G12 ECZ ADDITIONAL MATHEMATICS 2016 PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2016 Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

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EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/1

Paper 1

Monday **14 NOVEMBER 2016**

Additional Material(s):

Answer Booklet; Graph paper (1 Sheet)
Electronic calculators

Time: 2 hours

Instructions to Candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $2y + 3x = 12$ cuts the x-axis and y-axis at A and B respectively. Calculate the coordinates of the mid-point of A and B. [4]

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2 Solve the simultaneous equations

$2y - x = 4,$

$xy = 4 + 3x.$ [5]

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3 Find the range of values of k for which the line $y = -3x + 2k + 2$ does not meet the curve $y = x^2 + 3x - 5$. [4]

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4 A function f is defined by $f{:}x \longrightarrow \dfrac{3x-1}{2}$.

Find

(a) the value of a given that $ff(a) = \dfrac{5}{8}$, [3]

(b) the value of x such that $f^{-1}(x) = f(x)$. [3]

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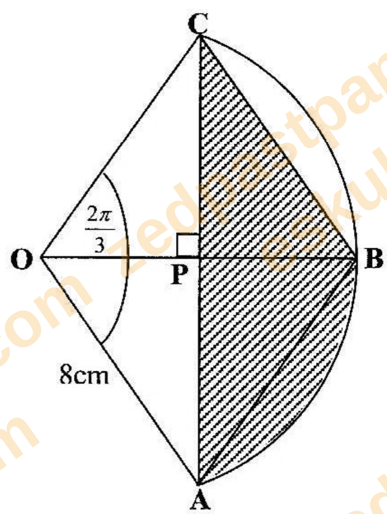
5 In the diagram below, **OABC** is a rhombus. **O** is the centre of a circle through **A**, **B** and **C**. $\angle$**AOC** $= \dfrac{2\pi}{3}$ and **OA** = 8cm. The diagonals **OB** and **AC** intersect at right angles at **P**.



Find

(a) the perimeter of the shaded region. [3]

(b) the area of the shaded region. [2]

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6 Prove the identity $\dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta} \equiv \tan\theta - \cot\theta$. [4]

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7 **(a)** The first three terms in the expansion of $(1 + ax)^n$ are $1 - 16x + 112x^2$. Find the values of a and n. [4]

(b) Given that the coefficient of x^3 in the expansion of $(3 + kx)(1 - x)^7$ is zero, find the value of k. [5]

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8 **(a)** Find an expression for $\dfrac{dy}{dx}$ of the equation $y = (2a^2 + bx)^4$, where a and b are real. [3]

(b) Two variables x and y are related by the equation $y = \left(\dfrac{x}{2} - 4\right)^3$. Obtain an expression for $\dfrac{dy}{dx}$ and find the approximate change in y as x increases from 4 to 4.02. [6]

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9 Find all the angles between 0° and 360° which satisfy the equation $|\tan x + 1| = 3$. [5]

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10 **(a)** The straight lines $\boldsymbol{a} = (2\boldsymbol{i} + \boldsymbol{j}) + \lambda(\boldsymbol{i} + \boldsymbol{j})$ and $\boldsymbol{b} = (5\boldsymbol{i} + \boldsymbol{j}) + \mu(2\boldsymbol{i} - \boldsymbol{j})$ intersect at the point **P**. Find the coordinates of **P**. [4]

(b) The position vectors of the points **A** and **B** with respect to O are $4\mathbf{i} - 3\mathbf{j}$ and $-\mathbf{i} + 2\mathbf{j}$ respectively. Find the position vector of the point **C** such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. [5]

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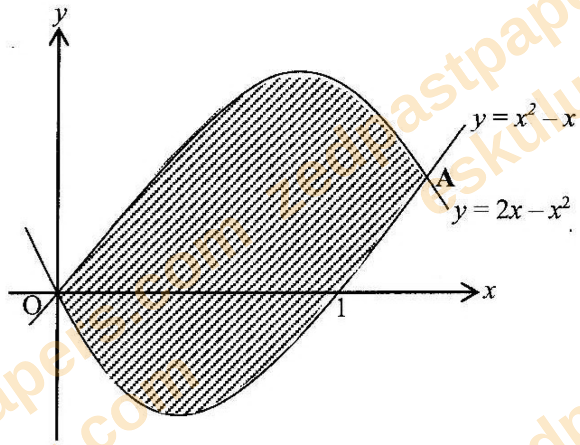
11 **(a)** Evaluate $\int_0^{\pi} \sin x \, dx$. [2]

(b) The diagram below shows part of the curve $y = 2x - x^2$ intersecting the curve $y = x^2 - x$ at O and A.



Find

(i) the coordinates of A. [2]

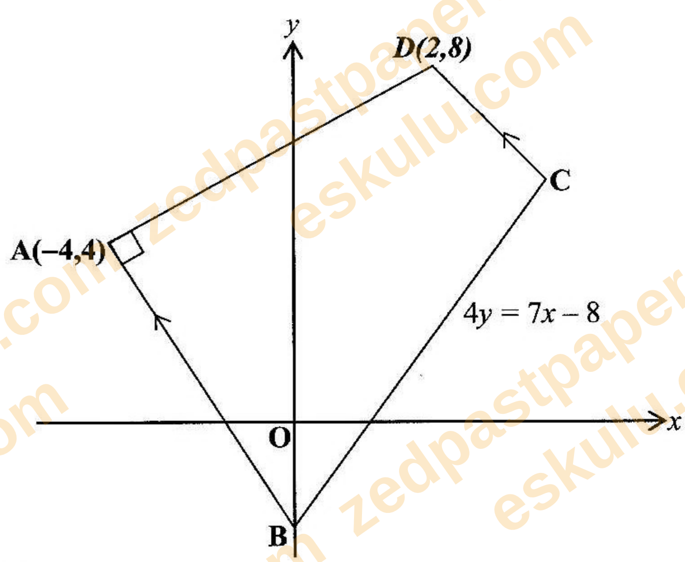
(ii) the area of the shaded region. [6]

Answer only one of the following alternatives.

12 **Either**

Solutions to this question by accurate drawing will not be accepted.

In the diagram below, **ABCD** is a trapezium. **AB** is parallel to **DC** and $\angle$**DAB** = 90°. The coordinates of **A** and **D** are (–4, 4) and (2, 8) respectively. The equation of **BC** is $4y = 7x - 8$.



Find

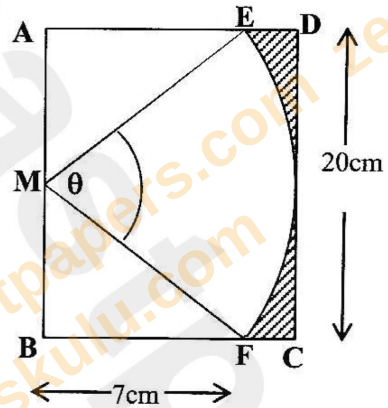
(a) the equation of AB, [4]

(b) the equation of DC, [3]

(c) the coordinates of C. [3]

Or

The diagram below shows a rectangle **ABCD**. **M** is the mid point of **AB** and **AB** = **DC** = 20cm. The points **E** and **F** are on **AD** and **BC** respectively, such that **AE** = **BF** = 7cm. **EF** is an arc of a circle centre **M**, such that angle **EMF** is θ radians.



Find

(a) the value of θ, [3]

(b) the perimeter of the shaded region, [4]

(c) the area of the shaded region. [3]

Question 1. Line $2y + 3x = 12$ cuts the x-axis at A and y-axis at B. Find the midpoint of AB.

x-intercept A: set $y = 0 \Rightarrow 3x = 12 \Rightarrow x = 4$. So A = (4, 0).

y-intercept B: set $x = 0 \Rightarrow 2y = 12 \Rightarrow y = 6$. So B = (0, 6).

Midpoint = $((4+0)/2, (0+6)/2) = (2, 3)$.

Answer: Midpoint of AB = (2, 3)

Question 2. Solve simultaneously: $2y - x = 4$ and $xy = 4 + 3x$.

From linear equation: $x = 2y - 4$.

Substitute into $xy = 4 + 3x$: $(2y-4)y = 4 + 3(2y-4) = 4 + 6y - 12 = 6y - 8$.

$2y^2 - 4y = 6y - 8 \Rightarrow 2y^2 - 10y + 8 = 0 \Rightarrow y^2 - 5y + 4 = 0$.

Factorise: $(y - 1)(y - 4) = 0 \Rightarrow y = 1$ or $y = 4$.

$y = 1$: $x = 2(1) - 4 = -2$. Check: $(-2)(1) = -2 = 4 + 3(-2) = -2$. ✓

$y = 4$: $x = 2(4) - 4 = 4$. Check: $(4)(4) = 16 = 4 + 3(4) = 16$. ✓

Answer: Solutions: (x, y) = (-2, 1) and (x, y) = (4, 4)

Question 3. Find the range of k for which $y = -3x + 2k + 2$ does not meet $y = x^2 + 3x - 5$.

Equate: $x^2 + 3x - 5 = -3x + 2k + 2 \Rightarrow x^2 + 6x - 7 - 2k = 0$.

No real roots: discriminant < 0 $\Rightarrow b^2 - 4ac < 0$.

$a=1$, $b=6$, $c=-(7+2k)$. Discriminant $= 36 + 4(7+2k) = 36 + 28 + 8k = 64 + 8k < 0$.

$8k < -64 \Rightarrow k < -8$.

Answer: $k < -8$

Question 4. Function $f: x \rightarrow (3x - 1)/(2x - 1)$.

(a) Find a such that $f(a) = 2/3$:

$(3a-1)/(2a-1) = 2/3 \Rightarrow 3(3a-1) = 2(2a-1) \Rightarrow 9a - 3 = 4a - 2 \Rightarrow 5a = 1$.

Answer: $a = 1/5$

(b) Find x such that $f^{-1}(x) = f(x)$:

Derive f^{-1}: let $y = (3x-1)/(2x-1)$. Then $y(2x-1) = 3x-1 \Rightarrow 2xy - y = 3x - 1$.

$x(2y - 3) = y - 1 \Rightarrow f^{-1}(x) = (x-1)/(2x-3)$.

Set $(x-1)/(2x-3) = (3x-1)/(2x-1)$: cross-multiply.

$(x-1)(2x-1) = (3x-1)(2x-3) \Rightarrow 2x^2 - 3x + 1 = 6x^2 - 11x + 3$.

$4x^2 - 8x + 2 = 0 \Rightarrow 2x^2 - 4x + 1 = 0$.

x = (4 +/- sqrt(16-8))/4 = (4 +/- 2*sqrt(2))/4 = (2 +/- sqrt(2))/2.

Answer: x = (2 + sqrt(2))/2 or x = (2 - sqrt(2))/2

Question 5. Rhombus OABC with O as centre of circle through A, B, C. Angle AOC = 2*pi/3, OA = 8cm.

The circle has centre O and radius OA = OB = OC = 8cm. Angle AOC = 2*pi/3.

Chord AC: by cosine rule in triangle OAC: AC^2 = 8^2 + 8^2 - 2(64)cos(2*pi/3)

= 128 - 128(-1/2) = 128 + 64 = 192. AC = 8*sqrt(3).

Arc AC (centre O, radius 8, angle 2*pi/3): arc length = 8*(2*pi/3) = 16*pi/3.

(a) Perimeter of shaded region = arc AC + chord AC.

Answer: Perimeter = 16*pi/3 + 8*sqrt(3) cm (approx 30.6 cm)

(b) Area of shaded region = sector area - triangle area.

Sector area = (1/2)(8^2)(2*pi/3) = 64*pi/3.

Triangle OAC area = (1/2)(8)(8)sin(2*pi/3) = 32*(sqrt(3)/2) = 16*sqrt(3).

Answer: Area = 64*pi/3 - 16*sqrt(3) cm^2 (approx 39.3 cm^2)

Question 6. Prove the identity: (2sin^2(theta) - 3)/(sin(theta)cos(theta)) = tan(theta) - cot(theta) - 2sec(theta)cosec(theta).

Start from RHS:

tan(theta) - cot(theta) - 2*sec(theta)*cosec(theta)

= sin/cos - cos/sin - 2/(sin*cos)

= (sin^2 - cos^2 - 2) / (sin*cos).

Now: sin^2 - cos^2 - 2 = sin^2 - (1 - sin^2) - 2 = 2*sin^2 - 3.

So RHS = (2sin^2(theta) - 3) / (sin(theta)*cos(theta)) = LHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Binomial expansions.

(a) First three terms of (1+ax)^n are 1 - 10x + 40x^2.

Comparing: C(n,1)a = -10 => na = -10 ... (i)

C(n,2)a^2 = 40 => n(n-1)a^2/2 = 40 ... (ii)

Try a = -2, n = 5: na = -10 ✓. C(5,2)(4) = 10*4 = 40 ✓.

Answer: n = 5, a = -2 (expansion is (1 - 2x)^5)

(b) In (3 + kx)(1 - 2x)^5, the coefficient of x^3 = 0. Find k.

From (1-2x)^5: coefficient of x^2 = C(5,2)(-2)^2 = 10*4 = 40.

coefficient of x^3 = C(5,3)(-2)^3 = 10*(-8) = -80.

Coefficient of x^3 in product = 3*(-80) + k*40 = -240 + 40k = 0.

40k = 240 => k = 6.

Answer: k = 6

Question 8. Differentiation and small increments.

(a) y = (2a + 3x)^b where a and b are real constants. Find dy/dx.

Using chain rule: dy/dx = b*(2a+3x)^(b-1) * 3 = 3b(2a + 3x)^(b-1).

Answer: dy/dx = 3b(2a + 3x)^(b-1)

(b) y = (2/x - 4)^2. Approximate change as x increases from 4 to 4.02.

Let u = 2x^(-1) - 4. dy/dx = 2u * (-2x^(-2)) = -4(2/x - 4)/x^2.

At x = 4: u = 2/4 - 4 = -3.5. dy/dx = -4(-3.5)/16 = 14/16 = 7/8.

delta_x = 0.02. delta_y ≈ (7/8)(0.02) = 7/400 = 0.0175.

Answer: Approximate increase in y ≈ 0.0175

Question 9. Find all angles in [0°, 360°] satisfying |tan(x) + 1| = 3.

Case 1: tan(x) + 1 = 3 => tan(x) = 2.

x = arctan(2) ≈ 63.4° or x = 63.4° + 180° = 243.4°.

Case 2: tan(x) + 1 = -3 => tan(x) = -4.

Reference angle: arctan(4) ≈ 76.0°.

tan negative in Q2 and Q4: x = 180° - 76.0° = 104.0° or x = 360° - 76.0° = 284.0°.

Answer: x ≈ 63.4°, 104.0°, 243.4°, 284.0°

Question 10. Vectors — intersection and position vector.

(a) Lines a = (2i+j) + s(3i+j) and b = (5i+j) + t(2i-j) intersect at P.

Equate components:

x: 2 + 3s = 5 + 2t ...(i)

y: 1 + s = 1 - t => s = -t ...(ii)

Sub (ii) into (i): 2 + 3s = 5 - 2s => 5s = 3 => s = 3/5.

x = 2 + 3(3/5) = 2 + 9/5 = 19/5. y = 1 + 3/5 = 8/5.

Answer: P = (19/5, 8/5)

(b) Position vectors OA = 2i + j and OB = 5i + j. Find OC such that AC→ = 3*AB→.

AB = OB - OA = (5-2)i + (1-1)j = 3i.

AC = 3*AB = 9i.

OC = OA + AC = (2i+j) + 9i = 11i + j.

Answer: Position vector of C = 11i + j

Question 11. Definite integration and area between curves.

(a) Evaluate integral of sin(x) dx from 0 to pi.

= [-cos(x)] from 0 to pi = -cos(pi) + cos(0) = 1 + 1 = 2.

Answer: Integral = 2

(b) Curves y = 2x - x^2 and y = x^2 - 2x intersect at O and A.

(b)(i) Set 2x - x^2 = x^2 - 2x => 4x = 2x^2 => 2x(x-2) = 0 => x = 0 or x = 2.

At x = 0: y = 0 (point O). At x = 2: y = 0 (point A).

Answer: A = (2, 0)

(b)(ii) For 0 < x < 2, y = 2x-x^2 is above y = x^2-2x (upper - lower = 4x-2x^2 > 0).

Area = integral from 0 to 2 of (4x - 2x^2) dx = [2x^2 - 2x^3/3] from 0 to 2.

= 2(4) - 2(8)/3 = 8 - 16/3 = 24/3 - 16/3 = 8/3.

Answer: Area of shaded region = 8/3 square units

Question 12. EITHER/OR — Answer ONE only.

=== EITHER: Trapezium ABCD. AB parallel to DC, angle DAB = 90°. A=(-4,4), D=(2,8). BC: 4y-7x+8=0. ===

(a) Equation of AB:

Slope of AD = (8-4)/(2-(-4)) = 4/6 = 2/3.

AB is perpendicular to AD (since angle DAB = 90°): slope of AB = -3/2.

Through A(-4, 4): y - 4 = -3/2*(x+4) => 3x + 2y + 4 = 0.

Answer: AB: 3x + 2y + 4 = 0

(b) Equation of DC:

DC is parallel to AB (slope -3/2), through D(2, 8):

y - 8 = -3/2*(x-2) => 3x + 2y - 22 = 0.

Answer: DC: 3x + 2y - 22 = 0

(c) Coordinates of C = intersection of DC and BC:

3x + 2y = 22 and 4y - 7x + 8 = 0 => 4y = 7x - 8 => 2y = 7x/2 - 4.

3x + 7x/2 - 4 = 22 => 6x + 7x = 52 => 13x = 52 => x = 4.

y = (7*4 - 8)/4 = 20/4 = 5.

Answer: C = (4, 5)

Verify: B = intersection of AB and BC: 13x = 0 => x = 0, y = -2. B = (0, -2).

AB parallel DC: both have slope -3/2. ✓ Angle DAB = 90°: AD slope 2/3, AB slope -3/2, product = -1. ✓

=== OR: Rectangle ABCD, AB = DC = 20cm. M = midpoint of AB. AE = BF = 7cm. Arc EF centre M. ===

Setup: A=(0,20), B=(0,0), M=(0,10). E=(7,20) on AD, F=(7,0) on BC (each 7cm from A/B).

ME = sqrt(7^2 + 10^2) = sqrt(49+100) = sqrt(149). MF = sqrt(149) (same by symmetry).

Chord EF = 20cm (vertical from (7,0) to (7,20)).

(a) Angle EMF by cosine rule:

cos(theta) = (ME^2 + MF^2 - EF^2)/(2*ME*MF) = (149+149-400)/(2*149) = -102/298 = -51/149.

Answer: theta = arccos(-51/149) ≈ 1.92 radians (approx 110°)

(b) Perimeter of shaded region = arc EF + chord EF.

Arc EF = sqrt(149) * theta ≈ 12.21 * 1.92 ≈ 23.4 cm.

Answer: Perimeter = sqrt(149)*theta + 20 ≈ 43.4 cm

(c) Area of shaded region = sector area - triangle EMF area.

Sector area = (1/2)(149)(theta).

Triangle area = (1/2)*|EF|*(distance from M to chord EF) = (1/2)(20)(7) = 70.

Area = 149*theta/2 - 70.

Answer: Area = 149*theta/2 - 70 ≈ 73.0 cm^2

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