G12 ECZ ADDITIONAL MATHEMATICS 2012 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2012 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

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EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate
and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4030/2

PAPER 2

Thursday 15 NOVEMBER 2012

Additional materials:
Answer Booklet
Mathematical tables/Electronic calculators
Graph paper (1 sheet)

TIME: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones should not be brought in the examination room.

Check the formulae overleaf

This question paper consists of 5 printed pages.

Mathematical Formulae

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

2 SERIES

Arithmetic $S_n = \frac{1}{2} n \{2a + (n-1)d\}$

Geometric $S_n = \frac{a(1-r^n)}{1-r}$ $(r \neq 1)$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

3 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\text{cosec}^2 A = 1 + \cot^2 A.$

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc \cos A.$

$\Delta = \frac{1}{2} bc \sin A$

4 STATISTICS

Mean and standard deviation

Ungrouped data

Mean $(\bar{x}) = \frac{\sum x}{n}$, SD $= \sqrt{\left\{\frac{\sum (x-\bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$

Grouped data

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$, SD $= \sqrt{\left\{\frac{\sum f(x-\bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$

1 Solve the simultaneous equations

$2a + b - c = 5,$

$a + 4b + 2c = 16,$

$15a + 6b - 2c = 12.$ [6]

2 (a) The surface area of a rectangular block of ice, $A\text{cm}^2$, is given by $A = 6x^2 + \frac{216}{x}$. Find an expression for $\frac{dA}{dx}$. [2]

(b) Given that the block of ice in (a) is melting in such a way that A is decreasing at a constant rate of $0.14\text{cm}^2/\text{s}$, calculate the rate of decrease of x at the instant when $x = 4$. [4]

3 (a) Find the range of values of x for which $2x^2 - x - 1 > 2x + 1$. [3]

(b) Express $2x^2 - 5x + 16$ in the form $a(x - b)^2 + c$, where a, b and c are constants. Hence find the minimum value of $2x^2 - 5x + 16$. [4]

4 Solve the equations

(a) $5^{x-1} = 13$, [3]

(b) $\lg(2x) - \lg(x - 3) = 1$. [4]

5 The expression $f(x) = cx^3 + 8x^2 + dx + 6$ is exactly divisible by $x^2 - 2x - 3$.

(a) Find the value of c and d. [4]

(b) Find the remaining factor of the expression. [2]

(c) If f(x) is not divisible by $x + 2$, find the remainder. [2]

6 (a) In how many ways can 7 red marbles and 3 green marbles be put in a straight line, if

(i) there are no restrictions, [2]

(ii) green marbles should not be next to each other? [3]

(b) A group of 6 pupils is to be chosen from 10 boys and 8 girls. Find the number of ways of choosing at least 4 boys. [3]

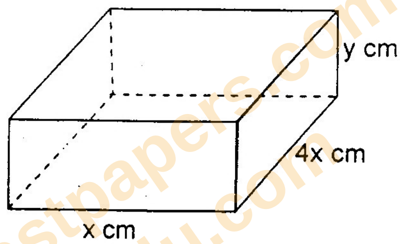
7 Find all the angles between 0° and 360° which satisfy the equation:

(a) $\tan\theta = -1.333$, [2]

(b) $\cos 3y = -0.5$, [3]

(c) $3\cos^2 z + 4\sin z = 4$. [4]

8 A pupil was given a 60 cm piece of wire to design a rectangular box for storing pens. The designer came up with the following diagram measuring 4x cm by x cm by y cm.



Given that the whole piece of wire was used,

(a) express the length of y in terms of x. [1]

(b) find an expression for the volume enclosed by the framework in terms of x. [2]

(c) calculate the value of x which makes the volume stationary. [3]

(d) show that this volume is maximum and find this volume. [3]

9 (a) The third term of a geometric progression of positive terms is 18 and the fifth term is 162.

(i) Find the common ratio and the first term. [2]

(ii) Write an expression for the n^{th} term of the progression. [1]

(iii) Find the sum of the first eight terms of the progression. [2]

(b) Find the geometric mean of $\frac{1}{64}$ and $\frac{1}{16}$. [1]

(c) Find the number of terms of the arithmetic progression 15, 19, 23, ... that must be taken for the sum to be equal to 444. [4]

10 The table below shows the frequency with which words of various lengths appear in the first paragraph of a particular book.

No. of letters	1 - 3	4 - 6	7 - 9	10 - 12	13 – 15
No. of words	35	33	34	13	5

Find the estimate of

(a) the mean, [2]

(b) the variance, [6]

(c) the standard deviation. [2]

11 A particle starts from rest at a point O and moves in a straight line with velocity v m/s given by $v = 6t - 3t^2$, where t is the time in seconds after the start.

Find

(a) the acceleration of the particle when it is next at instantaneous rest, [3]

(b) the maximum velocity, [3]

(c) the distance travelled to that position when the particle was next at instantaneous rest. [4]

12 **Answer only one of the following alternatives:**

EITHER

The curve $y = e^{-2x} - 2$ meets the x-axis at A and the y-axis at B.

(a) Find the coordinates of A and B. [3]

(b) Sketch the graph of $y = e^{-2x} - 2$ for the domain $-1 \le x \le 1$. [3]

(c) Find the equation of the straight line which must be drawn on the graph of $y = e^{-2x} - 2$ to obtain a solution of the equation $x = \ln\left(\frac{1}{\sqrt{2x+3}}\right)$. [3]

(d) State the range of $f: x \rightarrow e^{-2x} - 2$. [1]

OR

(a) Grace would like to give a sum of money to a charity each year for 10 years. She decides to give \$120 in the first year, and to increase her contribution by \$10 each year.

(i) How much does she give in the last year? [3]

(ii) How much does the charity receive from her altogether? [3]

(b) A geometric series has first term 27 and common ratio $\frac{4}{3}$. Find the least number of terms the series can have if its sum exceeds 729. [4]

Question 1. Solve the simultaneous equations: a+b-c=5, a+4b-2c=16, 5a+6b-2c=12.

From E3-E2: 4a+2b=-4 => 2a+b=-2 ...(A)

From E1: c=a+b-5. Substitute into E2: a+4b-2(a+b-5)=16 => -a+2b=-6 ...(B)

Wait — use A and B directly: A gives b=-2-2a. Into E1: a+(-2-2a)-c=5 => c=-a-7.

Sub b and c into E2: a+4(-2-2a)-2(-a-7)=16 => a-8-8a+2a+14=16 => -5a=10 => a=-2.

b = -2-2(-2) = -2+4 = 2. c = -(-2)-7 = 2-7 = -5.

Verify E3: 5(-2)+6(2)-2(-5) = -10+12+10 = 12. ✓

Answer: a = -2, b = 2, c = -5

Question 2. Surface area of a rectangular block of ice. A cm^2 given by A = 6x^2 + 216/x.

(a) Differentiate A = 6x^2 + 216x^{-1} with respect to x:

dA/dx = 12x - 216/x^2.

Answer: (a) dA/dx = 12x - 216/x^2

(b) A is decreasing at 4 cm^2/s. Find rate of change of x when x = 4.

By chain rule: dA/dt = (dA/dx)*(dx/dt).

At x=4: dA/dx = 12(4) - 216/16 = 48 - 27/2 = 69/2.

dA/dt = -4 (decreasing): -4 = (69/2)*(dx/dt).

dx/dt = -8/69 cm/s.

Answer: (b) Rate at which x is decreasing = 8/69 cm/s (approx 0.116 cm/s)

Question 3. Express 2x^2 - 5x + 16 in the form a(x-b)^2 + c. Find range of x where this exceeds 2x+1.

(a) Complete the square:

2x^2-5x+16 = 2(x^2 - 5x/2) + 16 = 2(x - 5/4)^2 - 2(25/16) + 16

= 2(x - 5/4)^2 - 25/8 + 128/8 = 2(x - 5/4)^2 + 103/8.

Answer: (a) a = 2, b = 5/4, c = 103/8. Minimum value = 103/8 at x = 5/4

(b) Find range of x where 2x^2-5x+16 > 2x+1:

=> 2x^2 - 7x + 15 > 0.

Discriminant = 49 - 4(2)(15) = 49 - 120 = -71 < 0.

Since discriminant < 0 and leading coefficient 2 > 0, the quadratic is always positive.

Answer: (b) The inequality holds for all real values of x.

Question 4. Solve the equations.

(a) $x^{2/3} = 4$.

Raise both sides to the power 3/2: $x = 4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$.

Also check: $(-8)^{2/3} = ((-8)^{1/3})^2 = (-2)^2 = 4$. So $x = -8$ is also valid.

Answer: (a) x = 8 or x = -8

(b) $\lg(2x) - \lg(x-3) = 1$.

$\lg(2x/(x-3)) = 1 \Rightarrow 2x/(x-3) = 10 \Rightarrow 2x = 10(x-3) = 10x - 30$.

$-8x = -30 \Rightarrow x = 15/4 = 3.75$.

Check: domain requires $x > 3$; $15/4 = 3.75 > 3$. ✓

Answer: (b) x = 15/4

Question 5. $f(x) = cx^3 + 8x^2 + dx + 6$ is exactly divisible by $x^2 - 2x - 3$.

$x^2 - 2x - 3 = (x-3)(x+1)$, so $f(3)=0$ and $f(-1)=0$.

$f(3) = 27c + 72 + 3d + 6 = 0 \Rightarrow 27c + 3d = -78 \Rightarrow 9c + d = -26$ ...(A)

$f(-1) = -c + 8 - d + 6 = 0 \Rightarrow c + d = 14$ ...(B)

(A)-(B): $8c = -40 \Rightarrow c = -5$.

$d = 14 - (-5) = 19$.

Answer: (i) c = -5, d = 19

(ii) $f(x) = -5x^3 + 8x^2 + 19x + 6$. Divide by (x^2-2x-3):

$f(x) = (x^2 - 2x - 3)(-5x + q)$. Expand: $-5x^3 + (q+10)x^2 + (-2q+15)x - 3q$.

Match x^2: $q+10=8 \Rightarrow q=-2$. Check x term: $4+15=19$ ✓. Check const: 6 ✓.

Answer: (ii) Third (remaining) factor: -(5x + 2)

So $f(x) = -(x-3)(x+1)(5x+2)$.

(iii) Remainder when f(x) divided by (x+2):

$f(-2) = -5(-8)+8(4)+19(-2)+6 = 40+32-38+6 = 40$.

Answer: (iii) Remainder = 40

Question 6. Arrangements of marbles and selection of pupils.

(a)(i) 7 red and 3 green marbles in a line, no restrictions (identical within colour):

Choose 3 positions from 10 for green marbles: $C(10,3) = 10!/(3!*7!) = 120$.

Answer: (a)(i) 120 arrangements

(a)(ii) Green marbles must not be adjacent:

First arrange 7 red (1 way). This creates 8 gaps: _R_R_R_R_R_R_R_

Choose 3 of 8 gaps for green (one per gap ensures no adjacency): $C(8,3) = 56$.

Answer: (a)(ii) 56 arrangements

(b) Choose 6 from 10 boys and 8 girls with at least 4 boys:

= C(10,4)*C(8,2) + C(10,5)*C(8,1) + C(10,6)*C(8,0)

= 210*28 + 252*8 + 210*1

= 5880 + 2016 + 210 = 8106.

Answer: (b) 8106 ways

Question 7. Find all angles between 0 and 360 degrees satisfying:

(a) tan(theta) = -1.333 ≈ -4/3.

Reference angle: arctan(4/3) ≈ 53.13°. Tan is negative in 2nd and 4th quadrants.

theta = 180° - 53.13° = 126.87° or theta = 360° - 53.13° = 306.87°.

Answer: (a) theta ≈ 126.9° and 306.9°

(b) cos(3y) = -0.5 = -1/2 for y in [0°, 360°], so 3y in [0°, 1080°].

cos(angle) = -1/2 at 120° and 240° (in basic range).

3y = 120°, 240°, 480°, 600°, 840°, 960°.

y = 40°, 80°, 160°, 200°, 280°, 320°.

Answer: (b) y = 40°, 80°, 160°, 200°, 280°, 320°

(c) 3cos^2(theta) + 4sin(theta) = 4. Replace cos^2 = 1 - sin^2:

3(1-sin^2) + 4sin = 4 => -3sin^2 + 4sin - 1 = 0 => 3sin^2 - 4sin + 1 = 0.

(3sin-1)(sin-1) = 0 => sin(theta) = 1/3 or sin(theta) = 1.

sin=1: theta=90°. sin=1/3: theta=arcsin(1/3)≈19.5° or 180°-19.5°=160.5°.

Answer: (c) theta ≈ 19.5°, 90°, 160.5°

Question 8. Rectangular wire frame 4x cm by x cm by y cm. Total wire = 60 cm.

Frame has 4 edges of each dimension: 4(4x) + 4(x) + 4(y) = 60.

16x + 4x + 4y = 60 => 20x + 4y = 60 => 4y = 60-20x.

Answer: (a) y = (60 - 20x)/4 = 15 - 5x

(b) Volume V = 4x * x * y = 4x^2*(15-5x) = 60x^2 - 20x^3.

Answer: (b) V = 60x^2 - 20x^3

(c) dV/dx = 120x - 60x^2 = 60x(2-x) = 0 => x = 0 or x = 2.

x=0 gives V=0 (degenerate), so the relevant stationary point is x = 2.

Answer: (c) V is stationary when x = 2

(d) d^2V/dx^2 = 120 - 120x. At x=2: 120-240 = -120 < 0 => maximum.

V_max = 60(4) - 20(8) = 240 - 160 = 80 cm^3.

Answer: (d) Maximum volume = 80 cm^3 (confirmed by negative second derivative)

Question 9. Geometric and arithmetic progressions.

(a) GP of positive terms: T_3=18, T_5=162.

T_5/T_3 = r^2 = 162/18 = 9 => r = 3 (positive terms).

T_3 = ar^2 = 9a = 18 => a = 2.

Answer: (a)(i) Common ratio r = 3, first term a = 2

(a)(ii) T_n = ar^{n-1} = 2*3^{n-1}.

Answer: (a)(ii) T_n = 2 x 3^{n-1}

(a)(iii) S_8 = a(r^8-1)/(r-1) = 2(3^8-1)/(3-1) = (3^8-1) = 6561-1 = 6560.

Answer: (a)(iii) S_8 = 6560

(b) AP: 15, 19, 23, ... (a=15, d=4). Find n such that S_n = 444.

S_n = n/2*(2*15 + (n-1)*4) = n/2*(30+4n-4) = n(13+2n).

n(13+2n) = 444 => 2n^2+13n-444 = 0.

Discriminant = 169+3552 = 3721 = 61^2.

n = (-13+61)/4 = 48/4 = 12. (Reject n=(-13-61)/4 < 0.)

Answer: (b) n = 12 terms

Last (12th) term: T_12 = 15+11*4 = 59.

Answer: (b) Last term T_12 = 59

Question 10. Frequency table: word lengths in first paragraph of a book.

Classes: 1-3, 4-6, 7-9, 10-12, 13-15.

Midpoints (x): 2, 5, 8, 11, 14.

Frequencies (f): 35, 33, 34, 13, 5. Total n = 120.

fx: 70, 165, 272, 143, 70. Sum = 720.

fx^2: 140, 825, 2176, 1573, 980. Sum = 5694.

(a) Mean = Sigma(fx)/Sigma(f) = 720/120 = 6.

Answer: (a) Mean = 6 letters

(b) Variance = Sigma(fx^2)/n - mean^2 = 5694/120 - 36 = 47.45 - 36 = 11.45.

Answer: (b) Variance = 11.45

(c) Standard deviation = sqrt(11.45) ≈ 3.38.

Answer: (c) Standard deviation ≈ 3.38 letters

Question 11. Particle from rest at O with velocity v = 6t - 3t^2 m/s.

Particle next at instantaneous rest: v = 0 => 6t-3t^2 = 3t(2-t)=0 => t=0 or t=2.

t=2 is the 'next' instantaneous rest (t=0 is the start).

(a) Acceleration: $a(t) = dv/dt = 6-6t$. At t=2: $a = 6-12 = -6$ m/s^2.

Answer: (a) Acceleration when next at rest = -6 m/s^2 (decelerating at 6 m/s^2)

(b) Maximum velocity: $dv/dt = 0 \Rightarrow 6-6t=0 \Rightarrow t=1$.

$d^2v/dt^2 = -6 < 0$. Confirmed maximum. $v_{max} = 6(1)-3(1)^2 = 3$ m/s.

Answer: (b) Maximum velocity = 3 m/s (at t = 1 s)

(c) Distance from O at t=2:

s = integral from 0 to 2 of $(6t-3t^2)\,dt = [3t^2 - t^3]$ from 0 to 2

$= (12-8) - 0 = 4$ m.

Answer: (c) Distance from O when at rest = 4 m

Question 12. EITHER: Exponential curve y = e^{2x} - 2.

(a) A = intersection with x-axis (y=0): $e^{2x}-2=0 \Rightarrow e^{2x}=2 \Rightarrow x = (\ln 2)/2$.

B = intersection with y-axis (x=0): $y = e^0-2 = 1-2 = -1$.

Answer: (a) A = ((ln 2)/2, 0) and B = (0, -1)

(b) Sketch for $-1 \le x \le 1$:

At x=-1: $y = e^{-2}-2 \approx -1.86$. At x=0: y=-1. At $x\approx 0.347$: y=0. At x=1: $y=e^2-2\approx 5.39$.

Curve is exponential, passing through B=(0,-1), cutting x-axis at A=((ln2)/2, 0),

increasing steeply for x > 0. Approaches y=-2 from above as $x \to -\infty$.

Answer: (b) Exponential curve through B(0,-1) and A((ln2)/2, 0), increasing for x in [-1,1]

(c) Find line to draw on $y = e^{2x}-2$ to solve $x = \ln(\sqrt{x+3})$.

From $x = \ln(\sqrt{x+3})$: $2x = \ln(x+3) \Rightarrow e^{2x} = x+3 \Rightarrow e^{2x}-2 = x+1$.

So the line y = x+1 drawn on $y = e^{2x}-2$ gives the desired intersections.

Answer: (c) Draw the straight line y = x + 1

(d) Range of $f: x \to -e^{2x} - 2$. Since $e^{2x} > 0$, $-e^{2x} < 0$, so $-e^{2x}-2 < -2$.

Answer: (d) Range of f is f < -2 (i.e. (-∞, -2))

Question 12or. OR: Grace's charity donations and geometric series.

(a) Grace gives $120 in year 1 and increases by $10 per year for 10 years.

AP: first term a=120, common difference d=10, n=10.

(a)(i) Last year (10th): $T_{10} = 120 + 9*10 = 120 + 90 = \$210$.

Answer: (a)(i) Amount in last year = $210

(a)(ii) Total: $S_{10} = 10/2*(120+210) = 5*330 = \$1650$.

Answer: (a)(ii) Charity receives $1650 altogether

(b) GP: first term 27, common ratio 3/2. Find least n for $S_n > 729$.

$S_n = 27*((3/2)^n - 1)/(3/2-1) = 54*((3/2)^n - 1)$.

$S_n > 729$: $(3/2)^n > 14.5$.

$n*\ln(3/2) > \ln(14.5) \Rightarrow n > \ln(14.5)/\ln(1.5) \approx 6.60$. So $n \geq 7$.

Check n=6: $(3/2)^6 = 729/64 \approx 11.39$. $S_6 = 54*10.39 \approx 561 < 729$. ✗

Check n=7: $(3/2)^7 = 2187/128 \approx 17.09$. $S_7 = 54*16.09 \approx 869 > 729$. ✓

Answer: (b) Minimum number of terms = 7

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