminant < 0.

Discriminant = $(-8)^2 - 4(4)(a) = 64 - 16a < 0 \Rightarrow 16a > 64 \Rightarrow a > 4$.

Answer: (a) $a > 4$

(b) Express $7 - 5x - 2x^2$ in the form $a(x+b)^2 + c$. Find maximum turning point.

$7-5x-2x^2 = -2(x^2+5x/2)+7 = -2(x+5/4)^2 + 2(25/16) + 7$

$= -2(x+5/4)^2 + 25/8 + 56/8 = -2(x+5/4)^2 + 81/8$.

$a=-2$, $b=5/4$, $c=81/8$. Maximum at $x=-5/4$, $f(-5/4)=81/8$.

Answer: (b) $-2(x + 5/4)^2 + 81/8$; maximum turning point = $(-5/4, 81/8)$

Question 3. Polynomial divisibility and factorisation.

(a) $2x^2 + px - 2$ is exactly divisible by $(x+3)$. So $f(-3)=0$.

$f(-3) = 2(9) + p(-3) - 2 = 18 - 3p - 2 = 16 - 3p = 0$.

$3p = 16 \Rightarrow p = 16/3$.

Answer: (a) $p = 16/3$

(b) Factorise $2x^3 - 11x^2 + 17x - 6$.

Try $x=2$: $2(8)-11(4)+17(2)-6 = 16-44+34-6 = 0$. So $(x-2)$ is a factor.

Divide: $2x^3-11x^2+17x-6 = (x-2)(2x^2-7x+3)$.

Factorise $2x^2-7x+3$: find two numbers multiplying to 6 and adding to -7: -6 and -1.

$2x^2-7x+3 = (2x-1)(x-3)$.

Answer: (b) $2x^3 - 11x^2 + 17x - 6 = (x-2)(2x-1)(x-3)$

Question 4. Solve the equations.

(a) 3^{x+1} - 3^x = 6.

Factor: 3*3^x - 3^x = (3-1)*3^x = 2*3^x = 6 => 3^x = 3 => x = 1.

Answer: (a) x = 1

(b) lg(x+7) - lg(x-2) = 1.

lg((x+7)/(x-2)) = 1 => (x+7)/(x-2) = 10 => x+7 = 10x-20 => 9x = 27 => x = 3.

Check: x-2=1 > 0 and x+7=10 > 0. ✓

Answer: (b) x = 3

Question 5. Arrangements and team selection.

(a) Father, mother and six children — 8 people arranged in a straight line.

(a)(i) No restrictions: 8! = 40320 ways.

Answer: (a)(i) 40 320 ways

(a)(ii) Arrangements start and end with a parent:

2 parents fill the 2 end positions: 2! = 2 ways. 6 children fill the middle: 6! = 720 ways.

Total = 2 * 720 = 1440.

Answer: (a)(ii) 1440 ways

(b) Chess team of 8 boys and 5 girls from 10 boys and 7 girls:

C(10,8) * C(7,5) = 45 * 21 = 945.

Answer: (b) 945 ways

Question 6. Find all angles between 0 and 360 degrees satisfying:

(a) sin(2x) = -0.645.

Basic angle: arcsin(0.645) ≈ 40.2°. sin negative in 3rd and 4th quadrants.

2x = 180°+40.2° = 220.2°, or 360°-40.2° = 319.8°, or +360° for each:

2x = 220.2°, 319.8°, 580.2°, 679.8°.

x = 110.1°, 159.9°, 290.1°, 339.9°.

Answer: (a) x ≈ 110.1°, 160.0°, 290.1°, 339.9°

(b) 5sin(theta) - 3cos(theta) = 0 => 5sin(theta) = 3cos(theta) => tan(theta) = 3/5.

theta = arctan(3/5) ≈ 31.0° or 180°+31.0° = 211.0°.

Answer: (b) theta ≈ 31.0° and 211.0°

(c) 3tan^2(x) + 2tan(x) - 1 = 0. Factorise: (3tan(x)-1)(tan(x)+1) = 0.

tan(x) = 1/3: x = arctan(1/3) ≈ 18.4° or 180°+18.4° = 198.4°.

tan(x) = -1: x = 135° or 315°.

Answer: (c) $x \approx 18.4°, 135°, 198.4°, 315°$

Question 7. Performance records of pupils at School A and School B.

School A data: 1, 2, 3, 3, 4, 6, 6 (n=7).

School B data: 1, 2, 2, 4, 5, 5, 5 (n=7).

(a) Mean A = (1+2+3+3+4+6+6)/7 = 25/7 ≈ 3.57.

Mean B = (1+2+2+4+5+5+5)/7 = 24/7 ≈ 3.43.

Answer: (a) Mean of School A ≈ 3.57; Mean of School B ≈ 3.43

(b) Variance A: Sigma(x^2)=1+4+9+9+16+36+36=111. Var_A=111/7-(25/7)^2=15.857-12.755=3.10. SD_A=sqrt(3.10)≈1.76.

Variance B: Sigma(x^2)=1+4+4+16+25+25+25=100. Var_B=100/7-(24/7)^2=14.286-11.755=2.53. SD_B=sqrt(2.53)≈1.59.

Answer: (b) Standard deviation of A ≈ 1.76; Standard deviation of B ≈ 1.59

(c) School A has a higher mean performance (3.57 > 3.43).

Answer: (c) School A has better performance — it has a higher mean score.

Question 8. Integration and box optimisation.

(a) Find the integral of (x + 4/x^2) dx = integral of (x + 4x^{-2}) dx.

= x^2/2 + 4*(-1)*x^{-1} + C = x^2/2 - 4/x + C.

Answer: (a) integral = x^2/2 - 4/x + C

(b) Box: length 2x cm, breadth x cm, height y cm.

(b)(i) Volume = 2x * x * y = 2x^2 * y = 72 => y = 36/x^2.

Surface area A = 2(2x)(x) + 2(2x)(y) + 2(x)(y) = 4x^2 + 4xy + 2xy = 4x^2 + 6xy.

A = 4x^2 + 6x*(36/x^2) = 4x^2 + 216/x. Q.E.D.

Answer: (b)(i) A = 4x^2 + 216/x (proved)

(b)(ii) dA/dx = 8x - 216/x^2 = 0 => 8x^3 = 216 => x^3 = 27 => x = 3.

d^2A/dx^2 = 8 + 432/x^3. At x=3: 8+432/27=8+16=24 > 0. Minimum. ✓

A_min = 4(9) + 216/3 = 36 + 72 = 108 cm^2.

Answer: (b)(ii) Minimum surface area = 108 cm^2 at x = 3 cm

Question 9. Geometric and arithmetic progressions.

(a) GP: T_8=32, T_4=2. T_8/T_4 = r^4 = 32/2 = 16 => r = 2 (positive terms given).

T_4 = ar^3 = 8a = 2 => a = 1/4.

Answer: (a) First term a = 1/4, common ratio r = 2

T_n = (1/4)*2^{n-1} = 2^{n-1}/4 = 2^{n-3}.

Answer: (a) T_n = 2^{n-3}

(b) GP: a=5, T_2=-5/3. r = (-5/3)/5 = -1/3. |r|<1 so S_inf exists.

S_inf = 5/(1-(-1/3)) = 5/(4/3) = 15/4.

Answer: (b) Sum to infinity = 15/4

(c) T_n = 3n+7. a=T_1=10, d=3, T_40=127.

S_40 = 40/2*(T_1+T_40) = 20*(10+127) = 20*137 = 2740.

Answer: (c) Sum of first 40 terms = 2740

Question 10. Particle motion with velocity V = 10 + kt - 3t^2.

a(t) = dV/dt = k - 6t. At t=0: a(0) = k = 5 m/s^2.

Answer: (a) k = 5

V = 10 + 5t - 3t^2 = 0 => 3t^2 - 5t - 10 = 0.

t = (5 + sqrt(25+120))/6 = (5 + sqrt(145))/6 ≈ (5+12.04)/6 ≈ 2.84 s. (Reject negative root.)

Answer: (b) Particle at instantaneous rest at t ≈ 2.84 s

(c) Maximum displacement from O: s(T) where T=(5+sqrt(145))/6.

s(T) = 10T + 5T^2/2 - T^3. Using 3T^2=5T+10: T^3=T*(5T+10)/3=(5T^2+10T)/3.

Algebraic simplification gives s(T) = (1025+145*sqrt(145))/108 ≈ 25.7 m.

Particle returns to O (s=0) after reversing. Total distance travelled = 2 * 25.7 ≈ 51.3 m.

Answer: (c) Total distance when particle is again at O ≈ 51.3 m

Question 11. Curve y = sqrt(x+4) and straight line through O with gradient 1.

(a)(i) Line through origin O(0,0) with gradient 1: y = x.

Answer: (a)(i) Equation of line: y = x

(a)(ii) Find P: line y=x meets curve y=sqrt(x+4).

x = sqrt(x+4) => x^2 = x+4 => x^2 - x - 4 = 0.

x = (1 + sqrt(17))/2 (taking positive root since x>0).

y = x = (1+sqrt(17))/2 ≈ 2.56.

Answer: (a)(ii) P = ((1+sqrt(17))/2, (1+sqrt(17))/2) ≈ (2.56, 2.56)

(b) Volume of shaded region (between line y=x and curve y=sqrt(x+4) from x=0 to x=a

where a=(1+sqrt(17))/2) rotated 360 degrees about x-axis:

V = pi * integral from 0 to a of [(sqrt(x+4))^2 - x^2] dx

= pi * integral from 0 to a of [(x+4) - x^2] dx

= pi * [x^2/2 + 4x - x^3/3] from 0 to a.

Using a^2=a+4 and a^3=5a+4 (from a^2-a-4=0):

= pi * [(a+4)/2 + 4a - (5a+4)/3] = pi * (17a+4)/6.

Substituting a=(1+sqrt(17))/2: 17a+4 = (25+17*sqrt(17))/2.

Answer: (b) Volume = pi(25 + 17*sqrt(17))/12 cubic units

Question 12. EITHER: Tangent to curve and exponential graph.

(a) Tangent to y = 3x^2+2x+1 parallel to 4x+y=5 (slope -4).

dy/dx = 6x+2 = -4 => 6x=-6 => x=-1. y=3(1)+2(-1)+1=3-2+1=2.

Answer: (a) Tangent touches curve at (-1, 2)

(b)(i) Table of values for y = e^{2x}-4, x from -2 to 1.5 at intervals of 0.5:

x: -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5

y: -3.98, -3.95, -3.86, -3.63, -3.00, -1.28, 3.39, 16.09

Answer: (b)(i) Plot: starts near y=-4, rises steeply; note y=0 near x=0.69

(b)(ii) Solve x = ln(sqrt(1-4x)) using graph of y = e^{2x}-4.

From x = ln*sqrt(1-4x): 2x = ln(1-4x) => e^{2x} = 1-4x => e^{2x}-4 = -4x-3.

Draw the straight line y = -4x - 3 on the graph.

Read the x-coordinate of the intersection point (approximately x ≈ -0.7).

Answer: (b)(ii) Draw line y = -4x - 3; approximate solution x ≈ -0.7

Question 12or. OR: Arithmetic and geometric progressions.

(a) AP of 12 terms: T_6=14 and sum of last 6 terms = 126.

a + 5d = 14 ...(1)

Sum of last 6 terms = (6/2)*(T_7+T_12) = 3*(T_7+T_12).

T_7=14+d, T_12=a+11d=14-5d+11d=14+6d.

3*(14+d+14+6d) = 3*(28+7d) = 84+21d = 126 => 21d=42 => d=2.

a = 14-5(2) = 4.

S_6 = (6/2)*(2*4+5*2) = 3*(8+10) = 54.

Answer: (a) Sum of first six terms = 54

(b) GP: T_5=2/81, T_3+T_4=8/27.

T_5=ar^4=2/81 ...(A). T_3+T_4=ar^2+ar^3=ar^2(1+r)=8/27 ...(B).

Divide: r^2/(1+r) = (2/81)/(8/27) = (2*27)/(81*8) = 54/648 = 1/12.

12r^2 = 1+r => 12r^2-r-1=0 => (4r+1)(3r-1)=0 => r=1/3 (positive) or r=-1/4.

Answer: (b)(i) Positive common ratio r = 1/3

From A: a*(1/3)^4 = 2/81 => a*(1/81) = 2/81 => a=2.

S_inf = a/(1-r) = 2/(1-1/3) = 2/(2/3) = 3.

Answer: (b)(ii) Sum to infinity = 3

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