G12 ECZ ADDITIONAL MATHEMATICS 2020 SPECIMEN PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2020 Specimen Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

1 Find the equation of a line perpendicular to the line $5x - 2y - 11 = 0$ passing through the point $(2, -3)$. [4]

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2 Solve the simultaneous equations [5]

$2x + y = 7,$

$xy - x^2 = 4.$

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3 Find the range of values of k, where $k > 0$, for which the line $y = kx - 2$ does not meet the curve $x^2 - 4y + 1 = 0$. [4]

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4 The functions g and h are defined by $g: x \rightarrow \frac{1}{2x+1}, x \neq -\frac{1}{2}$ and $h: x \rightarrow x + 3$ respectively.

Find

(a) $hg(x)$, [2]

(b) the values of x for which $hg(x) = h(x)$. [4]

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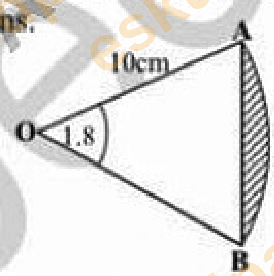
5 In the diagram below, OAB is a sector of a circle with centre O, radius 10cm and angle AOB = 1.8 radians.



Find

(a) the perimeter of the shaded region, [3]

(b) the area of the shaded region. [2]

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6 Prove the identity

$\tan\theta + \cot\theta \equiv \sec\theta \operatorname{cosec}\theta.$ [4]

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7 (a) Find the term independent of x in the expansion of $\left(3x + \frac{1}{x}\right)^{10}$. [4]

(b) Find the coefficient of x^2 in the expansion of $(1 + x)^7 (1 - 2x)^4$. [5]

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8 (a) Find $\frac{dy}{dx}$ for the equation $y = \frac{3x-5}{x^2-4x}$. [4]

(b) Two variables x and y are related by the equation $y = 2x^2 - 3x$. Obtain an expression for $\frac{dy}{dx}$ and find the approximate change in y as x increases from 6 to 6.02. [5]

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9 Find all the angles between 0° and 360° which satisfy the equation

$$|3\sin\theta - 1| = 2.$$ [5]

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10 (a) A and B are points with position vectors $3i + j$ and $10i + j$ respectively. Find the position vector of a point which divides $\overrightarrow{AB}$ in the ratio 3:4. [4]

(b) The position vectors of P and Q are $ai + 2j$ and $3i + j$. Given that the angle between $\underset{\sim}{p}$ and $\underset{\sim}{q}$ is 45°, find two possible values of a. [5]

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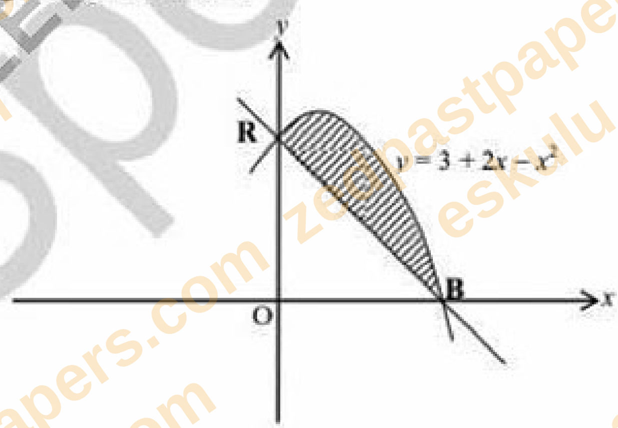
11 (a) Evaluate $\int_1^3 (3x-4)\,dx$. [2]

(b) The diagram below shows part of the curve $y = 3 + 2x - x^2$ which meets a straight line at B and R.



Find

(i) the coordinates of B, [2]

(ii) the equation of BR, [1]

(iii) the area of the shaded region. [5]

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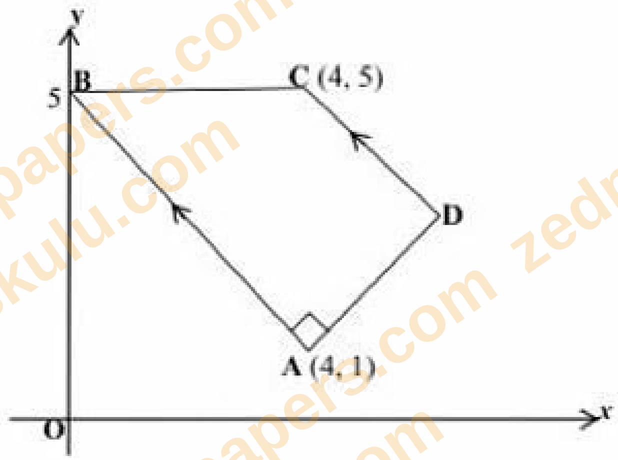
12 In the diagram below, ABCD is a trapezium with A(4, 1) and C(4, 5). The point B lies on the y-axis and angle DAB = 90°.



Find

(a) the equation of line AB, [3]

(b) the equation of line AD, [3]

(c) the coordinates of D. [4]

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/1

Paper 1

Specimen

Additional Materials:

Answer Booklet
Silent electronic calculator (Non programmable)

Time: 2 hours

Instructions to Candidates

Write your **name, centre number** and **candidate number** in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** the questions.

Write your answers in the **Answer Booklet provided**.

If you use more than one Answer Booklet, **fasten** the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Question 1. Find the equation of the line perpendicular to $5x - 2y - 11 = 0$ passing through $(2, -3)$.

Slope of given line: $5x-2y-11=0 \Rightarrow y=(5x-11)/2$, slope $= 5/2$.

Perpendicular slope $= -2/5$ (negative reciprocal).

Line through $(2,-3)$: $y+3 = (-2/5)(x-2) \Rightarrow 5y+15 = -2x+4 \Rightarrow 2x+5y+11 = 0$.

Answer: Equation: $2x + 5y + 11 = 0$

Question 2. Solve the simultaneous equations: $2x + y = 7$ and $xy - x^2 = 4$.

From linear: $y = 7-2x$. Substitute into $xy-x^2=4$.

$x(7-2x)-x^2 = 4 \Rightarrow 7x-2x^2-x^2 = 4 \Rightarrow -3x^2+7x-4 = 0 \Rightarrow 3x^2-7x+4 = 0$.

$(3x-4)(x-1) = 0 \Rightarrow x = 4/3$ or $x = 1$.

$x=4/3$: $y = 7-8/3 = 13/3$. $x=1$: $y = 7-2 = 5$.

Answer: Solutions: $(x, y) = (4/3, 13/3)$ and $(x, y) = (1, 5)$

Question 3. Find the range of values of k ($k > 0$) for which $y = kx - 2$ does not meet $x^2 - 4y + 1 = 0$.

Substitute $y = kx-2$ into $x^2-4y+1=0$:

$x^2 - 4(kx-2) + 1 = 0 \Rightarrow x^2 - 4kx + 9 = 0$.

For no intersection: discriminant < 0. $\Delta = (4k)^2 - 4(1)(9) = 16k^2 - 36 < 0$.

$16k^2 < 36 \Rightarrow k^2 < 9/4 \Rightarrow k < 3/2$. With $k>0$:

Answer: $0 < k < 3/2$

Question 4. Functions g and h defined by $g: x \to 3/(x+1)$, $x \neq -1$, and $h: x \to x + 3$.

(a) Find $hg(x)$.

$hg(x) = h(g(x)) = g(x)+3 = 3/(x+1) + 3 = (3+3(x+1))/(x+1) = (3x+6)/(x+1)$.

Answer: (a) $hg(x) = 3(x+2) / (x+1)$

(b) Find the values of x for which $h(x) = g(x)$.

$x+3 = 3/(x+1) \Rightarrow (x+3)(x+1) = 3 \Rightarrow x^2+4x+3 = 3 \Rightarrow x^2+4x = 0$.

$x(x+4) = 0 \Rightarrow x = 0$ or $x = -4$.

Answer: (b) $x = 0$ or $x = -4$

Question 5. Sector OAB, centre O, radius 10cm, angle AOB = 1.8 radians.

(a) Perimeter of shaded sector OAB:

Perimeter = OA + arc AB + OB = r + r*theta + r = r(2+theta).

= 10(2 + 1.8) = 10 * 3.8 = 38 cm.

Answer: (a) Perimeter = 38 cm

(b) Area of shaded sector OAB:

Area = (1/2)*r^2*theta = (1/2)*(10^2)*(1.8) = (1/2)*100*1.8 = 90 cm^2.

Answer: (b) Area = 90 cm^2

Question 6. Prove the identity: tan(theta) + cot(theta) = sec(theta) * cosec(theta).

LHS = sin(theta)/cos(theta) + cos(theta)/sin(theta).

= [sin^2(theta) + cos^2(theta)] / [sin(theta)*cos(theta)].

= 1 / [sin(theta)*cos(theta)] (using sin^2+cos^2=1).

= (1/cos(theta)) * (1/sin(theta)) = sec(theta) * cosec(theta) = RHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Binomial expansions.

(a) Find the term independent of x in the expansion of (2x + 1/x^2)^9.

General term: T_{r+1} = C(9,r)*(2x)^(9-r)*(1/x^2)^r = C(9,r)*2^(9-r)*x^(9-r)*x^(-2r).

= C(9,r)*2^(9-r)*x^(9-3r).

For term independent of x: 9-3r = 0 => r = 3.

T_4 = C(9,3)*2^6 = 84*64 = 5376.

Answer: Term independent of x = 5376

(b) Find the coefficient of x^5 in the expansion of (1+x+x^2)(1-2x)^5.

(1-2x)^5 terms: x^3 coeff = -C(5,3)*2^3 = -80; x^4 coeff = C(5,4)*2^4 = 80; x^5 coeff = -C(5,5)*2^5 = -32.

Coefficient of x^5 in (1+x+x^2)(1-2x)^5:

= 1*(-32) + 1*80 + 1*(-80) = -32+80-80 = -32.

Answer: Coefficient of x^5 = -32

Question 8. Differentiation and small increments.

(a) Find dy/dx for y = (3x-5)/(x^2-4).

Quotient rule: u=3x-5 (u'=3), v=x^2-4 (v'=2x).

dy/dx = [3(x^2-4)-(3x-5)*2x] / (x^2-4)^2

= [3x^2-12-6x^2+10x] / (x^2-4)^2 = (-3x^2+10x-12) / (x^2-4)^2.

Answer: (a) dy/dx = (-3x^2 + 10x - 12) / (x^2 - 4)^2

(b) x and y are related by y = 2x^2 - 3x. Find approximate change as x increases from 6 to 6.02.

dy/dx = 4x-3. At x=6: dy/dx = 24-3 = 21.

delta_x = 0.02. Approximate change: delta_y ≈ 21 * 0.02 = 0.42.

Answer: (b) Approximate increase in y ≈ 0.42

Question 9. Find all angles between 0° and 360° satisfying |3sin(theta) - 1| = 2.

Case 1: 3sin(theta)-1 = 2 => sin(theta) = 1 => theta = 90°.

Case 2: 3sin(theta)-1 = -2 => sin(theta) = -1/3.

theta = 180° + arcsin(1/3) ≈ 180° + 19.5° = 199.5°.

Or theta = 360° - arcsin(1/3) ≈ 360° - 19.5° = 340.5°.

Answer: theta = 90°, approx 199.5°, approx 340.5°

Question 10. Vectors.

(a) A and B have position vectors 5i+j and 4i-j. Find position vector of P dividing AB in ratio 3:4.

For AP:PB = 3:4 internally: OP = (4*OA + 3*OB) / (3+4) = (4*OA + 3*OB) / 7.

= (4(5i+j) + 3(4i-j)) / 7 = (20i+4j+12i-3j) / 7 = (32i+j) / 7.

Answer: (a) Position vector of P = (32i + j) / 7

(b) P and Q have position vectors 3i+2j and ni+j. Angle between OP and OQ is 45°. Find n.

cos(45°) = OP.OQ / (|OP|*|OQ|) = (3n+2) / (sqrt(13)*sqrt(n^2+1)) = 1/sqrt(2).

Square both sides: 2(3n+2)^2 = 13(n^2+1).

2(9n^2+12n+4) = 13n^2+13 => 18n^2+24n+8 = 13n^2+13 => 5n^2+24n-5 = 0.

(5n-1)(n+5) = 0 => n=1/5 or n=-5.

Check: n=1/5 gives cos(angle)=1/sqrt(2) => 45°. n=-5 gives angle=135° (rejected).

Answer: (b) n = 1/5

Question 11. Integration and area.

(a) Evaluate the integral from 1 to 4 of (3x - 6/x^2) dx.

= [3x^2/2 + 6/x] from 1 to 4.

At x=4: 3(16)/2 + 6/4 = 24 + 3/2 = 51/2.

At x=1: 3/2 + 6 = 15/2.

= 51/2 - 15/2 = 36/2 = 18.

Answer: (a) Integral = 18

(b) Curve y=3+2x-x^2 meets a straight line at B and R.

(b)(i) Curve meets x-axis (y=0): 3+2x-x^2=0 => x^2-2x-3=0 => (x-3)(x+1)=0.

x=-1 gives B; x=3 gives R. At B: y=0.

Answer: (b)(i) B = (-1, 0)

(b)(ii) The line BR is the x-axis.

Answer: (b)(ii) Equation of BR: y = 0

(b)(iii) Area of shaded region = integral from -1 to 3 of (3+2x-x^2) dx.

= [3x + x^2 - x^3/3] from -1 to 3.

At x=3: 9+9-9 = 9.

At x=-1: -3+1+1/3 = -2+1/3 = -5/3.

Area = 9-(-5/3) = 9+5/3 = 27/3+5/3 = 32/3.

Answer: (b)(iii) Area = 32/3 square units

Question 12. Trapezium ABCD with A(4,1), C(4,5). B lies on y-axis, angle DAB = 90°.

(a) Find equation of line AB.

Note: A(4,1) and C(4,5) share x=4. For trapezium with AB || DC:

B is on y-axis. Try B=(0,5): slope AB = (5-1)/(0-4) = -1.

Line AB through A(4,1) slope -1: y-1=-(x-4) => x+y=5.

Answer: (a) Equation of AB: x + y = 5 (or y = -x + 5)

(b) Find equation of line AD.

Angle DAB=90°: slope of AD = negative reciprocal of slope AB = -1/(-1) = 1.

Line AD through A(4,1) slope 1: y-1=x-4 => y = x-3.

Answer: (b) Equation of AD: y = x - 3

(c) Find coordinates of D.

DC is parallel to AB (slope -1) through C(4,5): y-5=-(x-4) => y=-x+9.

D = intersection of AD (y=x-3) and DC (y=-x+9):

x-3=-x+9 => 2x=12 => x=6. y=6-3=3.

Verify: slope CD = (3-5)/(6-4) = -1 = slope AB. ✓ AB || DC. ✓

Answer: (c) D = (6, 3)

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