G12 ECZ
ADDITIONAL MATHEMATICS
2016 GCE
PAPER 1
SOLUTIONS

Grade 12 ECZ Additional Mathematics 2016 GCE Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

Additional Mathematics 4030/1

Paper 1

Tuesday 26 JULY 2016

Additional materials:

Answer Booklet
Graph paper (1 Sheet)
Mathematical tables/Electronic calculators (non-programmable)

Time: 2 hours

Instructions to candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non-programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The straight line $y + 3 = 2x$ meets the curve $x^2 - 3x = y^2 - 21$ at **P** and **Q**.
Calculate the coordinates of **P** and **Q**. [5]

2 Find the equation of the line perpendicular to the line $2x + 4y + 3 = 0$ passing through the point $(-5, 2)$. [4]

3 Functions h and g are defined by

$h: x \to 3x - 2,$

$g: x \to \dfrac{x}{x-1},\ x \neq 1.$

Find

(a) $hg(x)$, [2]

(b) $gh(x)$, [1]

(c) the value of x for which $hg(x) = gh(x)$. [3]

4 Find the range of values of k for which the line $y = kx + 4$ does not meet the curve $2x^2 + xy = -6$. [4]

5 **(a)** Find the mid-term in the expansion of $(2 - 3x^2)^8$. [4]

(b) Find the term in x^3 in the expansion of $(1 + 2x)^3(1 - \frac{3}{2}x)^4$. [5]

6 Prove the identity

$$\frac{\text{Sin}^2\theta(1-\tan^2\theta)}{\text{Cos}^2\theta - \sin^2\theta} \equiv \tan^2\theta.$$ [4]

7 On the same diagram, sketch the graphs of $y = 1 + \sin 2x$ and $y = 2 - \dfrac{x}{\pi}$ for the domain 0 to 2π. Hence state the number of solutions to the equation

$1 + \sin 2x = 2 - \dfrac{x}{\pi}$. [5]

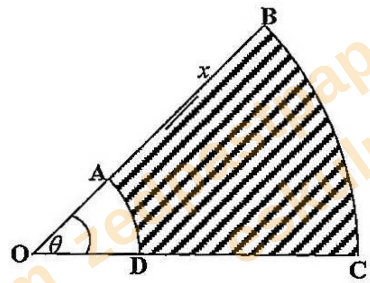
8 P and Q have position vectors $2\boldsymbol{i} - \boldsymbol{j}$ and $a\boldsymbol{i} + 4\boldsymbol{j}$ respectively. The cosine of the angle between p and q is $\dfrac{-2}{\sqrt{5}}$.

Find

(a) the value of a, [4]

(b) the unit vector in the direction of $\overrightarrow{PQ}$. [3]

9 The diagram below shows two sectors **OBC** and **OAD** with centre **O** and $\angle$**BOC** $= \theta$ radians. The radius of sector **OBC** is 3cm and **AB** $= x$cm.



(a) Show that the area of the shaded region is $\frac{1}{2}x\theta(6-x)$. [4]

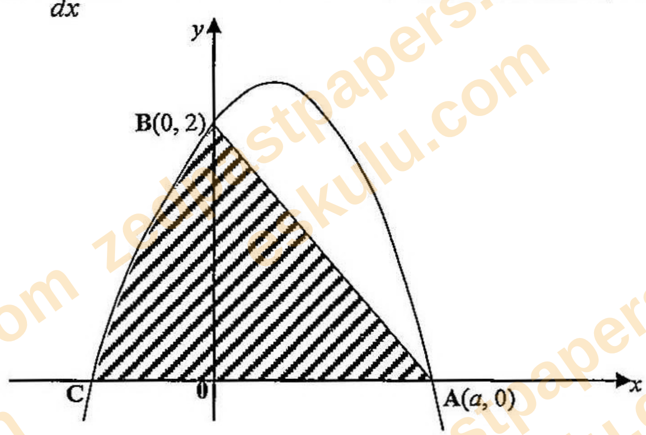
(b) Given that the area of the shaded region is 8cm^2 and $\theta = 2$ radians, find **OA**. [3]

10 (a) Find the point on the curve $y = \frac{1}{2}x^2 + \frac{8}{x}$ at which the tangent is parallel to the x-axis. [3]

(b) The radius of a cylinder is increased by 5%. Given that the height is constant, determine the percentage increase in the volume. [6]

11 (a) Find p, if $\int_p^2 (8-4x)\,dx = 18$. [3]

(b) The diagram below shows a straight line AB and a curve with a gradient function $\frac{dy}{dx} = 3 - 4x$. The curve intersects the axes at A, B and C.



Find

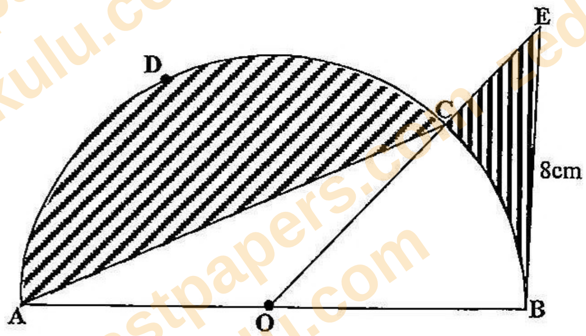
(i) the coordinates of the points **A** and **C**, [3]

(ii) the area of the shaded part. [4]

12 **Answer only one of the following alternatives:**

EITHER

In the diagram below, **ABCD** is a semi-circle with centre **O**. **AB** is the diameter and **BE** is a tangent to the circle at **B**. **BE** = 8cm and **OC**:**CE** = 3:2.



Find

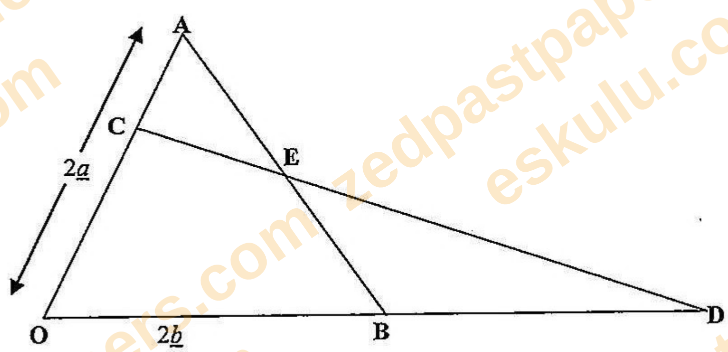
(a) $B\hat{O}E$ in radians, [2]

(b) the radius of the circle, [2]

(c) the total area of the shaded regions. [6]

OR

In the diagram below, the position vectors of **A** and **B** relative to the origin **O** are $2\underline{a}$ and $2\underline{b}$ respectively. The point **C** lies on **OA** such that **OC** = 2**CA**. The point **D** lies on **OB** produced, such that **OB** = **BD**. The lines **AB** and **CD** meet at **E**. $\overrightarrow{CE} = \lambda\overrightarrow{CD}$ and $\overrightarrow{AE} = \mu\overrightarrow{AB}$.



(a) Express $\overrightarrow{OE}$ in terms of

(i) $\underline{a}$, $\underline{b}$ and λ, [3]

(ii) $\underline{a}$, $\underline{b}$ and μ, [3]

(b) Hence or otherwise, find the value of λ and of μ. [4]

Question 1. The line $y + 3 = 2x$ meets the curve $x^2 - 3x = y^2 - 21$ at points P and Q. Find P and Q.

From the line: $y = 2x - 3$. Substitute into the curve equation.

$x^2 - 3x = (2x-3)^2 - 21 = 4x^2 - 12x + 9 - 21 = 4x^2 - 12x - 12$.

Rearrange: $0 = 3x^2 - 9x - 12 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x-4)(x+1) = 0$.

$x = 4$: $y = 2(4)-3 = 5$. $x = -1$: $y = 2(-1)-3 = -5$.

Answer: P = (-1, -5) and Q = (4, 5)

Question 2. Find the equation of the line perpendicular to $2x + 4y + 3 = 0$ passing through (-5, 2).

Rearrange $2x + 4y + 3 = 0$: $y = -x/2 - 3/4$. Slope of given line = $-1/2$.

Perpendicular slope = $+2$ (negative reciprocal).

Line through (-5, 2) with slope 2: $y - 2 = 2(x + 5) \Rightarrow y = 2x + 12$.

Answer: Equation: $y = 2x + 12$ (or equivalently $2x - y + 12 = 0$)

Question 3. Functions h and g: $h(x) = 3x - 2$, $g(x) = x/(x - 1)$, $x \neq 1$.

(a) hg(x): first apply g, then h.

$g(x) = x/(x-1)$. $h(g(x)) = 3 * [x/(x-1)] - 2 = (3x - 2(x-1))/(x-1) = (x+2)/(x-1)$.

Answer: $hg(x) = (x + 2) / (x - 1)$

(b) Inverse g^{-1}: let $y = x/(x-1)$. Then $y(x-1) = x \Rightarrow yx - y = x \Rightarrow x(y-1) = y$.

$x = y/(y-1)$. So $g^{-1}(x) = x/(x-1)$. Note: g is its own inverse (an involution).

Answer: $g^{-1}(x) = x / (x - 1)$

(c) Solve $hg(x) = h(x)$: $(x+2)/(x-1) = 3x - 2$.

$x + 2 = (3x-2)(x-1) = 3x^2 - 5x + 2$.

$3x^2 - 6x = 0 \Rightarrow 3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$.

Answer: $x = 0$ or $x = 2$

Question 4. Find the range of values of a for which $y = x + 4$ does not meet $2x^2 + xy + a = 4$.

Rearrange the curve: $2x^2 + xy = 4 - a$. Substitute $y = x + 4$:

$2x^2 + x(x+4) = 4 - a \Rightarrow 3x^2 + 4x + (a - 4) = 0$.

No real intersection: discriminant < 0.

$\Delta = 16 - 4(3)(a-4) = 16 - 12a + 48 = 64 - 12a < 0$.

$64 < 12a \Rightarrow a > 64/12 = 16/3$.

Answer: $a > 16/3$

Question 5. Binomial expansions.

(a) Find the middle (5th) term in the expansion of (2 - 3y)^8.

There are 9 terms; middle term is T_5 (r=4).

T_5 = C(8,4) * 2^4 * (-3y)^4 = 70 * 16 * 81 * y^4.

Answer: Middle term = 90720 y^4

(b) Find the term in x^3 in the expansion of (1 + 2x^2)(1 - x/2)^7.

From (1 - x/2)^7, coefficient of x^3 = C(7,3)*(-1/2)^3 = 35*(-1/8) = -35/8.

From (1 - x/2)^7, coefficient of x^1 = C(7,1)*(-1/2) = -7/2.

Contribution from 1 * x^3 term: -35/8.

Contribution from 2x^2 * x^1 term: 2 * (-7/2) = -7.

Total coefficient of x^3 = -35/8 - 7 = -35/8 - 56/8 = -91/8.

Answer: Term in x^3 = (-91/8) x^3

Question 6. Prove the identity: tan^2(A) - sin^2(A) = tan^2(A) * sin^2(A).

LHS = sin^2(A)/cos^2(A) - sin^2(A).

= sin^2(A) * [1/cos^2(A) - 1].

= sin^2(A) * [(1 - cos^2(A)) / cos^2(A)].

= sin^2(A) * [sin^2(A) / cos^2(A)] (since 1 - cos^2(A) = sin^2(A)).

= sin^2(A) * tan^2(A) = tan^2(A) * sin^2(A) = RHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Sketch y = 1 + sin(2x) and y = 2 - x/pi for 0 <= x <= 2*pi on the same axes. State the number of solutions of 1 + sin(2x) = 2 - x/pi.

y = 1 + sin(2x): oscillates between 0 and 2 with period pi, starting at y=1 (x=0).

Peaks at x = pi/4 (y=2) and x = 5*pi/4 (y=2). Troughs at x=3*pi/4 (y=0) and x=7*pi/4 (y=0).

y = 2 - x/pi: straight line from (0, 2) down to (2*pi, 0) with slope -1/pi.

At x=pi: line gives y=1, sinusoid gives 1+sin(2pi)=1. The curves meet here: (pi, 1) is one intersection.

Near x=0: both start at y=1 (sinusoid) and y=2 (line), line is above.

Counting crossings: in [0, pi] the line drops while the sinusoid dips to 0 and rises — 2 crossings.

In [pi, 2*pi] the sinusoid makes a full cycle back to 1 while line drops from 1 to 0 — 2 more crossings.

Answer: Number of solutions = 4

Question 8. P and Q have position vectors p = 2i - j and q = -4i + j. Find the cosine of the angle between p and q, and the unit vector in the direction of PQ.

(a) Cosine of angle between p and q:

p . q = (2)(-4) + (-1)(1) = -8 - 1 = -9.

|p| = sqrt(4 + 1) = sqrt(5). |q| = sqrt(16 + 1) = sqrt(17).

cos(theta) = (p . q) / (|p| |q|) = -9 / (sqrt(5) * sqrt(17)) = -9/sqrt(85).

Answer: cos(theta) = -9/sqrt(85) [= -9*sqrt(85)/85]

(b) Unit vector in direction of PQ:

PQ = q - p = (-4 - 2)i + (1 - (-1))j = -6i + 2j.

|PQ| = sqrt(36 + 4) = sqrt(40) = 2*sqrt(10).

Unit vector = (-6i + 2j) / (2*sqrt(10)) = (-3/sqrt(10))i + (1/sqrt(10))j.

Answer: Unit vector in direction PQ = (-3/sqrt(10)) i + (1/sqrt(10)) j [= (-3*sqrt(10)/10) i + (sqrt(10)/10) j]

Question 9. Two sectors OBC and OAD share centre O and angle theta radians. Radius of sector OBC = 3cm. A lies between O and B with AB = alpha cm (so OA = 3 - alpha).

(a) Shaded region = sector OBC minus sector OAD.

Area(OBC) = (1/2)(3^2)theta = 9*theta/2.

Area(OAD) = (1/2)(3 - alpha)^2 * theta.

Shaded area = (theta/2)[9 - (3-alpha)^2] = (theta/2)[9 - 9 + 6*alpha - alpha^2]

= (theta/2)(6*alpha - alpha^2) = (alpha*theta/2)(6 - alpha).

Answer: Area of shaded region = (alpha * theta / 2)(6 - alpha) Q.E.D.

(b) Given area = 6 cm^2 and theta = 2:

(alpha * 2 / 2)(6 - alpha) = 6 => alpha(6 - alpha) = 6 => alpha^2 - 6*alpha + 6 = 0.

alpha = (6 - sqrt(36 - 24))/2 = (6 - sqrt(12))/2 = 3 - sqrt(3) (taking the root < 3).

OA = 3 - alpha = 3 - (3 - sqrt(3)) = sqrt(3) cm.

Answer: OA = sqrt(3) cm (approx 1.73 cm)

Question 10. Differentiation and percentage change.

(a) Find the points on y = x^3/3 - x at which the tangent is parallel to the x-axis.

dy/dx = x^2 - 1 = 0 => x = 1 or x = -1.

At x = 1: y = 1/3 - 1 = -2/3. Point (1, -2/3).

At x = -1: y = -1/3 + 1 = 2/3. Point (-1, 2/3).

Answer: Points: (1, -2/3) and (-1, 2/3)

(b) Cylinder radius increased by 5%, height h constant. Find % increase in volume.

Original V = pi * r^2 * h. New V = pi * (1.05r)^2 * h = 1.1025 * pi * r^2 * h.

Percentage increase = (1.1025 - 1) * 100% = 10.25%.

Answer: Volume increases by 10.25%

Question 11. Integration and area under a curve.

(a) Find p given the integral of (8 - 4x) from -1 to p equals 18.

[8x - 2x^2] from -1 to p = (8p - 2p^2) - (8(-1) - 2(-1)^2) = (8p - 2p^2) - (-8 - 2).

= 8p - 2p^2 + 10 = 18 => 8p - 2p^2 = 8 => p^2 - 4p + 4 = 0 => (p - 2)^2 = 0.

Answer: p = 2

(b) The curve y = -x^2 + 3x + 4 = -(x - 4)(x + 1) intersects the axes at three points.

(b)(i) y-axis (x=0): y = 4. So A = (0, 4).

x-axis: -(x-4)(x+1) = 0 => x = 4 (positive root). So C = (4, 0). [B = (-1, 0)]

Answer: A = (0, 4) and C = (4, 0)

(b)(ii) Area of shaded region (arch of curve from B to C, between curve and x-axis):

Area = integral from -1 to 4 of (-x^2 + 3x + 4) dx.

= [-x^3/3 + 3x^2/2 + 4x] from -1 to 4.

At x=4: -64/3 + 24 + 16 = -64/3 + 40 = 56/3.

At x=-1: 1/3 + 3/2 - 4 = (2 + 9 - 24)/6 = -13/6.

Area = 56/3 - (-13/6) = 112/6 + 13/6 = 125/6.

Answer: Area of shaded region = 125/6 square units (approx 20.8 sq units)

Question 12. EITHER/OR — Answer ONE only.

=== EITHER: ABCD is a semicircle, centre O, diameter AB. BE is a tangent at B. BE = 8cm. OC:CE = 3:2. ===

Let radius = r. O is centre, so OB = OC = r (C is on the circle, O and E collinear with C between them).

OC:CE = r:CE = 3:2 => CE = 2r/3. So OE = OC + CE = r + 2r/3 = 5r/3.

In right-triangle OBE (right angle at B, since BE is tangent perpendicular to radius OB):

OE^2 = OB^2 + BE^2 => (5r/3)^2 = r^2 + 64 => 25r^2/9 - r^2 = 64 => 16r^2/9 = 64 => r^2 = 36.

Answer: (b) Radius of circle = 6 cm

(a) Angle BOE: in right triangle OBE with OB=6, BE=8, OE=10:

sin(BOE) = BE/OE = 8/10 = 4/5. Angle BOE = arcsin(4/5).

Answer: (a) Angle BOE = arcsin(4/5) ≈ 0.927 radians

(c) Total area of shaded regions (triangle OBE minus sector OBC, where angle BOC = angle BOE):

Area of triangle OBE = (1/2)(6)(8) = 24 cm^2.

Angle BOC = arcsin(4/5) ≈ 0.9273 rad.

Area of sector OBC = (1/2)(6^2)(arcsin(4/5)) = 18 * arcsin(4/5) ≈ 16.69 cm^2.

Answer: (c) Total shaded area = 24 - 18*arcsin(4/5) ≈ 7.3 cm^2

=== OR: O is origin. OA = 2a, OB = 2b. D on OB produced with OB = BD, so OD = 4b. ===

C is the midpoint of OA, so OC = a. Lines AB and CD meet at E. AE = lambda*AB, CE = mu*CD.

(a)(i) Express OE using line AB:

OE = OA + lambda*(OB - OA) = 2a + lambda*(2b - 2a) = 2(1-lambda)*a + 2*lambda*b.

Answer: OE = 2(1-lambda)a + 2*lambda*b

(a)(ii) Express OE using line CD:

OE = OC + mu*(OD - OC) = a + mu*(4b - a) = (1-mu)*a + 4*mu*b.

Answer: OE = (1-mu)a + 4*mu*b

(b) Equate the two expressions (a, b are non-parallel, so equate coefficients):

a-component: 2(1-lambda) = 1-mu => 2 - 2*lambda = 1 - mu => mu = 2*lambda - 1 ... (i)

b-component: 2*lambda = 4*mu => lambda = 2*mu ... (ii)

Sub (ii) into (i): mu = 2(2*mu) - 1 = 4*mu - 1 => 1 = 3*mu => mu = 1/3.

lambda = 2*(1/3) = 2/3.

Answer: lambda = 2/3 and mu = 1/3

OE = 2(1/3)*a + 2(2/3)*b = (2/3)a + (4/3)b.

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