G12 ECZ ADDITIONAL MATHEMATICS 2021 GCE PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2021 GCE Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

4030-1-000340

Additional Mathematics

4030/1

Paper 1

2021

Additional Materials:

Answer Booklet
Silent electronic calculator (Non programmable)

Time: 2 hours

Marks: 80

Instructions to Candidates

1. Write the **centre number** and your **examination number** on **every page** of the separate **Answer Booklet** provided.
2. There are **twelve** questions in this paper. Answer **all** questions.
3. Write your answers in the separate Answer Booklet provided.
4. If you use more than one Answer Booklet, fasten the Answer Booklets together.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for Candidates

1. The number of marks is shown in brackets [ ] at the end of each question or part question.
2. **The use of a non programmable electronic calculator is expected, where appropriate.**
3. You are reminded of the need for clear presentation in your answers.
4. Cell phones and other electronic devices are **not allowed** in the examination room.
5. Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\mathrm{Sin}^2 A + \cos^2 A = 1$

$\mathrm{Sec}^2 A = 1 + \tan^2 A$

$\mathrm{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that the coordinates of the midpoint of the points $(1-p, 1+t)$ and $(2p, 4t+3)$ are $(3, 7)$, find the value of p and of t. [4]

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2 Solve the simultaneous equations

$2y = x + 5,$

$x^2 + xy = -1.$ [5]

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3 Find the values of k for which $x + 2y = k$ is a tangent to the curve $y^2 + 4x - 20 = 0$. [4]

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4 The functions f and g are defined by $f{:}x \mapsto \dfrac{x+5}{2x}$, $x \neq 0$ and $g{:}x \mapsto 2x + 4$, where x is a real number.

Find

(a) $(fg)^{-1}(x)$, [4]

(b) the value of x for which $(fg)^{-1}(x) = -3$. [2]

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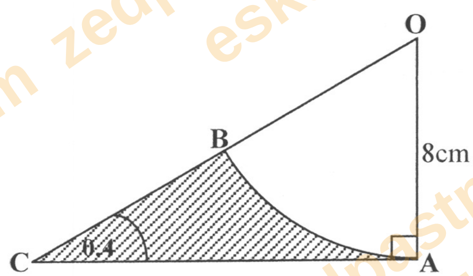
5 In the diagram below, triangle OCA is right angled at A, angle ACO is 0.4 radians and OBA is a sector of a circle centre O and radius 8 cm.



Calculate

(a) the length of AC, [2]

(b) the area of the shaded region. [3]

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6 Prove the identity

$$\frac{1}{(\sec\theta - \tan\theta)^2} \equiv \frac{1+\sin\theta}{1-\sin\theta}.$$ [4]

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7 **(a)** Find the coefficient of x in the expansion of $\left(x^2 - \dfrac{4}{x}\right)^5$. [4]

(b) Obtain the first four terms in the expansion of $(1 + p)^8$ in ascending powers of p. Hence find the coefficient of x in the expansion of $(1 + x + 2x^2)^8$. [5]

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8 (a) Find the gradient function of $y = \dfrac{2x^2}{3x^2+6}$. [3]

(b) Find the equation of the normal to the curve $y = \dfrac{3}{2x-5}$, $x \neq \dfrac{5}{2}$, at a point where $x = 2$. [6]

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9 On the same diagram, sketch the graphs of $y = \left|\cos\left(\dfrac{x}{2}\right) - 1\right|$ and $y = 1$ for the domain $0 \leq x \leq 2\pi$. Hence state the number of solutions for which $\left|\cos\left(\dfrac{x}{2}\right) - 1\right| = 1$. [5]

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10 (a) Find the vector equation of the perpendicular bisector of the line joining the points (–3, –9) and (11, 5). [4]

(b) The position vectors of P, Q and R are $3i - 2j$, $ni + j$ and $2i - 8j$ respectively. Find the value of n for which P, Q and R are collinear. [5]

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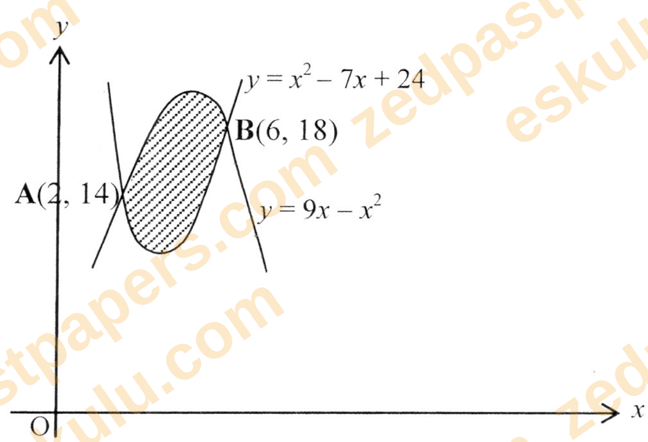
11 (a) Find $\displaystyle\int\left(x^3 - \frac{6}{x^2}\right)dx$. [2]

(b) The diagram below shows part of the curves $y = x^2 - 7x + 24$ and $y = 9x - x^2$ intersecting at A(2, 14) and B(6, 18).

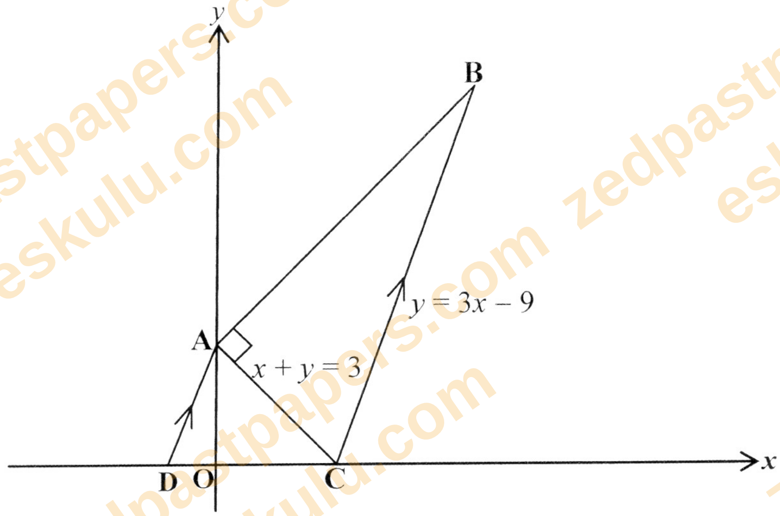


Find

(i) the tangent to the curve $y = x^2 - 7x + 24$ at the point B, [4]

(ii) the area of the shaded region. [4]

12 In the diagram below, ABCD is a trapezium in which angle BAC = 90° and AD is parallel to CB. The points C and D lie on the x-axis and A lies on the y-axis. The equations of lines CB and AC are $y = 3x - 9$ and $x + y = 3$ respectively.



Find

(a) the equation of line AB, [3]

(b) the coordinates of the points D and B, [4]

(c) the area of the trapezium ABCD. [3]

Question 1. The midpoint of P(-1, t+1) and Q(2p, 4p-1) is (3, 7p). Find p and t.

x-coordinate: (-1 + 2p)/2 = 3 => -1 + 2p = 6 => 2p = 7 => p = 7/2.

y-coordinate: (t+1 + 4p-1)/2 = 7p => (t+4p)/2 = 7p => t + 4p = 14p => t = 10p.

t = 10 * (7/2) = 35.

Answer: p = 7/2 and t = 35

Question 2. Solve the simultaneous equations: 2y - x = 3 and y^2 + xy = 6.

From linear equation: x = 2y - 3. Substitute into y^2 + xy = 6.

y^2 + (2y-3)y = 6 => y^2 + 2y^2 - 3y = 6 => 3y^2 - 3y - 6 = 0.

y^2 - y - 2 = 0 => (y-2)(y+1) = 0 => y = 2 or y = -1.

y=2: x = 4-3 = 1. y=-1: x = -2-3 = -5.

Answer: Solutions: (x, y) = (1, 2) and (x, y) = (-5, -1)

Question 3. Find the value of k for which x + 2y = k is a tangent to the curve y^2 + 4x - 20 = 0.

From line: x = k - 2y. Substitute into y^2 + 4x - 20 = 0.

y^2 + 4(k-2y) - 20 = 0 => y^2 - 8y + (4k-20) = 0.

For tangency: discriminant = 0. Delta = 64 - 4(4k-20) = 64 - 16k + 80 = 144 - 16k = 0.

16k = 144 => k = 9.

Verify: y^2 - 8y + 16 = 0 => (y-4)^2 = 0 => y=4 (double root). ✓

Answer: k = 9

Question 4. Functions f: x → 3/(2x+1), x != -1/2, and g: x → 2x + 4.

(a) Find fg(-1).

Step 1: g(-1) = 2(-1) + 4 = 2.

Step 2: fg(-1) = f(g(-1)) = f(2) = 3/(2(2)+1) = 3/(4+1) = 3/5.

Answer: (a) fg(-1) = 3/5

(b) Find x for which g(x) = g^(-1)(x).

Find g^(-1): let y = 2x+4, so x = (y-4)/2. Hence g^(-1)(x) = (x-4)/2.

Set g(x) = g^(-1)(x): 2x+4 = (x-4)/2 => 4x+8 = x-4 => 3x = -12 => x = -4.

Answer: (b) x = -4

Question 5. Triangle OCA right-angled at A. Angle ACO = 0.4 rad. Sector OBA centre O, radius 8cm.

In right triangle OCA: right angle at A, angle ACO = 0.4 rad, OA = 8cm (radius).

(a) Length of AC: $\tan(\text{angle ACO}) = OA/AC \Rightarrow AC = OA/\tan(0.4) = 8/\tan(0.4)$.

$\tan(0.4) \approx 0.4228$. $AC \approx 18.93$cm.

Answer: (a) $AC = 8/\tan(0.4) \approx 18.9$ cm

(b) Area of shaded region = area of triangle OCA - area of sector OBA.

Sector angle at O = $\pi/2 - 0.4$ (angles in triangle sum to π, right angle at A).

Triangle OCA area = $(1/2)(OA)(AC) = (1/2)(8)(8/\tan 0.4) = 32/\tan(0.4)$.

Sector OBA area = $(1/2)(8^2)(\pi/2 - 0.4) = 32(\pi/2 - 0.4)$.

Shaded = $32/\tan(0.4) - 32(\pi/2 - 0.4) = 32[\cot(0.4) - \pi/2 + 0.4]$.

$\approx 32[2.365 - 1.571 + 0.4] \approx 32(1.194) \approx 38.2\ \text{cm}^2$.

Answer: (b) Area of shaded region $\approx 38.2\ \text{cm}^2$

Question 6. Prove the identity: $1/(\cos(\theta)(1+\tan(\theta))) + 1/(\cos(\theta)(1-\tan(\theta))) = 2\cos(\theta)/\cos(2\theta)$.

Combine fractions over common denominator $\cos(\theta)(1-\tan^2(\theta))$:

LHS = $[(1-\tan(\theta))+(1+\tan(\theta))] / [\cos(\theta)(1-\tan^2(\theta))]$

$= 2 / [\cos(\theta)(1-\tan^2(\theta))]$.

Use identity: $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = \cos^2(\theta)(1-\tan^2(\theta))$.

So $\cos(\theta)(1-\tan^2(\theta)) = \cos(2\theta)/\cos(\theta)$.

LHS = $2 / (\cos(2\theta)/\cos(\theta)) = 2\cos(\theta)/\cos(2\theta)$ = RHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Binomial expansions.

(a) Find the coefficient of x^5 in the expansion of $(1 - x/2)^8$.

General term: $T_{r+1} = C(8,r)\cdot(1)^{8-r}\cdot(-x/2)^r = C(8,r)\cdot(-1/2)^r \cdot x^r$.

For x^5: $r=5$. $T_6 = C(8,5)\cdot(-1/2)^5 \cdot x^5 = 56 \cdot (-1/32) \cdot x^5 = -56x^5/32$.

$= -7x^5/4$.

Answer: Coefficient of $x^5 = -7/4$

(b) Find the first four terms in the expansion of $(1+x+2x^2)(1-x/2)^8$.

First four terms of $(1-x/2)^8$:

$T1=1$, $T2=-4x$, $T3=C(8,2)(x/2)^2=28x^2/4=7x^2$, $T4=-C(8,3)(x/2)^3=-56x^3/8=-7x^3$.

Multiply $(1+x+2x^2)(1-4x+7x^2-7x^3+\ldots)$:

x^0: 1*1=1.

x^1: 1*(-4)+1*1=-3.

x^2: 1*7+1*(-4)+2*1=7-4+2=5.

x^3: 1*(-7)+1*7+2*(-4)=-7+7-8=-8.

First four terms: 1 - 3x + 5x^2 - 8x^3.

Answer: Coefficient of x^3 = -8

Question 8. Differentiation.

(a) Find gradient function of y = 2x^2 / (3x^2 + 6).

Using quotient rule: u=2x^2 (u'=4x), v=3x^2+6 (v'=6x).

dy/dx = (4x(3x^2+6) - 2x^2*6x) / (3x^2+6)^2

= (12x^3+24x-12x^3) / (3x^2+6)^2 = 24x / (3(x^2+2))^2 = 24x / (9(x^2+2)^2).

Answer: (a) dy/dx = 8x / (3(x^2+2)^2)

(b) Find equation of normal to y = 3/(2x-5) at x = 2.

At x=2: y = 3/(4-5) = 3/(-1) = -3. Point: (2, -3).

y = 3(2x-5)^(-1). dy/dx = -6/(2x-5)^2. At x=2: dy/dx = -6/(–1)^2 = -6.

Normal slope = 1/6 (negative reciprocal of -6).

Normal through (2,-3): y+3 = (1/6)(x-2) => 6y+18 = x-2 => x-6y = 20.

Answer: (b) Normal equation: x - 6y = 20

Question 9. Sketch y = |cos(x/2) - 1| and y = 1 for 0 <= x <= 2*pi. State number of solutions of |cos(x/2)-1| = 1.

For 0 <= x <= 2pi: x/2 ranges from 0 to pi, so cos(x/2) ranges from 1 to -1.

=> cos(x/2) - 1 ranges from 0 to -2, which is always <= 0.

=> |cos(x/2) - 1| = 1 - cos(x/2) (since cos(x/2)-1 <= 0).

y = 1 - cos(x/2): starts at (0,0), increases to (2pi, 2). Monotone curve.

Setting 1 - cos(x/2) = 1: cos(x/2) = 0 => x/2 = pi/2 => x = pi.

The line y = 1 intersects y = |cos(x/2)-1| at exactly one point: x = pi.

Answer: Number of solutions = 1 (at x = pi)

Question 10. Vectors.

(a) Find vector equation of perpendicular bisector of line joining P(-3,-9) and Q(11,5).

Midpoint M = ((-3+11)/2, (-9+5)/2) = (4, -2).

Direction of PQ = (14,14). Perpendicular direction (rotate 90°): (1,-1) (or (-1,1)).

Perpendicular bisector passes through M=(4,-2) with direction (1,-1):

Answer: (a) r = (4, -2) + t(1, -1)

(b) Position vectors: P=3i-2j, Q=ni+j, R=2i-8j. Find n for collinearity.

PQ = Q-P = (n-3)i + 3j. PR = R-P = -i - 6j.

For collinear: PQ = lambda*PR. From j: 3 = -6*lambda => lambda = -1/2.

From i: n-3 = -1*lambda = 1/2 => n = 7/2.

Answer: (b) n = 7/2

Question 11. Integration and areas.

(a) Find the integral of (x^3 - 6/x^2) dx.

= integral(x^3 dx) - integral(6*x^(-2) dx) = x^4/4 - 6*(-1)*x^(-1) + C.

Answer: (a) Integral = x^4/4 + 6/x + C

(b) Curves y=x^2-7x+24 and y=9x-x^2 intersect at A(2,14) and B(6,18).

(b)(i) Tangent to y=x^2-7x+24 at B(6,18):

dy/dx = 2x-7. At x=6: dy/dx = 12-7 = 5.

Tangent: y-18 = 5(x-6) => y = 5x - 12.

Answer: (b)(i) Tangent at B: y = 5x - 12

(b)(ii) Area of shaded region between the two curves from x=2 to x=6:

Area = integral from 2 to 6 of [(9x-x^2)-(x^2-7x+24)] dx = integral(16x-2x^2-24) dx.

= [8x^2 - 2x^3/3 - 24x] from 2 to 6.

At x=6: 8(36)-2(216)/3-24(6) = 288-144-144 = 0.

At x=2: 8(4)-2(8)/3-24(2) = 32-16/3-48 = -16-16/3 = -64/3.

Area = 0 - (-64/3) = 64/3.

Answer: (b)(ii) Area = 64/3 square units

Question 12. Trapezium ABCD. Angle BAC = 90°, AD || CB. C,D on x-axis, A on y-axis. CB: y=3x-9, AC: x+y=3.

(a) Find equation of line AB.

A is on y-axis (x=0) and on AC (x+y=3): A = (0, 3).

Slope of AC = -1 (from x+y=3). Since angle BAC=90°, slope of AB = 1 (negative reciprocal).

Line AB through A(0,3) with slope 1: y = x + 3.

Answer: (a) Equation of AB: y = x + 3

(b) Find coordinates of D and B.

B = intersection of AB (y=x+3) and CB (y=3x-9):

x+3 = 3x-9 => 12 = 2x => x=6. y=9. B = (6, 9).

D: on x-axis (y=0) and AD || CB (slope 3). Line AD through A(0,3) slope 3: y=3x+3.

At y=0: 3x+3=0 => x=-1. D = (-1, 0).

Answer: (b) D = (-1, 0) and B = (6, 9)

(c) Area of trapezium ABCD. Parallel sides: AD and CB.

|AD| = sqrt((-1-0)^2+(0-3)^2) = sqrt(1+9) = sqrt(10).

|CB| = sqrt((6-3)^2+(9-0)^2) = sqrt(9+81) = sqrt(90) = 3*sqrt(10).

Distance between parallel lines 3x-y+3=0 and 3x-y-9=0: h = |3-(-9)|/sqrt(9+1) = 12/sqrt(10).

Area = (1/2)(|AD|+|CB|)*h = (1/2)(sqrt(10)+3sqrt(10))*(12/sqrt(10)) = (1/2)(4sqrt(10))(12/sqrt(10)).

= (1/2)(4)(12) = 24 sq units.

Answer: (c) Area of trapezium ABCD = 24 square units

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