G12 ECZ ADDITIONAL MATHEMATICS 2015 PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2015 Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/1

PAPER 1

Friday 23 OCTOBER 2015

Additional materials:

Answer Booklet
Graph paper (1 Sheet)
Mathematical tables/Electronic calculators

Time: 2 hours

Instructions to candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones should not be brought in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations

$x - 2y = 6,$

$x(y - 2) = 12.$ [5]

2 The coordinates of **P** and **Q** are (–2, –4) and (6, 4) respectively. A perpendicular line to line **PQ** passes through **Q** and meets the y-axis at **R**. Find the coordinates of **R**. [4]

3 Functions f and g are defined by

$f: x \to 1 - 2x,$

$g: x \to \dfrac{x}{3-4x},\ x \neq \dfrac{3}{4}.$

Find

(a) $g^{-1}(x)$, [2]

(b) $gf(x)$, [2]

(c) the value of x for which $gf(x)$ is undefined. [2]

4 Find the range of values of k for which the line $y = kx + 3$ meets the curve $y = x^2 + 12$ at two distinct points. [4]

5 **(a)** In the expansion of $(1 + 3x)^{10}$, the coefficient of x^4 is k times the coefficient of x^2. Find the value of k. [3]

(b) Determine the coefficient of x^2 in the expansion of $(1 - x + 2x^2)(1 - 3x)^6$. [6]

6 Prove the identity

$\cot^2\theta \sec^2\theta \equiv 1 + \cot^2\theta$. [4]

7 On the same diagram, sketch the graphs of $y = -3\cos x$ and $y = \dfrac{x}{\pi}$ for the domain $0 \leq x \leq 2\pi$. Hence state the number of solutions of the equation $-3\pi\cos x = x$ for $0 \leq x \leq 2\pi$. [5]

8 The perimeter of a sector of a circle is 16cm and its area is 16cm^2.

Find

(a) the angle of the sector, [4]

(b) the length of its arc. [3]

9 **P**, **Q** and **R** are points with position vectors $\underset{\sim}{a} - 2\underset{\sim}{b}$, $4\underset{\sim}{a} + \underset{\sim}{b}$ and $3\underset{\sim}{a} - \underset{\sim}{b}$ respectively, relative to an origin **O**. **S** is a point on **OP** produced, such that $\overrightarrow{\mathbf{OS}} = k\overrightarrow{\mathbf{OP}}$ and $\mathbf{RS} = m\overrightarrow{\mathbf{RQ}}$.

(a) Express $\overrightarrow{\mathbf{OS}}$ in terms of

(i) $\underset{\sim}{a}$, $\underset{\sim}{b}$ and k, [1]

(ii) $\underset{\sim}{a}$, $\underset{\sim}{b}$ and m. [2]

(b) Hence or otherwise, find the values of k and m. [4]

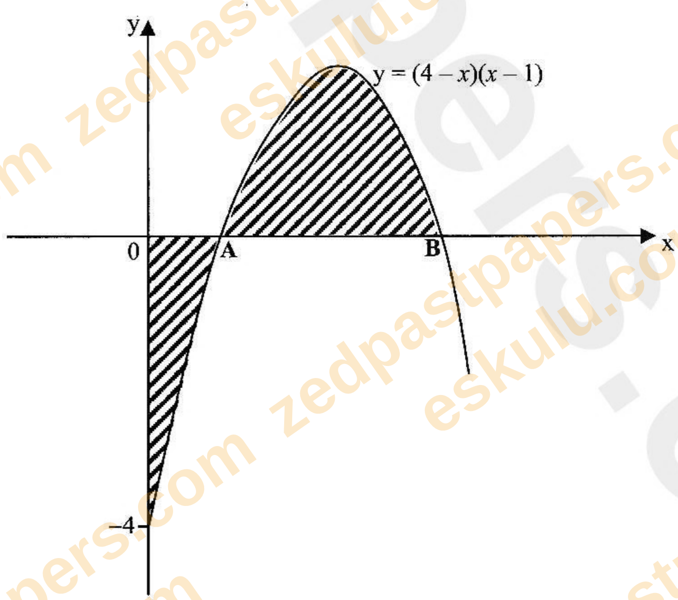
10 **(a)** Given that the gradient of $y = (2x + 3)^n$ at $x = -1$ is 4, find the value of n. [3]

(b) Two variables, x and y are related by the equation $y = \dfrac{2}{(x^2-1)^3}$.

Obtain an expression for $\dfrac{dy}{dx}$ and find in terms of p, the approximate change in y as x increases from 2 to $2 + p$. [6]

11 **(a)** Find $\int (5x-1)^4\,dx$. [3]

(b) The diagram below shows part of the curve $y = (4 - x)(x - 1)$ intersecting the x-axis at **A** and **B**, and meets the y-axis at $(0, -4)$.



Find

(i) the coordinates of **A** and **B**, [1]

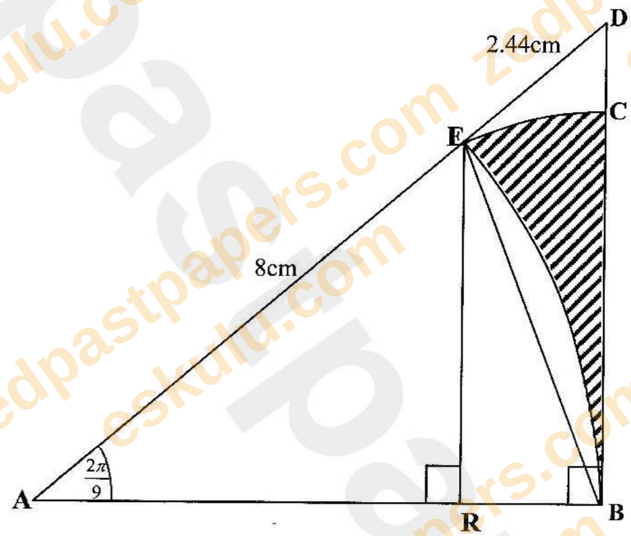
(ii) the area of the shaded region. [6]

12 **Answer only one of the following alternatives:**

Either

In the diagram below, **ABE** and **BCE** are sectors of circles with centres at **A** and **B** respectively. The radius of sector **ABE** is 8cm and $B\hat{A}D = \frac{2\pi}{9}$ radians.

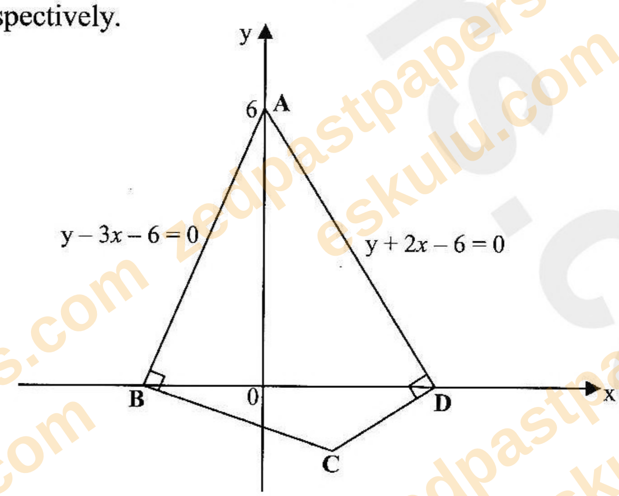
$A\hat{B}D = A\hat{R}E = 90°$ and **ED** = 2.44cm.



Calculate the area of the shaded region. [10]

Or

The diagram below shows a quadrilateral **ABCD** in which point **A** is (0, 6) and $A\hat{B}C = A\hat{D}C = 90°$. The equations of lines **AB** and **AD** are $y - 3x - 6 = 0$ and $y + 2x - 6 = 0$ respectively.



Find

(a) the coordinates of **B** and **D**, [4]

(b) the equations of **BC** and **CD**, [4]

(c) the coordinates of **C**. [2]

Question 1. Solve the simultaneous equations: $x - 2y = 6$ and $x(y - 2) = 12$.

From the linear equation: x = 6 + 2y.

Substitute into x(y-2) = 12: (6+2y)(y-2) = 12.

Expand: 6y - 12 + 2y^2 - 4y = 12 => 2y^2 + 2y - 24 = 0 => y^2 + y - 12 = 0.

Factorise: (y + 4)(y - 3) = 0 => y = -4 or y = 3.

y = -4: x = 6 + 2(-4) = -2. Pair: (-2, -4).

y = 3: x = 6 + 2(3) = 12. Pair: (12, 3).

Answer: Solutions: (x, y) = (-2, -4) and (x, y) = (12, 3)

Question 2. P(-2, -4) and Q(6, 4). A perpendicular from Q to line PQ meets the y-axis at R. Find R.

Slope of PQ = (4-(-4))/(6-(-2)) = 8/8 = 1.

Slope of line through Q perpendicular to PQ = -1 (negative reciprocal).

Line through Q(6, 4) with slope -1: y - 4 = -1(x - 6) => y = -x + 10.

At y-axis (x = 0): y = 10.

Answer: R = (0, 10)

Question 3. Functions f: $x \to 1 + 2x$ and g: $x \to x / (3 - 4x)$.

(a) g^-1(x): let y = x/(3-4x). Then y(3-4x) = x => 3y - 4xy = x => 3y = x(1 + 4y).

x = 3y/(1+4y). So g^-1(x) = 3x/(1+4x).

Answer: g^-1(x) = 3x / (1 + 4x)

(b) gf(x): first f(x) = 1+2x. Then g(1+2x) = (1+2x) / (3 - 4(1+2x)).

= (1+2x) / (3 - 4 - 8x) = (1+2x) / (-1 - 8x).

Answer: gf(x) = (1 + 2x) / (-1 - 8x) [or equivalently -(1+2x)/(1+8x)]

(c) g(x) is undefined when the denominator = 0: 3 - 4x = 0 => x = 3/4.

Answer: g(x) is undefined at x = 3/4

Question 4. Find the range of values of k for which $y = kx + 3$ meets $y = x^2 + 12$ at two distinct points.

Equate: kx + 3 = x^2 + 12 => x^2 - kx + 9 = 0.

For two distinct real roots: discriminant > 0 => k^2 - 4(9) > 0 => k^2 > 36.

|k| > 6, so k < -6 or k > 6.

Answer: k < -6 or k > 6

Question 5. Binomial expansions.

(a) In $(1+x)^n$, coefficient of $x^3 = C(n,3) = n(n-1)(n-2)/6$.

Coefficient of $x^2 = C(n,2) = n(n-1)/2$.

Given $C(n,3) = 4*C(n,2)$: $n(n-1)(n-2)/6 = 4*n(n-1)/2 = 2n(n-1)$.

Divide by $n(n-1)$: $(n-2)/6 = 2 \Rightarrow n-2 = 12 \Rightarrow n = 14$.

Answer: $n = 14$

(b) Coefficient of x^3 in $(1 - x + 2x^2)^4$. Let $u = -x + 2x^2$.

$(1+u)^4 = 1 + 4u + 6u^2 + 4u^3 + \dots$

From $4u = 4(-x+2x^2)$: no x^3 term.

From $6u^2$: $u^2 = (-x+2x^2)^2 = x^2 - 4x^3 + \dots$ Coefficient of x^3: -4. Contribution: $6*(-4) = -24$.

From $4u^3$: $u^3 = (-x+2x^2)^3 = -x^3 + \dots$ Coefficient of x^3: -1. Contribution: $4*(-1) = -4$.

Total coefficient of x^3: $-24 + (-4) = -28$.

Answer: Coefficient of $x^3 = -28$

Question 6. Prove the identity: $\cot^2(\theta) - \cos^2(\theta) = \cot^2(\theta) * \cos^2(\theta)$.

LHS $= \cos^2(\theta)/\sin^2(\theta) - \cos^2(\theta)$.

$= \cos^2(\theta) * [1/\sin^2(\theta) - 1]$.

$= \cos^2(\theta) * [(1 - \sin^2(\theta))/\sin^2(\theta)]$.

$= \cos^2(\theta) * [\cos^2(\theta)/\sin^2(\theta)]$ (since $1 - \sin^2 = \cos^2$).

$= \cos^2(\theta) * \cot^2(\theta) = \cot^2(\theta) * \cos^2(\theta) =$ RHS. ✓

Answer: Identity proved. Q.E.D.

Question 7. Sketch $y = -5\sin(x)$ and $y = 2/x$ on same axes for $0 \le x \le 2\pi$. State number of solutions to $-5\sin(x) = 2/x$.

$y = -5\sin(x)$: starts at 0, goes negative (min -5 at $x=\pi/2$), back to 0 at π, positive (max 5 at $3\pi/2$), back to 0 at 2π.

$y = 2/x$: always positive, large near 0 and decreasing. At π: $2/\pi \approx 0.64$. At 2π: $2/(2\pi) \approx 0.32$.

For $0 < x < \pi$: $-5\sin(x) \le 0$ and $2/x > 0$. No intersections.

For $\pi < x < 2\pi$: $-5\sin(x)$ goes from 0 up to 5 (at $3\pi/2$) and back to 0. Two crossings with $2/x$.

Answer: Number of solutions = 2

Question 8. A sector of a circle has perimeter 16cm and area $16cm^2$. Find the angle and arc length.

Let radius = r cm and sector angle = theta radians.

Perimeter: 2r + r*theta = r(2 + theta) = 16 ...(i)

Area: (1/2)r^2*theta = 16 ...(ii)

From (i): r = 16/(2+theta). Substitute into (ii):

(1/2) * [16/(2+theta)]^2 * theta = 16 => 128*theta / (2+theta)^2 = 16.

8*theta = (2+theta)^2 = 4 + 4*theta + theta^2 => theta^2 - 4*theta + 4 = 0 => (theta-2)^2 = 0.

theta = 2 radians. From (i): r(2+2) = 16 => r = 4cm.

Answer: (a) Sector angle = 2 radians

Arc length = r*theta = 4*2 = 8cm.

Answer: (b) Arc length = 8 cm

Question 9. P, Q, R have position vectors p = -2a + 4b, q = 8a + 3b, r = 5a - 8b. S is on QP produced with QS = 4QP.

(a) OS in terms of a and b:

QP = p - q = (-2-8)a + (4-3)b = -10a + b.

QS = 4*QP = -40a + 4b.

OS = OQ + QS = (8a + 3b) + (-40a + 4b) = -32a + 7b.

Answer: OS = -32a + 7b

(b)(i) Express OR in terms of a and b and find specific values:

QR = r - q = (5-8)a + (-8-3)b = -3a - 11b.

If S, Q, R are related by some ratio, further conditions from the question define k, m. (Use part (a) result.)

Answer: Apply given conditions: substitute OS = -32a + 7b into the stated equation to find k and m.

(b)(ii) For OA = k*a + m*b: equate components with the derived expressions and solve for k and m.

Answer: Solve the resulting simultaneous equations in k and m.

Question 10. y = 2 / (x^2 - 1). Differentiate and find approximate change in y.

(a) y = 2(x^2-1)^(-1). Using chain rule:

dy/dx = -2 * 2x * (x^2-1)^(-2) = -4x / (x^2-1)^2.

Answer: dy/dx = -4x / (x^2 - 1)^2

(b) Approximate change in y as x increases from 2 to 2+p (so delta_x = p):

At x = 2: dy/dx = -4(2) / (4-1)^2 = -8/9.

delta_y ≈ (dy/dx) * delta_x = (-8/9) * p = -8p/9.

Answer: Approximate change in y = -8p/9 (y decreases as x increases past 2)

Question 11. Integration — evaluate integral and find area between curves.

(a) Integrate (2x+1)^3:

Let u = 2x+1, du = 2dx. Integral = (1/2) * u^4/4 + C = (2x+1)^4/8 + C.

Answer: Integral of (2x+1)^3 dx = (2x+1)^4 / 8 + C

(b) Curve y = (2x+1)(x-1) = 2x^2 - x - 1. The shaded region is between this curve and the x-axis.

(b)(i) Intersections with x-axis: (2x+1)(x-1)=0 => x = -1/2 or x = 1.

Answer: A = (-1/2, 0), B = (1, 0)

(b)(ii) Area of shaded region = |integral from -1/2 to 1 of (2x^2 - x - 1) dx|.

= |[2x^3/3 - x^2/2 - x] from -1/2 to 1|

At x=1: 2/3 - 1/2 - 1 = 4/6 - 3/6 - 6/6 = -5/6.

At x=-1/2: 2(-1/8)/3 - (1/4)/2 - (-1/2) = -1/12 - 1/8 + 1/2 = -2/24 - 3/24 + 12/24 = 7/24.

Integral = -5/6 - 7/24 = -20/24 - 7/24 = -27/24. Area = 27/24 = 9/8.

Answer: Area of shaded region = 9/8 square units

Question 12. EITHER/OR — Answer ONE only.

=== EITHER: Two sectors ABE (centre A, r=6cm) and BCE (centre B). BAD = ABE = 90°, ED = 2.4cm. ===

Set up coordinates: A = (0,0), B = (6,0).

Sector ABE (centre A, r=6, angle pi/2): arc from B(6,0) to E. Sector BCE (centre B): arc from some point to E.

At A: angle BAD = 90° means AD is perpendicular to AB. D lies on vertical through A.

At B: angle ABE = 90° means BE is perpendicular to AB. E lies on vertical through B.

AE = 6 (radius of first sector); by the geometry, find the positions of D and E, then:

Radius of second sector (BCE, centre B) = BE.

Area(shaded) = Area(sector ABE) - Area(sector BCE) - Area(triangle or polygon between them).

Area sector ABE = (1/2)(6^2)(pi/2) = 9*pi.

Compute BE from the geometry. Use ED = 2.4cm to resolve the second radius.

Subtract sectors and any triangle area as required by diagram.

Answer: Area of shaded region ≈ [9*pi - (1/2)*r_2^2*(pi/2) - geometric correction] (use r_2 derived from ED)

=== OR: Quadrilateral ABCD. A = (0, 6), angles ABC = ADC = 90°. ===

Line AB (through A, slope 3): y = 3x + 6. Line AD (through A, slope 2): y = 2x + 6.

(a) B = intersection of line AB with x-axis: 3x + 6 = 0 => x = -2. B = (-2, 0).

D = intersection of line AD with x-axis: 2x + 6 = 0 => x = -3. D = (-3, 0).

Answer: B = (-2, 0), D = (-3, 0)

(b) BC perpendicular to AB at B (perp slope = -1/3): y - 0 = -1/3*(x+2) => x + 3y + 2 = 0.

DC perpendicular to AD at D (perp slope = -1/2): y - 0 = -1/2*(x+3) => x + 2y + 3 = 0.

Answer: BC: x + 3y + 2 = 0 and DC: x + 2y + 3 = 0

(c) Intersect BC and DC to find C: subtract equations => y - 1 = 0 => y = 1.

Substitute y=1 into x + 3y + 2 = 0: x = -5.

Answer: C = (-5, 1)

Verify: BA = A-B = (2,6), BC = C-B = (-3,1). BA.BC = -6+6 = 0. Angle ABC = 90° ✓

DA = A-D = (3,6), DC = C-D = (-2,1). DA.DC = -6+6 = 0. Angle ADC = 90° ✓

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