G12 ECZ ADDITIONAL MATHEMATICS 2016 SPECIMEN PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2016 Specimen Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

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4030/1/2

EXAMINATIONS COUNCIL OF ZAMBIA

EXAMINATIONS FOR SCHOOL CERTIFICATE AND
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

Additional Mathematics

SPECIMEN PAPERS FOR EXAMINATIONS STARTING FROM 2016 FOR INTERNAL CANDIDATES AND FROM 2017 FOR EXTERNAL CANDIDATES

Additional Mathematics (4030/1/2) – Preamble

The **Additional Mathematics (4030)** examination is part of the Senior Secondary – School Certificate Examination taken at the end of Grade 12. The School Certificate Examination is designed based on the three-year course, covering Grades 10, 11 and 12. The Ministry of General Education revised the senior secondary school curriculum (CDC 2013) and its implementation begun in 2014. For the purposes of the examination the Additional Mathematics teaching syllabus was analysed and an examinations syllabus was developed with accompanying specimen papers.

The purpose of the Additional Mathematics assessment is to measure learner achievements against the set competencies as outlined in the Grade 10–12 Syllabus. The examination will also serve the purpose of certification and progression to tertiary education.

Assessment Objectives

Candidates will be tested against the following objectives:

AO1 Knowledge and Comprehension

AO2 Application

AO3 Analysis

AO4 Synthesis and Evaluation

Test Design

The examination will consist of two theory papers as indicated in the table below.

Paper Name and Code	Paper Code	Duration	Number of questions	Marks	Weighting
Additional Mathematics Paper 1	(4030/1)	2 hours	12	80	45%
Additional Mathematics Paper 2	(4030/2)	2 hours 30 minutes	12	100	55%
			Total		100%

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/1

Paper 1

date OCTOBER 2016

Additional Materials:

Answer Booklet,
Electronic calculators

Time: 2 hours

Instructions to Candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for Candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A line passing through the point A(0, 1) is perpendicular to the line joining the points B(0, 6) and C(3, 0) at D. Find the coordinates of D. [4]

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2 Solve the simultaneous equations

$y - x = 3,$
$xy = 10.$ [5]

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3 The straight line $y = 2x + k$ is a tangent to the curve $y = 3x - x^2$. Find the value of k. [4]

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4 The function f is defined by $f(x) = \frac{bx - 1}{ax}$, $x \neq 0$, where a and b are real numbers.

If $f(2) = \frac{5}{2}$ and $f^{-1}(0) = -\frac{1}{3}$, find the value of a and of b. [6]

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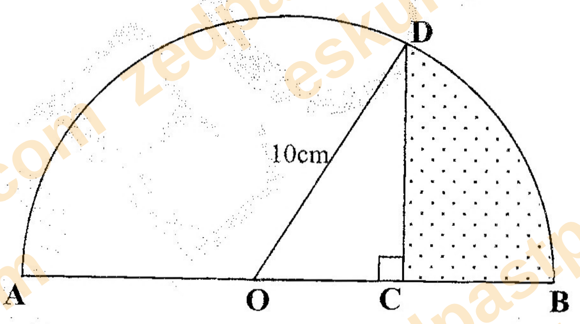
5 In the diagram below, ADB is a semi-circle with centre O and radius 10cm. DC is perpendicular to AB, where C is the midpoint of OB. The length of arc BD is $\frac{10\pi}{3}$ cm.



(a) Find angle DOC, in radians. [1]

(b) Find the area of the shaded part. [4]

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6 Prove the identity $\frac{\cos x}{\sec x - \tan x} \equiv 1 + \sin x$. [4]

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7 **(a)** The first three terms in the expansion of $(1 + px)^n$ in ascending powers of x are $1 + 14x + 84x^2$. Find the value of n and of p. [4]

(b) Find the coefficient of x^2 in the expansion of $(2 - x)^6 (1 + x)^3$. [5]

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8 **(a)** Find the derivative of $y = \dfrac{2x^2}{2x+1}$. [3]

(b) Find the equation of the normal to the curve $y = 2x^2 - x^3$ at $(-1, 3)$. [6]

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9 Find the derivative of $y = \cos(3x - 1)$. Hence, find the gradient of y at a point where $x = 0.5$ radians. [5]

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10 **(a)** The position vectors of A and B are $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ respectively.

Find

(i) a unit vector in the direction of vector AB, [2]

(ii) a vector equation through A and B. [2]

(b) The position vectors of A and B are $2\mathbf{i} + 3\mathbf{j}$ and $3\mathbf{i} + \mathbf{j}$ respectively.
Find the size of angle OAB in degrees. [5]

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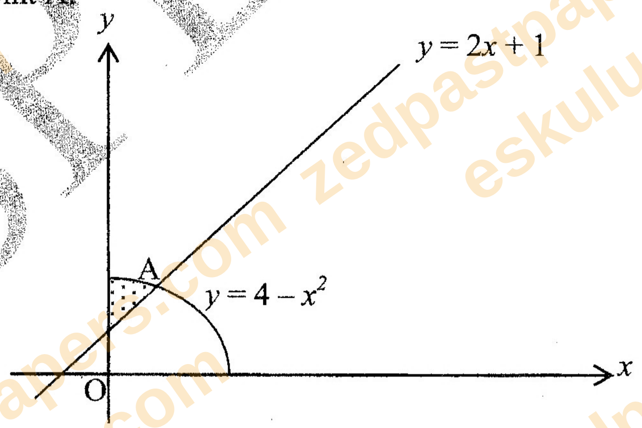
11 **(a)** Find $\int \left(\sqrt{3x+2}\right) dx$. [3]

(b) The diagram below shows part of the curve $y = 4 - x^2$ which meets the line $y = 2x + 1$ at point A.



Calculate

(i) the x-coordinate of the point A, [3]

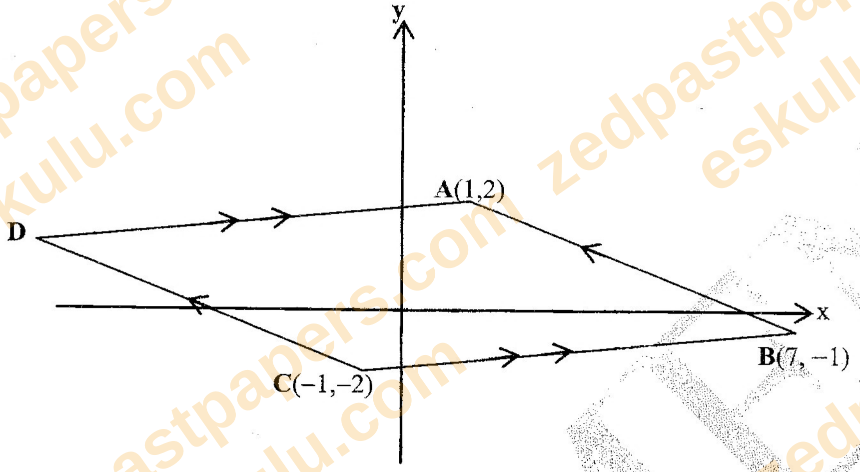
(ii) the area of the shaded region. [4]

Answer only one of the following alternatives.

12 Either

Solutions by scale drawing will not be accepted.

In the diagram below, **ABCD** is a parallelogram with **A**(1, 2), **B**(7, −1) and **C**(−1, −2).



Find

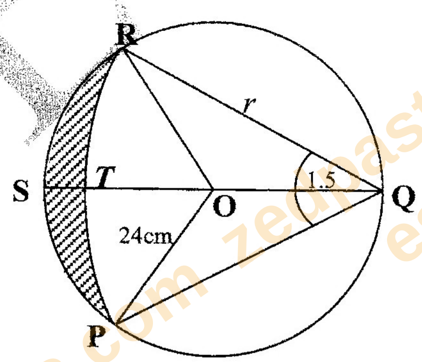
(a) the equation of **AD** and **CD**, [6]

(b) the coordinates of **D**, [2]

(c) the area of the parallelogram. [2]

Or

In the diagram below **PSRQ** is a circle with centre **O** and radius 24cm. **RTP** is an arc of a circle centre **Q**, radius r and **RQP** = 1.5 radians.



Find

(a) the radius r, [2]

(b) the perimeter of the shaded region, [3]

(c) the area of the shaded region. [5]

Question 1. A line through A(0, 1) perpendicular to BC meets BC at D, where B and C are given points. Find the coordinates of D.

Line BC: slope = (0 - 6)/(3 - 0) = -2 [using B(0, 6) and C(3, 0)].

Perpendicular slope = 1/2 (negative reciprocal).

Line through A(0, 1) with slope 1/2: y = (1/2)x + 1.

Line BC through B(0, 6): y = -2x + 6.

Intersection: (1/2)x + 1 = -2x + 6 => (5/2)x = 5 => x = 2. Then y = 1/2(2) + 1 = 2.

Answer: D = (2, 2)

Question 2. Solve the simultaneous equations: y - x = 3 and xy = 10.

From y - x = 3: y = x + 3. Substitute into xy = 10: x(x + 3) = 10.

x^2 + 3x - 10 = 0 => (x + 5)(x - 2) = 0 => x = -5 or x = 2.

x = -5: y = -5 + 3 = -2. x = 2: y = 2 + 3 = 5.

Answer: Solutions: (x, y) = (-5, -2) and (x, y) = (2, 5)

Question 3. The line y = 2x + k is a tangent to the curve y = 3x - x^2. Find the value of k.

At the tangent point, the line and curve meet with equal values and equal gradients.

Set equal: 2x + k = 3x - x^2 => x^2 - x + k = 0.

For a tangent (exactly one intersection point): discriminant = 0.

Delta = (-1)^2 - 4(1)(k) = 1 - 4k = 0 => k = 1/4.

Answer: k = 1/4

Question 4. A function f is defined by f(x) = (bx - 1)/(ax), x != 0, where a and b are real numbers. Given f(2) = 1/2 and f'(-1) = 1/3, find a and b.

Rewrite: f(x) = b/a - 1/(ax). Differentiate: f'(x) = 1/(ax^2).

f'(-1) = 1/(a(-1)^2) = 1/a = 1/3 => a = 3.

f(2) = (2b - 1)/(2 * 3) = (2b - 1)/6 = 1/2 => 2b - 1 = 3 => b = 2.

Verify: f(x) = (2x-1)/(3x). f(2) = 3/6 = 1/2 ✓. f'(x) = 1/(3x^2). f'(-1) = 1/3 ✓

Answer: a = 3, b = 2

Question 5. ADB is a semicircle with centre O and radius 10cm. DC is perpendicular to AB, where C is the midpoint of OB. Arc BD = 10*pi/3 cm.

Set O at origin, A=(-10,0), B=(10,0). C = midpoint of OB = (5, 0).

Angle BOD = arc BD / radius = (10*pi/3)/10 = pi/3 rad.

D = (10 cos(pi/3), 10 sin(pi/3)) = (5, 5*sqrt(3)). DC goes from (5, 5*sqrt(3)) to (5,0) = vertical. ✓ Perpendicular to AB.

(a) Angle DOC: OD = (5, 5*sqrt(3)), OC = (5, 0).

cos(DOC) = OD.OC / (|OD||OC|) = 25 / (10 * 5) = 1/2. Angle DOC = pi/3.

Answer: (a) Angle DOC = pi/3 radians

(b) Shaded region bounded by arc BD, segment DC, and segment CB.

Using Green's theorem (or sector minus triangle):

Arc contribution (B to D, angle pi/3, radius 10): (1/2)*r^2*theta = (1/2)(100)(pi/3) = 50*pi/3.

Segment D to C: contributes -(1/2)*|OC|*|DC| = -(1/2)(5)(5*sqrt(3)) = -25*sqrt(3)/2.

Segment C to B: contributes 0 (both on x-axis).

Answer: (b) Area of shaded region = 50*pi/3 - 25*sqrt(3)/2 cm^2 (approx 30.7 cm^2)

Question 6. Prove the identity: cos(z) / (sec(z) - tan(z)) = 1 + sin(z).

LHS: cos(z) / (sec(z) - tan(z)).

Write sec = 1/cos and tan = sin/cos: denom = (1 - sin(z))/cos(z).

LHS = cos(z) / [(1 - sin(z))/cos(z)] = cos^2(z) / (1 - sin(z)).

Factor: cos^2(z) = 1 - sin^2(z) = (1-sin(z))(1+sin(z)).

LHS = (1-sin(z))(1+sin(z)) / (1 - sin(z)) = 1 + sin(z) = RHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Binomial expansions of (1 + px)^n.

(a) The first three terms are 1 + 14x + 84x^2. Find n and p.

C(n,1)*p = np = 14 ...(i)

C(n,2)*p^2 = n(n-1)p^2/2 = 84 ...(ii)

From (i): p = 14/n. Substitute into (ii): n(n-1)(196/n^2)/2 = 84 => 196(n-1)/2n = 84.

98(n-1)/n = 84 => 98n - 98 = 84n => 14n = 98 => n = 7.

p = 14/7 = 2. Check: (1+2x)^7: term 1 = 1, term 2 = 14x, term 3 = 21*4x^2 = 84x^2 ✓.

Answer: n = 7, p = 2

(b) Coefficient of x^5 in (2 - x^2)(1 + 2x)^7.

From (1+2x)^7: coeff of x^5 = C(7,5)*2^5 = 21*32 = 672.

From (1+2x)^7: coeff of x^3 = C(7,3)*2^3 = 35*8 = 280.

Coeff of x^5 in product = 2*(coeff x^5) + (-1)*(coeff x^3) = 2*672 - 280 = 1344 - 280.

Answer: Coefficient of x^5 = 1064

Question 8. Differentiation.

(a) Find dy/dx for $y = 2x^3 / (3x^2 - 1)$.

Using quotient rule: $u = 2x^3$ ($u' = 6x^2$), $v = 3x^2-1$ ($v' = 6x$).

$dy/dx = (u'v - uv')/v^2 = (6x^2(3x^2-1) - 2x^3 \cdot 6x) / (3x^2-1)^2$.

$= (18x^4 - 6x^2 - 12x^4) / (3x^2-1)^2 = (6x^4 - 6x^2) / (3x^2-1)^2$.

Answer: $dy/dx = 6x^2(x^2 - 1) / (3x^2 - 1)^2$

(b) Find the equation of the normal to the curve $y = x^3 - 2x^2$ at $(-1, -3)$.

Check: $y(-1) = -1 - 2 = -3$ ✓.

$dy/dx = 3x^2 - 4x$. At $x = -1$: gradient $= 3(1) - 4(-1) = 3 + 4 = 7$.

Normal gradient $= -1/7$ (negative reciprocal).

Normal through $(-1, -3)$: $y + 3 = (-1/7)(x + 1) \Rightarrow 7y + 21 = -x - 1$.

Answer: Normal: $x + 7y + 22 = 0$

Question 9. Find the derivative of y = cos(3x - 1). Hence find the gradient of y at x = 0.5 radians.

Using chain rule: $dy/dx = -\sin(3x-1) \cdot 3 = -3\sin(3x-1)$.

Answer: $dy/dx = -3 \sin(3x - 1)$

At $x = 0.5$: gradient $= -3\sin(3(0.5) - 1) = -3\sin(1.5 - 1) = -3\sin(0.5)$.

$\sin(0.5) \approx 0.4794$.

Answer: Gradient at $x = 0.5$ rad $= -3\sin(0.5) \approx -1.44$

Question 10. Vectors. OA = (3, 4), OB = (-1, 2).

(a)(i) Unit vector in direction of AB:

$AB = OB - OA = (-4, -2)$. $|AB| = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$.

Answer: Unit vector of AB $= (-4/(2\sqrt{5}), -2/(2\sqrt{5})) = (-2/\sqrt{5}, -1/\sqrt{5})$ [$= (-2\sqrt{5}/5, -\sqrt{5}/5)$]

(a)(ii) Vector equation of line through A and B:

Answer: $r = (3, 4) + t(-4, -2)$ [or equivalently $r = (3, 4) + t(-2, -1)$]

(b) Angle OAB: vectors AO = OA - OA... wait: $AO = O - A = (-3, -4)$. $AB = (-4, -2)$.

$\cos(OAB) = AO \cdot AB / (|AO||AB|) = ((-3)(-4) + (-4)(-2)) / (5 \cdot 2\sqrt{5})$.

$= (12 + 8) / (10\sqrt{5}) = 20 / (10\sqrt{5}) = 2/\sqrt{5}$.

Angle OAB $= \arccos(2/\sqrt{5}) \approx 26.6$ degrees.

Answer: Angle OAB $= \arccos(2/\sqrt{5}) \approx 26.6$ degrees

Question 11. Integration and area.

(a) Find the integral of (3*sqrt(x) + 2x) dx.

= integral of (3*x^(1/2) + 2x) dx = 3*(2/3)*x^(3/2) + x^2 + C = 2x^(3/2) + x^2 + C.

Answer: Integral = 2x^(3/2) + x^2 + C [= 2*sqrt(x^3) + x^2 + C]

(b) Curve y = 4 - x^2 meets line y = 2x + 1 at A. Find x-coordinate of A and the shaded area.

(b)(i) Set equal: 4 - x^2 = 2x + 1 => x^2 + 2x - 3 = 0 => (x+3)(x-1) = 0.

x = -3 or x = 1. From the diagram, A is the positive intersection.

Answer: x-coordinate of A = 1 (A = (1, 3))

(b)(ii) Shaded area between curve (above) and line (below) from x = -3 to x = 1:

Area = integral from -3 to 1 of [(4-x^2) - (2x+1)] dx = integral of (3 - 2x - x^2) dx.

= [3x - x^2 - x^3/3] from -3 to 1.

At x=1: 3 - 1 - 1/3 = 5/3.

At x=-3: -9 - 9 - (-27/3) = -9 - 9 + 9 = -9.

Area = 5/3 - (-9) = 5/3 + 9 = 5/3 + 27/3 = 32/3.

Answer: Area of shaded region = 32/3 square units (approx 10.7 sq units)

Question 12. EITHER/OR — Answer ONE only.

=== EITHER: ABCD is a parallelogram with A(1, 2), B(7, -1) and C(-1, -2). ===

For a parallelogram ABCD: AB is parallel to DC, and AD is parallel to BC.

Slope AB = (-1-2)/(7-1) = -3/6 = -1/2. So DC also has slope -1/2.

Slope BC = (-2-(-1))/(-1-7) = (-1)/(-8) = 1/8. So AD also has slope 1/8.

(a) Equation of AD (through A(1,2), slope 1/8):

y - 2 = (1/8)(x - 1) => 8y - 16 = x - 1 => x - 8y + 15 = 0.

Answer: AD: x - 8y + 15 = 0

Equation of CD (through C(-1,-2), slope -1/2):

y + 2 = (-1/2)(x + 1) => 2y + 4 = -x - 1 => x + 2y + 5 = 0.

Answer: CD: x + 2y + 5 = 0

(b) D = intersection of AD and CD:

From AD: x = 8y - 15. Substitute into CD: (8y-15) + 2y + 5 = 0 => 10y = 10 => y = 1.

x = 8(1) - 15 = -7.

Answer: D = (-7, 1)

(c) Area of parallelogram = |AB x AD| (magnitude of cross product).

AB = (6, -3, 0), AD = (-8, -1, 0). Cross product magnitude = |6*(-1) - (-3)*(-8)| = |-6 - 24| = 30.

Answer: Area of parallelogram ABCD = 30 square units

=== OR: Circle centre O, radius 24cm. Q is a point on the circle. Arc RTP has centre Q, radius r. Angle RQP = 1.5 rad. ===

Since Q is on the big circle and angle RQP is the inscribed angle subtending chord RP,

the central angle ROP = 2 * angle RQP = 2 * 1.5 = 3 radians.

Chord RP (from big circle): RP = 2 * 24 * sin(3/2) = 48 * sin(1.5).

Chord RP (from small arc, radius r, central angle 1.5 rad): RP = 2r * sin(0.75).

Since sin(1.5) = 2*sin(0.75)*cos(0.75): 48*2*sin(0.75)*cos(0.75) = 2r*sin(0.75).

Divide both sides by 2*sin(0.75): 48*cos(0.75) = r.

Answer: (a) r = 48 * cos(0.75) ≈ 35.1 cm

(b) Perimeter of shaded region = arc RTP + arc RSP.

Arc RTP = r * 1.5 = 48cos(0.75) * 1.5 = 72cos(0.75) ≈ 52.7 cm.

Arc RSP (on big circle, angle 2*pi - 3 rad): = 24*(2*pi - 3) ≈ 24*3.283 ≈ 78.8 cm.

Answer: (b) Perimeter ≈ 72cos(0.75) + 24(2*pi - 3) ≈ 131.5 cm

(c) Area of shaded region = Sector RSP (big circle) - Sector RQP (small arc).

Sector RSP = (1/2)(24^2)(2*pi - 3) = 288(2*pi - 3) ≈ 946 cm^2.

Sector RQP = (1/2)r^2 * 1.5 = 0.75*(48cos(0.75))^2 = 0.75*2304*cos^2(0.75).

cos(0.75) ≈ 0.7317. Sector RQP ≈ 0.75*2304*0.5354 ≈ 924 cm^2.

Answer: (c) Area of shaded region ≈ 288(2*pi - 3) - 0.75*(48*cos(0.75))^2 ≈ 22 cm^2

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