G12 ECZ ADDITIONAL MATHEMATICS AUG 2017 GCE PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics August 2017 GCE Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

Additional Mathematics 4030/1

Paper 1

Tuesday **15 AUGUST 2017**

Additional Materials:

Answer Booklet
Silent electronic calculator (non programmable)

Time: 2 hours

Instructions to Candidates

Write your **name, centre number** and **candidate number** in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the **Answer Booklet provided.**

If you use more than one Answer Booklet, **fasten** the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the equation of a line parallel to the line $3x - 4y = 12$ passing through the point $(-2, 5)$. [4]

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2 Solve the simultaneous equations

$x - 2y = 1,$

$y^2 - 3xy + 8 = 0.$ [5]

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3 Given that the line $y - 2x = m$ is a tangent to the curve $x^2 + xy = -12$, find the possible values of m. [4]

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4 Functions f and g are defined by $f : x \longrightarrow \dfrac{3}{2x+1}, x \neq -\dfrac{1}{2}$

and $g : x \longrightarrow 2x + 3.$

Find

(a) the value of t, given that $fg(t) = \dfrac{2}{5}$, [3]

(b) the values of x for which $g(x) = f(x)$. [3]

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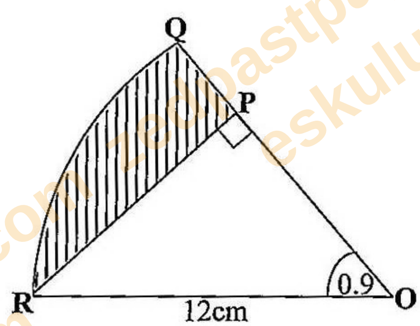
5 In the diagram below, ORQ is a sector of a circle with centre O and radius 12cm. RP is perpendicular to OQ at P and angle POR = 0.9 radians.



Find

(a) the perimeter of the shaded region, [3]

(b) the area of the shaded region. [2]

[Turn over

6 Prove the identity $\frac{1-\cos^2\theta}{\sec^2\theta-1} \equiv 1-\sin^2\theta$. [4]

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7 (a) Given that $540x^6$ is the middle term in the expansion of $(a+x^n)^6$ where a is a real number, find the value of a and of n. [4]

(b) Find the term in x^3 in the expansion of $(1+x+x^2)(1-x)^8$. [5]

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8 (a) A curve has the equation $y=\frac{-8}{2x-1}$. Find the value of x, for which $\frac{dy}{dx}=1, x>0$. [4]

(b) Find the equation of the normal to the curve $y=\frac{2x+4}{x-2}$ at the point where $x=4$. [5]

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9 Find all the angles between 0° and 360° which satisfy the equation $|\sin x - 0.6| = 0.3$. [5]

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10 (a) The coordinates of the points A and B are $(-1, 4)$ and $(2, 3)$ respectively. Find

(i) the magnitude of $\overrightarrow{AB}$, [3]

(ii) a vector equation of the line AB. [1]

(b) Find the angle between the vectors $3\mathbf{i}+4\mathbf{j}$ and $5\mathbf{i}+12\mathbf{j}$. [5]

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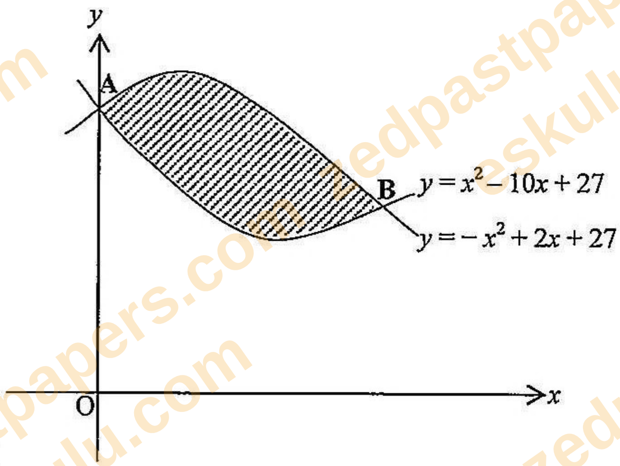
11 **(a)** Find the value of n if $\int_0^3 (x^2 + nx)\,dx = 27$. [3]

(b) The diagram below shows part of the curve $y = x^2 - 10x + 27$ intersecting the curve $y = -x^2 + 2x + 27$ at A and B.



Find

(i) the x-coordinate of point B, [3]

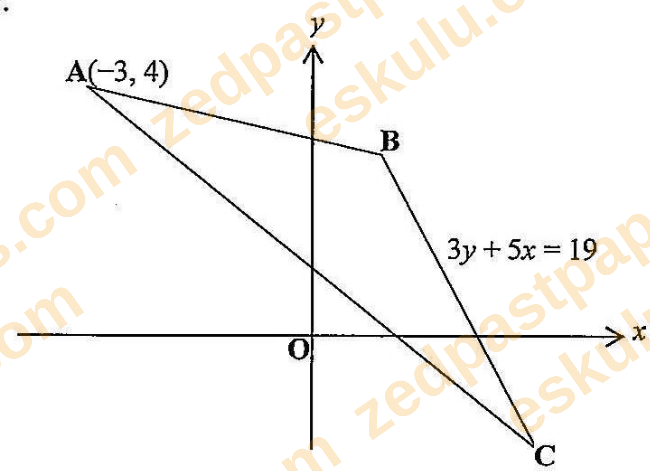
(ii) the area of the shaded region. [4]

Answer only one of the following alternatives.

12 **EITHER**

Solutions to this question by scale drawing will not be accepted.

In the diagram below, ABC is a triangle. The coordinates of A are $(-3, 4)$ and the equation of BC is $3y + 5x = 19$. The gradients of AB and AC are $-\frac{1}{5}$ and $-\frac{3}{4}$ respectively.



Find

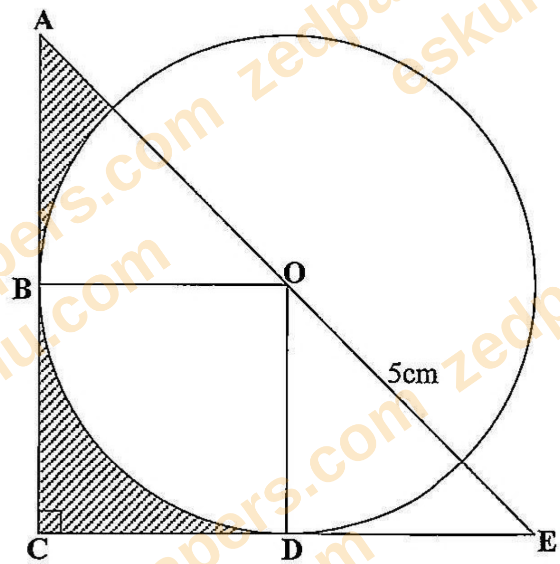
(a) the coordinates of B, [4]

(b) the magnitude of AC. [6]

[Turn over

OR

The diagram shows a circle, centre O, of radius 5cm. ACE is an isosceles triangle such that the adjacent sides AC and CE are tangents to the circle at B and D respectively. Angle ACE is $\frac{\pi}{2}$ radians.



Find

(a) the perimeter of the shaded region, [5]

(b) the area of the shaded region. [5]

Question 1. Find the equation of the line parallel to $3x - 4y + 12 = 0$ passing through $(-2, 5)$.

Parallel lines have the same slope. From $3x-4y+12=0$: slope = $3/4$.

Line through $(-2, 5)$ with slope $3/4$: $y - 5 = (3/4)(x + 2) \Rightarrow 4y - 20 = 3x + 6$.

$\Rightarrow 3x - 4y + 26 = 0$.

Answer: Equation: $3x - 4y + 26 = 0$

Question 2. Solve the simultaneous equations: $x - 2y = 1$ and $y^2 - 3xy + 8 = 0$.

From $x - 2y = 1$: $x = 2y + 1$. Substitute into $y^2 - 3xy + 8 = 0$.

$y^2 - 3(2y+1)y + 8 = 0 \Rightarrow y^2 - 6y^2 - 3y + 8 = 0 \Rightarrow -5y^2 - 3y + 8 = 0$.

$5y^2 + 3y - 8 = 0 \Rightarrow (5y + 8)(y - 1) = 0 \Rightarrow y = 1$ or $y = -8/5$.

$y = 1$: $x = 3$. $y = -8/5$: $x = 2(-8/5)+1 = -16/5+5/5 = -11/5$.

Answer: Solutions: $(x, y) = (3, 1)$ and $(x, y) = (-11/5, -8/5)$

Question 3. Given that $y - 2x = m$ is a tangent to the curve $x^2 + xy = -12$, find possible values of m.

From line: $y = 2x + m$. Substitute into $x^2 + xy = -12$.

$x^2 + x(2x+m) = -12 \Rightarrow x^2 + 2x^2 + mx + 12 = 0 \Rightarrow 3x^2 + mx + 12 = 0$.

For tangency: discriminant = 0. Delta = $m^2 - 4(3)(12) = m^2 - 144 = 0$.

$m^2 = 144 \Rightarrow m = 12$ or $m = -12$.

Answer: $m = 12$ or $m = -12$

Question 4. Functions f and g defined by $f: x \to 3/(2x+1)$, $x \neq -1/2$, and $g: x \to 2x + 3$.

(a) Find t given $fg(t) = 7/3$.

$fg(t) = f(g(t)) = f(2t+3) = 3 / (2(2t+3)+1) = 3 / (4t+7)$.

Set equal to $7/3$: $3/(4t+7) = 7/3 \Rightarrow 9 = 7(4t+7) = 28t+49 \Rightarrow 28t = -40$.

Answer: (a) $t = -10/7$

(b) Find values of x for which $g(x) = f(x)$.

$2x + 3 = 3/(2x+1) \Rightarrow (2x+3)(2x+1) = 3 \Rightarrow 4x^2 + 8x + 3 = 3 \Rightarrow 4x^2 + 8x = 0$.

$4x(x + 2) = 0 \Rightarrow x = 0$ or $x = -2$.

Answer: (b) $x = 0$ or $x = -2$

Question 5. Sector ORQ, centre O, radius 12cm. RP perpendicular to OQ at P. Angle POR = 0.9 rad.

Since P is on OQ: angle POR = angle ROQ = 0.9 rad (the sector angle).

In right triangle OPR (right angle at P): OP = OR*cos(0.9) = 12cos(0.9).

RP = OR*sin(0.9) = 12sin(0.9). PQ = OQ - OP = 12 - 12cos(0.9) = 12(1-cos0.9).

Arc QR (sector arc) = 12 * 0.9 = 10.8cm.

(a) Shaded region bounded by RP, PQ, and arc QR:

Perimeter = RP + PQ + arc QR = 12sin(0.9) + 12(1-cos0.9) + 10.8.

= 12(0.7833) + 12(1-0.6216) + 10.8 ≈ 9.40 + 4.54 + 10.8 ≈ 24.7cm.

Answer: (a) Perimeter of shaded region ≈ 24.7 cm

(b) Area of shaded region = sector area - triangle OPR.

Sector OQR area = (1/2)(12^2)(0.9) = 64.8cm^2.

Triangle OPR area = (1/2)*OP*RP = (1/2)(12cos0.9)(12sin0.9) = 72sin0.9*cos0.9 = 36sin(1.8).

≈ 36 * 0.9738 ≈ 35.1cm^2. Shaded = 64.8 - 35.1 ≈ 29.7cm^2.

Answer: (b) Area of shaded region ≈ 29.7 cm^2

Question 6. Prove the identity: (1 - cos^2(theta)) / (sec^2(theta) - 1) = 1 - sin^2(theta).

Note: 1 - cos^2(theta) = sin^2(theta).

Note: sec^2(theta) - 1 = tan^2(theta) = sin^2(theta)/cos^2(theta).

LHS = sin^2(theta) / (sin^2(theta)/cos^2(theta)) = sin^2(theta) * cos^2(theta)/sin^2(theta) = cos^2(theta).

cos^2(theta) = 1 - sin^2(theta) = RHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Binomial expansions.

(a) 540x^6 is the middle term in the expansion of (a + x^n)^6. Find a and n.

The 6-term expansion has middle term T_4 (r=3): T_4 = C(6,3)*a^3*(x^n)^3 = 20*a^3*x^(3n).

Match with 540x^6: 20a^3 = 540 => a^3 = 27 => a = 3.

And 3n = 6 => n = 2.

Answer: a = 3 and n = 2

(b) Find the term in x^3 in the expansion of (1 + x + x^2)(1 - x)^8.

(1-x)^8: coeff of x^1 = -8; coeff of x^2 = C(8,2) = 28; coeff of x^3 = -C(8,3) = -56.

Coeff of x^3 in (1+x+x^2)(1-x)^8:

= 1*(-56) + 1*(28) + 1*(-8) = -56 + 28 - 8 = -36.

Answer: Term in $x^3 = -36x^3$

Question 8. Differentiation.

(a) Curve $y = -8/(2x-1)$. Find x where $dy/dx = 1$, $x > 0$.

$y = -8(2x-1)^{-1}$. $dy/dx = -8 \cdot (-1)(2x-1)^{-2} \cdot 2 = 16/(2x-1)^2$.

Set $= 1$: $(2x-1)^2 = 16 \Rightarrow 2x-1 = \pm 4$.

$2x-1 = 4 \Rightarrow x = 5/2$ (valid, $x>0$). Or $2x-1=-4 \Rightarrow x=-3/2$ (rejected).

Answer: (a) $x = 5/2$

(b) Find the equation of the normal to $y = (2x+4)/(x-2)$ at $x = 4$.

At $x=4$: $y = (8+4)/2 = 6$. Point is $(4, 6)$.

Quotient rule: $dy/dx = [2(x-2)-(2x+4) \cdot 1]/(x-2)^2 = (2x-4-2x-4)/(x-2)^2 = -8/(x-2)^2$.

At $x=4$: $dy/dx = -8/4 = -2$. Normal slope $= 1/2$ (negative reciprocal).

Normal through $(4,6)$: $y-6 = (1/2)(x-4) \Rightarrow 2y-12 = x-4 \Rightarrow x-2y+8 = 0$.

Answer: (b) Normal equation: $x - 2y + 8 = 0$

Question 9. Find all angles between 0° and 360° satisfying $|\sin(x) - 0.6| = 0.3$.

Case 1: $\sin(x) - 0.6 = 0.3 \Rightarrow \sin(x) = 0.9$.

$x = \arcsin(0.9) \approx 64.2°$ or $x = 180° - 64.2° = 115.8°$.

Case 2: $\sin(x) - 0.6 = -0.3 \Rightarrow \sin(x) = 0.3$.

$x = \arcsin(0.3) \approx 17.5°$ or $x = 180° - 17.5° = 162.5°$.

Answer: $x \approx 17.5°, 64.2°, 115.8°, 162.5°$

Question 10. Vectors. A = (-1, 4), B = (2, 3).

(a)(i) Magnitude of vector AB.

$AB = B - A = (2-(-1), 3-4) = (3, -1)$. $|AB| = \sqrt{9+1} = \sqrt{10}$.

Answer: (a)(i) $|AB| = \sqrt{10}$

(a)(ii) Vector equation of line AB.

Direction vector $= (3,-1)$. A point on line: $A = (-1,4)$.

Answer: (a)(ii) $r = (-1, 4) + t(3, -1)$

(b) Angle between vectors $u = 3i + 4j$ and $v = 5i + 12j$.

$u \cdot v = 3 \cdot 5 + 4 \cdot 12 = 15 + 48 = 63$. $|u| = 5$. $|v| = 13$.

$\cos(\theta) = 63/(5 \cdot 13) = 63/65$. $\theta = \arccos(63/65) \approx 14.3°$.

Answer: (b) Angle $\approx 14.3°$ (or $\arccos(63/65)$)

Question 11. Integration and area.

(a) Find n given integral from 0 to n of (x^2 + 4x) dx = 27.

Integrate: [x^3/3 + 2x^2] from 0 to n = n^3/3 + 2n^2 = 27.

Try n = 3: 27/3 + 2(9) = 9 + 18 = 27. ✓

Answer: (a) n = 3

(b) Curves y = x^2-10x+27 and y = -x^2+2x+27 intersect at A and B.

(b)(i) Find x-coordinate of B: set equal: x^2-10x+27 = -x^2+2x+27.

=> 2x^2 - 12x = 0 => 2x(x-6) = 0 => x=0 (point A) or x=6 (point B).

Answer: (b)(i) x-coordinate of B = 6

(b)(ii) Area of shaded region (between curves, x=0 to x=6):

Area = integral_0^6 [(-x^2+2x+27)-(x^2-10x+27)] dx = integral_0^6 (-2x^2+12x) dx.

= [-2x^3/3 + 6x^2]_0^6 = -2(216)/3 + 6(36) = -144 + 216 = 72.

Answer: (b)(ii) Area of shaded region = 72 square units

Question 12. EITHER/OR — Answer ONE only.

=== EITHER: Triangle ABC with A(-3, 4), BC: 5x+3y=19. Gradient AB=-1/5, gradient AC=-3/2. ===

(a) Find coordinates of B:

Line AB through A(-3,4) with slope -1/5: y-4 = (-1/5)(x+3) => 5y-20 = -x-3 => x+5y = 17.

Intersect with BC (5x+3y=19): from x+5y=17, x=17-5y.

5(17-5y)+3y=19 => 85-25y+3y=19 => 85-22y=19 => 22y=66 => y=3. x=17-15=2.

Answer: (a) B = (2, 3)

(b) Find magnitude of AC:

Line AC through A(-3,4) with slope -3/2: y-4=(-3/2)(x+3) => 3x+2y=-1.

Intersect with BC (5x+3y=19): multiply 3x+2y=-1 by 3: 9x+6y=-3.

Multiply 5x+3y=19 by 2: 10x+6y=38. Subtract: x=41. Then 2y=-1-123=-124 => y=-62.

C = (41, -62). |AC| = sqrt((41+3)^2+(-62-4)^2) = sqrt(44^2+66^2) = sqrt(1936+4356).

= sqrt(6292) = sqrt(4*121*13) = 2*11*sqrt(13) = 22*sqrt(13).

Answer: (b) |AC| = 22*sqrt(13) ≈ 79.3 units

=== OR: Circle centre O, radius 5cm. Triangle ACE isosceles, AC and CE tangent at B and D. Angle ACE = pi/2. ===

Since angle ACE = pi/2, OC bisects the right angle: angle OCB = pi/4.

In right triangle OCB (right angle at B): CB = OB/tan(pi/4) = 5/1 = 5cm.

OC = OB/sin(pi/4) = 5*sqrt(2) cm. Angle BOC = pi/2 - pi/4 = pi/4 (each side).

Angle BOD = angle BOC + angle COD = pi/4 + pi/4 = pi/2.

Shaded region bounded by arc BD (near C), BC and CD:

(a) Perimeter = BC + arc BD + DC.

Arc BD = r * angle BOD = 5 * pi/2 = 5pi/2 cm.

Perimeter = 5 + 5pi/2 + 5 = 10 + 5pi/2 ≈ 10 + 7.85 ≈ 17.85cm.

Answer: (a) Perimeter of shaded region = 10 + 5pi/2 ≈ 17.85 cm

(b) Area of shaded region = area of kite OBDC - area of sector OBD.

Kite OBDC = triangle OBC + triangle OCD = (1/2)(5)(5) + (1/2)(5)(5) = 12.5+12.5 = 25cm^2.

Sector OBD area = (1/2)(5^2)(pi/2) = 25pi/4 cm^2.

Shaded area = 25 - 25pi/4 = 25(1-pi/4) ≈ 25(0.215) ≈ 5.37cm^2.

Answer: (b) Area of shaded region = 25 - 25pi/4 ≈ 5.37 cm^2

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