G12 ECZ ADDITIONAL MATHEMATICS 2010 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2010 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

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EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate
and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4030/2

PAPER 2

Thursday 11 November 2010 2 hours 30 minutes

Additional materials:
Answer Booklet
Mathematical tables/calculators

TIME: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones should not be brought in the examination room.

Check the formulae overleaf

Solve the simultaneous equations

$3x - y + z = 8$

$x + 2y - z = 6$

$2x + y + z = 9$ [6]

(a) Calculate the number of arrangements of the word 'DISCOVERY'. [1]

(b) A group of 7 pupils is to be chosen from 11 boys and 9 girls. Find the number of ways of choosing

(i) 7 pupils if there are no restrictions. [2]

(ii) at least 5 boys. [3]

(a) By using an appropriate substitution, find the values of x such that $2(4^x) + (4)^{-x} = 3$.

(b) $3\log_c 2 + \log_c 18 = 2$. Find c. [7]

The points P(-2,2), Q(4,4) and R(5,2) are vertices of a triangle. The perpendicular bisector of PQ and the line through P parallel to QR intersect at S.
Find the coordinates of S. [7]

(a) Find the range of the function $f : x \to 2x^2 + 3x + 4$ for the domain $-3 \le x \le 2$. [5]

(b) Find the range of values of c, given that for all values of x, $x^2 - 4x + c > 6$. [3]

The functions f and g are defined by

$$f : x \to \frac{ax+6}{x}, x \neq 0$$

$$g : x \to \frac{x+1}{x-3}, x \neq 3.$$

Find

(a) g^2, [3]

(b) g^{-1}, [2]

(c) the value of **a** for which $fg^2(4) = 6$. [3]

8 **(a)** Find the coefficient of x^3 in the expansion of

(i) $\left(2-\dfrac{3x}{2}\right)^8$, [2]

(ii) $(1+4x)\left(2-\dfrac{3x}{2}\right)^8$. [3]

(b) Find the term independent of x in the expansion of $\left(x-\dfrac{1}{2x^2}\right)^{15}$. [3]

9 **(a)** Two variables x and y are related by the equation $x^2y = 900$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ in terms of x. [2]

(ii) Hence, find the approximate change in y as x increases from 10 to 10 + p, where p is small. [3]

(b) A particle moves in a straight line, so that its velocity Vm/s and time t seconds after passing a point B is given by V = 8 – 4t.

Find the distance from B

(i) after 3 seconds,

(ii) when it comes to rest. [4]

Find all the angles between 0° and 360° for which

(i) $3\sin\theta = 2\cos^2\theta$, [4]

(ii) $5\tan x = \dfrac{2}{\cos^2 x} - 9$. [5]

10 Two machines, X and Y, are used to pack sweets. A sample of 10 packets was taken from each machine and the mass of each packet, measured to the nearest gram, was noted.

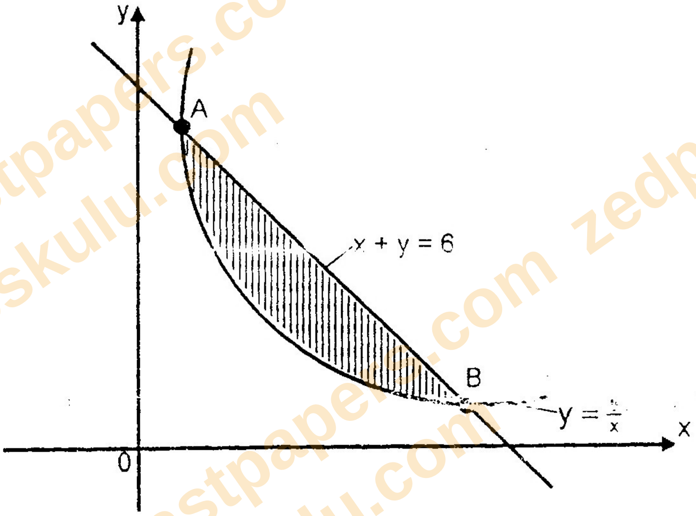
Machine X (mass in g)	196	198	198	199	200	200	201	201	202	205
Machine Y (mass in g)	192	194	195	198	200	201	203	204	206	207

(i) Find the standard deviation of the masses of the packets taken in the sample from each machine. [9]

(ii) Which machine is more reliable? [1]

(a) Find the equation of the normal to the curve $y = x^2 + 7x + 15$ at the point where $x = -4$. [4]

(b) The diagram below shows a line $x + y = 6$ and a curve $y = \frac{5}{x}$ intersecting at A and B.



(i) Find A and B. [2]

(ii) Determine the volume obtained by rotating the shaded area around the x-axis through 360°. [4]

Answer only one of the following alternatives:

EITHER

(a) Given that $y = \frac{8}{x^2} + \frac{x^3}{6}$, find the stationary value of y and determine whether it is a maximum or a minimum. [4]

(b) A cuboid has a total surface area of 150cm^2 and is such that its base is a square of side x cm.

(i) Show that the height, h, of the cuboid is given by $h = \frac{75 - x^2}{2x}$. [2]

(ii) Express the volume, v, of the cuboid in terms of x. [1]

(iii) Given that x can vary, find the value of x for which v has a stationary value. Find this value of v and determine whether it is a maximum or a minimum. [5]

OR

(a) The first three terms of a geometric progression are $x + 2$, $x - 4$ and $x - 6$. Find

(i) the value of x,

(ii) the sum to infinity of the geometric progression. [5]

(b) (i) Find the number of terms of the arithmetic progression 12, 16, 20,...., that must be taken for the sum to be equal to 672. [3]

(ii) The fourth term and the twenty second term of an arithmetic progression are 4 and 25 respectively. Determine the 50^{th} term of the progression. [4]

Question 1. Solve the simultaneous equations: $3x-y+z=8$, $x+2y-z=6$, $2x+y+z=9$.

Add equations (1) and (2): $4x+y=14$. ...(A)

Add equations (2) and (3): $3x+3y=15 \Rightarrow x+y=5$. ...(B)

(A)-(B): $3x=9 \Rightarrow x=3$. From (B): $y=5-3=2$.

From (1): $9-2+z=8 \Rightarrow z=1$.

Verify (3): $6+2+1=9$. ✓

Answer: $x = 3$, $y = 2$, $z = 1$

Question 2. Arrangements and combinations.

(a) Number of arrangements of the word DISCOVERY (9 distinct letters):

$= 9! = 362880$.

Answer: (a) 362 880 arrangements

(b)(i) Choose 7 pupils from 11 boys and 9 girls (total 20), no restrictions:

$C(20,7) = 20!/(7! \times 13!) = 77520$.

Answer: (b)(i) 77 520 ways

(b)(ii) At least 5 boys from 11 boys, with 7 chosen total:

$= C(11,5)\times C(9,2) + C(11,6)\times C(9,1) + C(11,7)\times C(9,0)$.

$= 462\times 36 + 462\times 9 + 330\times 1 = 16632+4158+330 = 21120$.

Answer: (b)(ii) 21 120 ways

Question 3. Equations with substitution and logarithms.

(a) Find x: $2(4^x)+4(2^x)=3$. Let $u=2^x$ so $4^x=u^2$:

$2u^2+4u-3=0$. $u = (-4 + \sqrt{16+24})/4 = (-4+\sqrt{40})/4 = (-2+\sqrt{10})/2$.

(Reject negative root since $u=2^x>0$.)

$x = \log_2((-2+\sqrt{10})/2) = \log_2(-2+\sqrt{10}) - 1$.

Answer: (a) $x = \log_2((-2 + \sqrt{10})/2) \approx -0.285$

(b) $3\log_c(2) + \log_c(18) = 2$. Combine: $\log_c(2^3 \times 18) = 2$.

$\log_c(144) = 2 \Rightarrow c^2 = 144 \Rightarrow c = 12$ ($c > 0$).

Answer: (b) $c = 12$

Question 4. P(2,4), Q(4,4) and R(5,2) are vertices of a triangle. The perpendicular bisector of PQ and the line through P parallel to QR intersect at S.

PQ is horizontal (both have $y=4$). Midpoint of PQ = (3,4).

Perpendicular bisector of PQ is the vertical line x = 3.

Slope of QR = (2-4)/(5-4) = -2. Line through P(2,4) with slope -2:

y - 4 = -2(x-2) => y = -2x + 8.

Find S: x=3 on line y=-2x+8: y = -6+8 = 2. S = (3, 2).

Answer: S = (3, 2)

Question 5. Function f: x → -2x^2+3x+4 for domain -3 <= x <= 2. Also inequality.

(a) Find the range of f on [-3, 2].

Complete square: f = -2(x^2-3x/2)+4 = -2(x-3/4)^2 + 2(9/16)+4 = -2(x-3/4)^2+41/8.

Vertex at x=3/4 (in domain): f_max = 41/8.

Check endpoints: f(-3) = -18-9+4 = -23. f(2) = -8+6+4 = 2.

Minimum: f(-3) = -23.

Answer: (a) Range: -23 <= f(x) <= 41/8

(b) Find range of c such that x^2+cx+c+3 > 0 for all x.

Discriminant must be negative: c^2-4(c+3) < 0 => c^2-4c-12 < 0.

(c-6)(c+2) < 0 => -2 < c < 6.

Answer: (b) -2 < c < 6

Question 6. Functions f: x → (ax+1)/x, x != 0, and g: x → x+3.

(a) Find g^2(x) = g(g(x)):

g(g(x)) = g(x+3) = (x+3)+3 = x+6.

Answer: (a) g^2(x) = x + 6

(b) Find g^(-1)(x):

y = x+3 => x = y-3. So g^(-1)(x) = x-3.

Answer: (b) g^(-1)(x) = x - 3

(c) Find a such that fg^(-1)(4) = 6:

g^(-1)(4) = 4-3 = 1. f(1) = (a*1+1)/1 = a+1.

Set a+1 = 6 => a = 5.

Answer: (c) a = 5

Question 7. Find all angles between 0 and 360 degrees for which:

(i) 3sin(theta)+1 = 2cos^2(theta).

Replace cos^2(theta)=1-sin^2(theta): 3sin+1 = 2-2sin^2.

2sin^2(theta)+3sin(theta)-1=0.

sin(theta) = (-3+sqrt(9+8))/4 = (-3+sqrt(17))/4 ≈ 0.2808. (Reject negative root as <-1.)

theta = arcsin(0.2808) ≈ 16.3° or 180°-16.3° = 163.7°.

Answer: (i) theta ≈ 16.3° and 163.7°

(ii) 5tan^2(x)-9tan(x)-2=0.

Factor: (5tan x+1)(tan x-2)=0 => tan x = -1/5 or tan x = 2.

tan x=2: x = arctan(2) ≈ 63.4° or 243.4°.

tan x=-1/5: x = 180°-arctan(1/5) ≈ 168.7° or 360°-arctan(1/5) ≈ 348.7°.

Answer: (ii) x ≈ 63.4°, 168.7°, 243.4°, 348.7°

Question 8. Binomial expansions.

(a)(i) Coefficient of x^2 in (2-3x)^8:

T_3 = C(8,2)*2^6*(-3x)^2 = 28*64*9*x^2 = 16128 x^2.

Answer: (a)(i) Coefficient of x^2 = 16128

(a)(ii) Coefficient of x^2 in (1+4x)(2-3x)^8:

= 1*(coeff x^2 in (2-3x)^8) + 4*(coeff x in (2-3x)^8).

Coeff x in (2-3x)^8: T_2 = C(8,1)*2^7*(-3) = 8*128*(-3) = -3072.

= 16128 + 4*(-3072) = 16128 - 12288 = 3840.

Answer: (a)(ii) Coefficient of x^2 = 3840

(b) Term independent of x in (x - 1/(2x))^12:

T_{r+1} = C(12,r)*x^{12-r}*(-1/(2x))^r = C(12,r)*(-1/2)^r * x^{12-2r}.

For constant: 12-2r=0 => r=6.

T_7 = C(12,6)*(-1/2)^6 = 924*(1/64) = 231/16.

Answer: (b) Term independent of x = 231/16

Question 9. Variables and particle motion.

(a)(i) x^2*y = 900. Differentiate implicitly with respect to x:

2x*y + x^2*(dy/dx) = 0 => dy/dx = -2y/x = -2*(900/x^2)/x = -1800/x^3.

Answer: (a)(i) dy/dx = -1800/x^3

(a)(ii) Approximate change as x increases from 10 to 10+p:

delta_y ≈ (dy/dx)*delta_x = (-1800/10^3)*p = -1.8p.

Answer: (a)(ii) Approximate change in y = -1.8p

(b) Particle from B with V = 8-4t (m/s).

(b)(i) Distance from B after 3 seconds (V=0 at t=2, so particle reverses):

s(t) = integral(8-4t)dt = 8t-2t^2. s(3) = 24-18 = 6m.

Answer: (b)(i) Distance from B after 3 s = 6 m

(b)(ii) Particle comes to rest: V=0 => 8-4t=0 => t=2.

s(2) = 16-8 = 8 m from B.

Answer: (b)(ii) Distance from B when at rest = 8 m (at t = 2 s)

Question 10. Two machines X and Y pack sweets. Sample of 10 packets each.

Machine X masses (g): 196,198,198,198,200,200,201,201,202,205.

Machine Y masses (g): 192,194,194,195,198,200,201,203,204,205.

Mean X = (196+198+198+198+200+200+201+201+202+205)/10 = 1999/10 = 199.9 g.

Mean Y = (192+194+194+195+198+200+201+203+204+205)/10 = 1986/10 = 198.6 g.

Var X: Sigma(x^2)/n - mean^2. Sigma(x^2) = 38416+3(39204)+2(40000)+2(40401)+40804+42025.

= 38416+117612+80000+80802+40804+42025 = 399659.

Var X = 399659/10 - 199.9^2 = 39965.9 - 39960.01 = 5.89. SD_X = sqrt(5.89) ≈ 2.43 g.

Var Y: Sigma(y^2) = 36864+37636+37636+38025+39204+40000+40401+41209+41616+42025 = 394616.

Var Y = 394616/10 - 198.6^2 = 39461.6 - 39441.96 = 19.64. SD_Y = sqrt(19.64) ≈ 4.43 g.

Answer: (i) SD of X ≈ 2.43 g; SD of Y ≈ 4.43 g

Answer: (ii) Machine X is more reliable (smaller standard deviation)

Question 11. Curve y = x^2+7x+15. Line x+y=5 and curve y=4/x.

(a) Find the equation of the normal to y=x^2+7x+15 at x=-4.

At x=-4: y = 16-28+15 = 3. Point (-4, 3).

dy/dx = 2x+7. At x=-4: gradient = -8+7 = -1.

Normal slope = 1 (negative reciprocal). Normal: y-3 = 1*(x+4) => y = x+7.

Answer: (a) Normal equation: y = x + 7

(b) Line x+y=5 (y=5-x) and curve y=4/x intersect at A and B.

(b)(i) Set 5-x = 4/x: x(5-x)=4 => x^2-5x+4=0 => (x-1)(x-4)=0.

x=1: y=4. x=4: y=1. So A=(1,4) and B=(4,1).

Answer: (b)(i) A = (1, 4) and B = (4, 1)

(b)(ii) Volume when shaded region rotated 360 degrees about x-axis (from x=1 to x=4):

V = pi * integral from 1 to 4 of [(5-x)^2 - (4/x)^2] dx.

= pi * integral[(25-10x+x^2) - 16/x^2] dx.

= pi * [25x - 5x^2 + x^3/3 + 16/x] from 1 to 4.

At x=4: 100 - 80 + 64/3 + 4 = 24 + 64/3 = 136/3.

At x=1: 25 - 5 + 1/3 + 16 = 36 + 1/3 = 109/3.

V = pi*(136/3 - 109/3) = pi*(27/3) = 9*pi.

Answer: (b)(ii) Volume = 9 pi cubic units

Question 12. EITHER: Stationary value and cuboid.

(a) Given y = 8x^2 - x^4. Find stationary values.

dy/dx = 16x - 4x^3 = 4x(4-x^2) = 0 => x=0 or x=+-2.

d^2y/dx^2 = 16-12x^2. At x=0: 16>0 => local minimum, y=0.

At x=+-2: 16-48=-32<0 => local maximum, y=8(4)-16=32-16=16.

Answer: (a) Max value = 16 (at x = +/-2); Min value = 0 (at x = 0)

(b) Cuboid: total surface area 150 cm^2, square base of side x cm.

(b)(i) Show h=(75-x^2)/(2x):

Surface area = 2x^2 + 4xh = 150 => 4xh = 150-2x^2 => h = (150-2x^2)/(4x) = (75-x^2)/(2x). Q.E.D.

Answer: (b)(i) h = (75-x^2)/(2x) proved.

(b)(ii) Volume V = x^2 * h = x^2*(75-x^2)/(2x) = x(75-x^2)/2 = (75x-x^3)/2.

Answer: (b)(ii) V = (75x - x^3)/2

(b)(iii) Find stationary value: dV/dx = (75-3x^2)/2 = 0 => x^2=25 => x=5.

d^2V/dx^2 = -3x. At x=5: -15 < 0 => maximum.

V_max = (75*5 - 125)/2 = (375-125)/2 = 250/2 = 125 cm^3.

Answer: (b)(iii) Maximum volume = 125 cm^3 (at x = 5 cm)

Question 12or. OR: Geometric and arithmetic progressions.

(a) GP: first three terms are x+2, x-4 and x-6.

Equal ratios: (x-4)/(x+2) = (x-6)/(x-4).

(x-4)^2 = (x+2)(x-6) => x^2-8x+16 = x^2-4x-12.

-4x = -28 => x = 7.

T1 = 9, T2 = 3, T3 = 1. Common ratio r = 1/3. |r|<1 so S_inf exists.

S_inf = T1/(1-r) = 9/(1-1/3) = 9/(2/3) = 27/2.

Answer: (a)(i) x = 7

Answer: (a)(ii) Sum to infinity = 27/2 = 13.5

(b)(i) AP: 12, 16, 20, ... (a=12, d=4). Find n for S_n=672.

S_n = n/2*(2*12+(n-1)*4) = n/2*(20+4n) = n(10+2n).

2n^2+10n = 672 => n^2+5n-336=0 => (n+21)(n-16)=0 => n=16.

Answer: (b)(i) n = 16 terms

(b)(ii) AP: T_10=4, T_22=25. Find T_50.

a+9d=4. ...(1) a+21d=25. ...(2) Subtract: 12d=21 => d=7/4.

$a = 4-9(7/4) = 4-63/4 = -47/4$.

$T_{50} = a+49d = -47/4 + 49*(7/4) = (-47+343)/4 = 296/4 = 74$.

Answer: (b)(ii) $T_{50} = 74$

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