G12 ECZ ADDITIONAL MATHEMATICS 2006 PAPER 2 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2006 Paper 2 (4030/2)

12 questions, 100 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate
and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4030/2

PAPER 2

Thursday 23 NOVEMBER 2006 2 hours 30 minutes

Additional materials:
Answer paper
Graph paper (3 sheets)
Mathematical tables/calculators

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the answer paper/answer booklet.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

All working must be shown clearly.

If the degree of accuracy is not specified in the question and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.

INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question.

Calculators may be used.

Cell phones should not be brought in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

The total number of marks is 100.

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\operatorname{cosec}^2 A = 1 + \cot^2 A.$

Formula for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc \cos A.$

$$\Delta = \frac{1}{2} bc \sin A$$

3 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \quad \text{SD} = \sqrt{\left\{\frac{\sum (x - \bar{x})^2}{n}\right\}} = \sqrt{\left\{\frac{\sum x^2}{n} - (\bar{x})^2\right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \quad \text{SD} = \sqrt{\left\{\frac{\sum f(x - \bar{x})^2}{\sum f}\right\}} = \sqrt{\left\{\frac{\sum fx^2}{\sum f} - (\bar{x})^2\right\}}$$

Pure Mathematics

Solve the simultaneous equations:

$-x + y + 3z = -9$

$2x - y - z = 11$

$x + 2y + 3z = -3$ [6]

Show that the equation $x^2 + (2-k)x + k = 3$ has real roots for all real values of k. [6]

Solve the equations

(a) $\log x + \log(5x+5) = 2$

(b) $2^{x+1} + 2^x = 9$ [6]

4 **(a)** In how many ways can 5 boys and 3 girls stand in a straight line, if

(i) there are no restrictions?

(ii) the boys stand next to each other? [4]

(b) A collection of 16 books contains one of Harry Potter's books. Mary is going to choose 6 of these books to take on holiday.

(i) In how many ways can she choose 6 books?

(ii) How many of these choices will include the Harry Potter book? [4]

5 **(a)** Given that the expansion of $(b+x)(1-2x)^n$ in ascending powers of x is $3 - 41x + cx^2 + \ldots\ldots$, find the values of the constants b, c and n. [6]

(b) Find the term independent of x in the expansion of $\left(2x - \frac{1}{2x^2}\right)^{12}$ [3]

6 The expression $x^3 + ax^2 + bx + 12$ is divisible by $x^2 + 2x - 3$.

(i) Find the value of a and b.

(ii) Find the remaining factor of the expression.

(iii) Hence, or otherwise, solve the equation $x^3 + ax^2 + bx = -12$. [9]

7 A particle moves in a straight line so that, at time t seconds after leaving a fixed point O, its velocity Vm/s is given by

$$V = \frac{27}{(2t+1)^2} - 3.$$

Find

(i) the initial velocity, [2]

(ii) the value of t for which the particle is at instantaneous rest, [3]

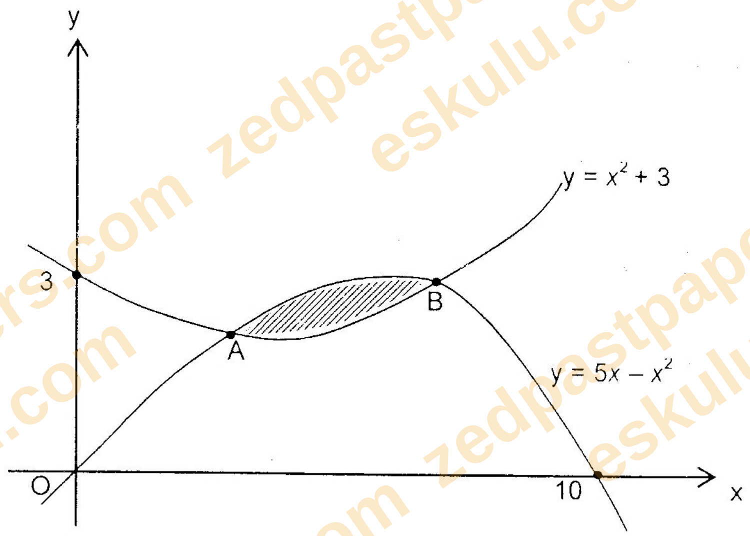
(iii) the initial acceleration of the particle. [4]

8 Find all the angles between 0° and 360° for which

(i) $2\sec^2 x = 8 - \tan x$,

(ii) $\cot^2 x + \operatorname{cosec} x = 11$. [8]

9 The diagram shows the intersection of two curves $y = x^2 + 3$ and $y = 5x - x^2$ at the points A and B.



(i) Find the x-coordinates of points A and B. [2]

(ii) Calculate the area of the shaded region. [3]

(iii) The shaded region is rotated through 360° about the x-axis to form a solid of revolution. Calculate the volume generated. [4]

10 The curve $y = 3 - e^{2x}$ meets the x-axis at A and the y-axis at B.

(i) Find the coordinates of A and B. [3]

(ii) Sketch the curve for the domain $-3 \leq x \leq 1$. [3]

(iii) Find the equation of the straight line which must be drawn on the graph of $y = 3 - e^{2x}$ to obtain a solution of the equation $x = \ln \sqrt{2 - x}$. [4]

11 A random sample of 97 people who own mobile phones was used to collect data on the amount of time they spent per day on their phones. The result are displayed in the table below.

Time spent per day (t minutes)	$0 \leq t < 5$	$5 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$30 \leq t < 40$	$40 \leq t < 70$
Number of people	11	20	32	18	10	6

(i) Calculate estimates of the mean and standard deviation of the time spent per day on these mobile phones.

(ii) On graph paper, draw a fully labelled histogram to represent the data. [10]

12 **Either**

(a) A geometric progression has a first term **a** and a common ratio **r**.
Given that the second term of the progression is 24 and the fifth term is $\frac{8}{9}$,
calculate the

(i) first term,

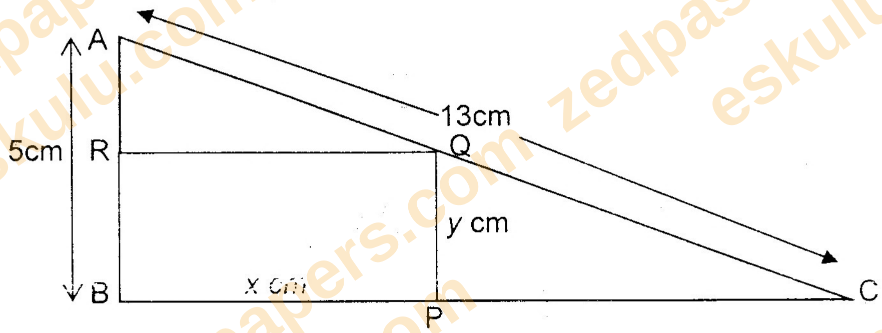
(ii) common ratio,

(iii) sum to infinity. [5]

(b) In an arithmetic progression T_n stands for the **n**th term and S_n stands for the sum of the first n terms. Given that $T_{15} = 2T_9$ and $S_{15} + S_9 = 279$,
find the first term and the common difference. [5]

Or

In the triangle ABC, angle ABC = 90°, AB = 5cm and AC = 13cm as shown in the diagram. The rectangle BPQR is such that its vertices P, Q and R lie on BC, CA and AB respectively.



Given that BP = x cm and PQ = y cm,

(a) show that $y = \dfrac{60 - 5x}{12}$, [4]

(b) express the area of the rectangle in terms of x and hence calculate the maximum value of this area as x varies. [6]

Question 1. Solve the simultaneous equations: -x+y+3z=-9, 2x-y-z=11, x+2y+3z=-3.

Add equations (1) and (2): (-x+y+3z)+(2x-y-z) = -9+11 => x+2z = 2. ...(A)

2×(2)+(3): (4x-2y-2z)+(x+2y+3z) = 22+(-3) => 5x+z = 19. ...(B)

From (A): x = 2-2z. Substitute into (B): 5(2-2z)+z=19 => 10-9z=19 => z=-1.

x = 2-2(-1) = 4. From (1): -4+y+3(-1)=-9 => y=-9+7 = -2.

Verify (2): 2(4)-(-2)-(-1) = 8+2+1 = 11 ✓. (3): 4+2(-2)+3(-1)=-3 ✓.

Answer: x = 4, y = -2, z = -1

Question 2. Show that x^2 - (2-k)x - (k+3) = 0 has real roots for all real values of k.

Rearrange equation to standard form: x^2 - (2-k)x - (k+3) = 0.

Here a=1, b=-(2-k)=(k-2), c=-(k+3).

Discriminant = b^2 - 4ac = (k-2)^2 - 4(1)(-(k+3)).

= (k-2)^2 + 4(k+3) = k^2-4k+4+4k+12 = k^2 + 16.

Since k^2 >= 0 for all real k, we have Delta = k^2+16 >= 16 > 0.

Since Delta > 0 for all real k, the equation always has two distinct real roots.

Answer: Delta = k^2 + 16 > 0 for all real k. Q.E.D.

Question 3. Solve the equations: (a) log x + log(5x+5) = 2; (b) 2^(x+1) + 2^x = 9.

(a) log x + log(5x+5) = 2 (base 10).

log[x(5x+5)] = 2 => x(5x+5) = 10^2 = 100 => 5x^2+5x-100 = 0 => x^2+x-20 = 0.

(x+5)(x-4) = 0 => x=-5 or x=4. Since log requires x>0 and 5x+5>0: reject x=-5.

Answer: (a) x = 4

(b) 2^(x+1) + 2^x = 9.

2×2^x + 2^x = 9 => 3×2^x = 9 => 2^x = 3 => x = log_2(3) = ln3/ln2.

Answer: (b) x = log_2(3) = ln3/ln2 ≈ 1.585

Question 4. Permutations and combinations.

(a) 5 boys and 3 girls in a straight line.

(a)(i) No restrictions: all 8 people arranged in line = 8! = 40320.

Answer: (a)(i) 40 320 ways

(a)(ii) Boys stand next to each other: treat 5 boys as one unit. 4 units (1+3) arranged: 4! ways. Boys within unit: 5! ways.

Total = 4! × 5! = 24 × 120 = 2880.

Answer: (a)(ii) 2 880 ways

(b) 16 books (6 Harry Potter), choose 6 to take on holiday.

(b)(i) Total ways to choose 6 books from 16: C(16,6) = 16!/(6!×10!) = 8008.

Answer: (b)(i) 8 008 ways

(b)(ii) Choices including the specific Harry Potter book: fix 1 HP book, choose 5 more from remaining 15.

C(15,5) = 15!/(5!×10!) = 3003.

Answer: (b)(ii) 3 003 ways

Question 5. Binomial expansion.

(a) Expansion of (3+8x)(1-2x)^n = 3-4x+cx^2+... Find n, b (coefficient of x), c.

(1-2x)^n = 1 + n(-2x) + C(n,2)(-2x)^2 + ... = 1-2nx+2n(n-1)x^2-...

(3+8x)(1-2x)^n: constant term = 3 ✓.

x coefficient: 3(-2n)+8 = -6n+8 = -4 => 6n = 12 => n = 2.

x^2 coefficient c: 3×C(2,2)×4 + 8×(-4) = 12-32 = -20.

Verify: (3+8x)(1-4x+4x^2) = 3-12x+12x^2+8x-32x^2+... = 3-4x-20x^2. ✓

Answer: n = 2, coefficient of x in first factor = 8, c = -20

(b) Find the term independent of x in (2x - 1/(2x))^8.

T_{r+1} = C(8,r)(2x)^(8-r)(-1/(2x))^r = C(8,r)*2^(8-r)*(-1)^r*(2)^(-r)*x^(8-r)*x^(-r).

= C(8,r)*2^(8-2r)*(-1)^r * x^(8-2r).

Constant term: 8-2r=0 => r=4.

T_5 = C(8,4)*2^(8-8)*(-1)^4 = 70*1*1 = 70.

Answer: (b) Term independent of x = 70

Question 6. The expression x^4 + ax^2 + bx + 12 is divisible by x^2 + 2x - 2.

(i) Find a and b. Perform long division: quotient = x^2+cx+d.

(x^2+2x-2)(x^2+cx+d) = x^4+(c+2)x^3+(d+2c-2)x^2+(2d-2c)x-2d.

Compare with x^4+ax^2+bx+12:

x^3: c+2=0 => c=-2.

x^0: -2d=12 => d=-6.

x^2: d+2c-2 = -6-4-2 = -12 => a=-12.

x^1: 2d-2c = -12+4 = -8 => b=-8.

Answer: (i) a = -12, b = -8

(ii) Other factor of x^4-12x^2-8x+12:

(x^2+2x-2)(x^2-2x-6) = x^4-12x^2-8x+12. ✓

Answer: (ii) Other factor: x^2 - 2x - 6

(iii) Solve x^4-12x^2-8x+12 = 0 using factors:

(x^2+2x-2)(x^2-2x-6) = 0.

x^2+2x-2=0: x = (-2 ± sqrt(4+8))/2 = -1 ± sqrt(3).

x^2-2x-6=0: x = (2 ± sqrt(4+24))/2 = 1 ± sqrt(7).

Answer: (iii) x = -1+sqrt(3), x = -1-sqrt(3), x = 1+sqrt(7), x = 1-sqrt(7)

Question 7. A particle moves so that velocity Vm/s at time t seconds is V = 27/(2t+1)^3 - 3.

(i) Initial velocity (t=0): V = 27/(2(0)+1)^3 - 3 = 27/1 - 3 = 24 m/s.

Answer: (i) Initial velocity = 24 m/s

(ii) At rest: V=0 => 27/(2t+1)^3 = 3 => (2t+1)^3 = 9 => 2t+1 = 9^(1/3) = cuberoot(9).

t = (9^(1/3)-1)/2 = (cuberoot(9)-1)/2 ≈ (2.080-1)/2 ≈ 0.540 s.

Answer: (ii) t = (cuberoot(9) - 1)/2 ≈ 0.540 seconds

(iii) Initial acceleration: dV/dt = 27×(-3)(2t+1)^(-4)×2 = -162(2t+1)^(-4).

At t=0: dV/dt = -162/(1)^4 = -162 m/s^2.

Answer: (iii) Initial acceleration = -162 m/s^2 (deceleration of 162 m/s^2)

Question 8. Find all angles between 0 and 360 degrees.

(i) 2sec^2(x) = 8 - tan(x).

Replace sec^2(x) = 1+tan^2(x): 2(1+tan^2 x) = 8-tan x.

2+2tan^2(x)+tan(x)-8=0 => 2tan^2(x)+tan(x)-6=0.

(2tan x-3)(tan x+2)=0 => tan x = 3/2 or tan x = -2.

tan x=3/2: x = arctan(3/2) ≈ 56.3° or 180°+56.3° = 236.3°.

tan x=-2: x = 180°-arctan(2) ≈ 116.6° or 360°-arctan(2) ≈ 296.6°.

Answer: (i) x ≈ 56.3°, 116.6°, 236.3°, 296.6°

(ii) cot^2(x) + cosec(x) = 11.

Replace cot^2(x) = cosec^2(x)-1: cosec^2(x)-1+cosec(x)=11.

cosec^2(x)+cosec(x)-12=0 => (cosec x+4)(cosec x-3)=0.

cosec x=-4 => sin x=-1/4: x=180°+arcsin(1/4) ≈ 194.5° or 360°-arcsin(1/4) ≈ 345.5°.

cosec x=3 => sin x=1/3: x=arcsin(1/3) ≈ 19.5° or 180°-arcsin(1/3) ≈ 160.5°.

Answer: (ii) x ≈ 19.5°, 160.5°, 194.5°, 345.5°

Question 9. Two curves y = x^2 + 3 and y = 5x - x^2 intersect at A and B.

(i) x-coordinates of A and B: x^2+3 = 5x-x^2 => 2x^2-5x+3=0 => (2x-3)(x-1)=0.

x=1 (point A) or x=3/2 (point B).

Answer: (i) x-coords: A has x=1, B has x=3/2

(ii) Area of shaded region (between the curves from x=1 to x=3/2):

Area = integral from 1 to 3/2 of [(5x-x^2)-(x^2+3)] dx = integral(5x-2x^2-3) dx.

= [5x^2/2 - 2x^3/3 - 3x] from 1 to 3/2.

At x=3/2: 45/8 - 9/4 - 9/2 = 45/8-18/8-36/8 = -9/8.

At x=1: 5/2 - 2/3 - 3 = 15/6-4/6-18/6 = -7/6.

Area = |-9/8 - (-7/6)| = |-27/24+28/24| = 1/24.

Answer: (ii) Area = 1/24 square units

(iii) Volume of revolution (360° about x-axis):

V = pi × integral from 1 to 3/2 of [(5x-x^2)^2-(x^2+3)^2] dx.

(5x-x^2)^2 = 25x^2-10x^3+x^4. (x^2+3)^2 = x^4+6x^2+9.

Difference = 19x^2-10x^3-9.

integral = [19x^3/3 - 5x^4/2 - 9x] from 1 to 3/2.

At x=3/2: 19(27/8)/3 - 5(81/16)/2 - 27/2 = 171/8-405/32-27/2 = 684/32-405/32-432/32 = -153/32.

At x=1: 19/3-5/2-9 = 38/6-15/6-54/6 = -31/6.

V = pi×[(-153/32)-(-31/6)] = pi×(-918/192+992/192) = pi×74/192 = 37pi/96.

Answer: (iii) Volume = 37pi/96 cubic units

Question 10. The curve y = 3 - e^x meets the x-axis at A and the y-axis at B.

(i) Find coordinates of A and B.

At A (y=0): 3-e^x=0 => e^x=3 => x=ln3. A=(ln3, 0).

At B (x=0): y=3-e^0=3-1=2. B=(0, 2).

Answer: (i) A = (ln3, 0), B = (0, 2)

(ii) Sketch for -3 <= x <= 1:

The curve y=3-e^x is strictly decreasing (dy/dx=-e^x<0) and concave down (d^2y/dx^2=-e^x<0).

At x=-3: y≈2.95. At x=0 (y-axis): y=2 (point B). At x=1: y≈0.28.

The curve decreases from top-left toward lower-right, passing through B(0,2).

A(ln3,0) lies just beyond x=1.1 (outside the sketch range).

Answer: (ii) Smooth decreasing concave-down curve through B(0,2); label B; A is indicated.

(iii) Find straight line needed to solve x = ln(2-x) on the graph.

From x=ln(2-x): e^x=2-x. On y=3-e^x: e^x=3-y. So 3-y=2-x => y=x+1.

Drawing y=x+1 on the same graph: its intersection with y=3-e^x gives x=ln(2-x).

Answer: (iii) Straight line: y = x + 1

Question 11. Statistics: mobile phone usage data (97 people).

Grouped frequency table (frequency density needed for unequal class widths):

Interval | Midpoint m | Freq f | fm | fm^2

0-5 | 2.5 | 11 | 27.5 | 68.75

5-10 | 7.5 | 20 | 150 | 1125

10-20 | 15 | 32 | 480 | 7200

20-30 | 25 | 18 | 450 | 11250

30-40 | 35 | 10 | 350 | 12250

40-70 | 55 | 6 | 330 | 18150

Totals: | | 97 | 1787.5 | 50043.75

(i) Mean = Sigma(fm)/Sigma(f) = 1787.5/97 ≈ 18.4 minutes.

Variance = Sigma(fm^2)/Sigma(f) - mean^2 = 50043.75/97 - (1787.5/97)^2.

= 515.9 - 339.6 = 176.3. SD = sqrt(176.3) ≈ 13.3 minutes.

Answer: (i) Mean ≈ 18.4 min, Standard deviation ≈ 13.3 min

(ii) Histogram (unequal class widths => use frequency density = freq/class width):

0-5 width 5: fd=2.20. 5-10 width 5: fd=4.00. 10-20 width 10: fd=3.20.

20-30 width 10: fd=1.80. 30-40 width 10: fd=1.00. 40-70 width 30: fd=0.20.

Draw rectangles with bases on x-axis (time) and heights = frequency density.

Answer: (ii) Histogram: x-axis = time (min), y-axis = frequency density; bars at the widths and heights above.

Question 12. EITHER: Series.

(a) GP: second term = 24, fifth term = 64/9. Find first term, common ratio, sum to infinity.

T_2 = ar = 24 and T_5 = ar^4 = 64/9.

Divide: r^3 = (64/9)/24 = 64/216 = 8/27 => r = 2/3.

a = 24/r = 24/(2/3) = 36.

Sum to infinity: S_inf = a/(1-r) = 36/(1-2/3) = 36/(1/3) = 108.

Answer: (a)(i) First term a = 36

Answer: (a)(ii) Common ratio r = 2/3

Answer: (a)(iii) Sum to infinity S_inf = 108

(b) AP: T_{10} = 27 and S_6 = 279. Find first term and common difference.

T_{10}: a+9d = 27. ...(1)

$S_6 = 6/2\times(2a+5d) = 3(2a+5d) = 279 \Rightarrow 2a+5d = 93$. ...(2)

From (1): $a = 27-9d$. Substitute into (2): $2(27-9d)+5d = 54-13d = 93 \Rightarrow d = -3$.

$a = 27-9(-3) = 27+27 = 54$.

Verify: $T_{10}=54-27=27$ ✓. $S_6=3(108-15)=3\times93=279$ ✓.

Answer: (b) First term a = 54, common difference d = -3

Question 12or. OR: Triangle ABC with angle ABC = 90 deg, AB = 5 cm, AC = 13 cm. Rectangle BPQR inscribed.

$BC = \sqrt{AC^2-AB^2} = \sqrt{169-25} = \sqrt{144} = 12$ cm.

Setup: B at right angle. P on BC, Q on hypotenuse AC, R on AB.

With BP=x, PQ=y: rectangle has corners B(0,0), P(x,0), Q(x,y), R(0,y).

(a) Show $y = (60-5x)/12$.

Line AC from A(0,5) to C(12,0): equation $x/12 + y/5 = 1 \Rightarrow 5x+12y=60$.

Q=(x,y) lies on AC: $5x+12y=60 \Rightarrow y=(60-5x)/12$. Q.E.D.

Answer: (a) y = (60-5x)/12 proved.

(b) Area of rectangle $= xy = x\times(60-5x)/12 = (60x-5x^2)/12$.

$dA/dx = (60-10x)/12$. Set =0: $60-10x=0 \Rightarrow x=6$.

$d^2A/dx^2 = -10/12 < 0 \Rightarrow$ maximum at x=6.

Maximum area $= (60\times6-5\times36)/12 = (360-180)/12 = 180/12 = 15$ cm^2.

Answer: (b) Maximum area = 15 cm^2 (when BP = 6 cm)

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