# G12 ECZ ADDITIONAL MATHEMATICS 2014 PAPER 1 SOLUTIONS

**Grade 12 ECZ Additional Mathematics 2014 Paper 1 (4030/1)**

12 questions, 80 marks — full worked solutions

*The full question paper appears first — worked solutions follow.*

EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate
and General Certificate of Education Ordinary Level

# ADDITIONAL MATHEMATICS 4030/1

PAPER 1

Monday 27 OCTOBER 2014

Additional materials:
Answer Booklet
Graph paper (1 Sheet)
Mathematical tables/Electronic calculators

TIME: 2 hours

## INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

**The use of a non programmable electronic calculator is expected, where appropriate.**
**Cell phones should not be brought in the examination room.**

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

## MATHEMATICS FORMULAE

### 1 ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

### TRIGONOMETRY

*Identities*

$\sin^2 A + \cos^2 A = 1$

$\sec^2 A = 1 + \tan^2 A$

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Solve the following simultaneous equations;

$3x^2 + xy - 2y^2 = 27$,

$y = -x + 2$. [5]

The line $2y = -x + 8$ is the perpendicular bisector of the line AB with A(2, −2).
Find the coordinates of the midpoint of AB. [4]

The functions f and $g$ are defined by $f(x) = x^2 - 2x - 24$ and $g(x) = \frac{8x}{3} + 5$.

(a) Find the range of f. [3]

(b) Sketch the functions $g$ and $g^{-1}$ on the same axes. [3]

Given that the line $y = 3 - kx$ is a tangent to the curve $x^2 + xy - 3 = 0$, find the value of $k$. [4]

(a) Find the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{10}$. [4]

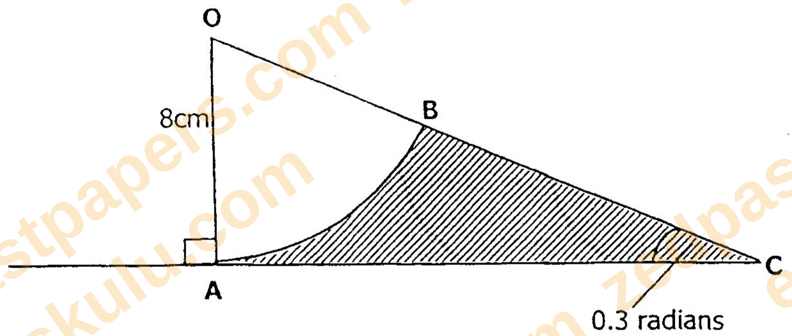
(b) Expand $(1 + a - a^2)^5$ in ascending powers of $a$ up to the term containing $a^2$. [5]

Prove the identity

$$\frac{1}{\sec\theta + \tan\theta} + \frac{1 + \sin\theta}{\cos\theta} \equiv 2\sec\theta.$$ [4]

Draw the graph of $y = |3\sin x|$ for the domain $0 \le x \le 2\pi$ and hence find the approximate solutions to the equation $|3\sin x| = 1.5$. [5]

In the diagram below, OAB is a sector of a circle with centre O and radius 8cm. The line AC is perpendicular to the radius OA and the line OC cuts the circle at B. Angle OCA is 0.3 radians.




Calculate

(a) the area of the shaded region, [2]

(b) the perimeter of the shaded region. [5]

**9** **(a)** Given that $\overrightarrow{OA}$ has magnitude 15 and has the same direction as $\begin{pmatrix} 24 \\ 18 \end{pmatrix}$, express $\overrightarrow{OA}$ as a column vector. [3]

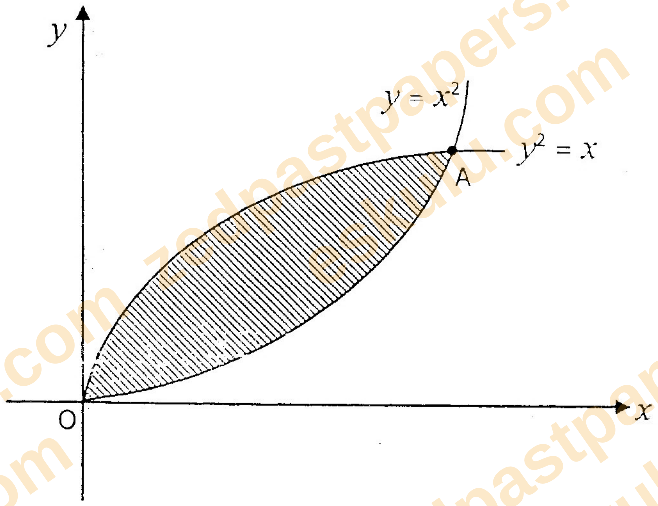
**(b)** The position vector of B relative to the origin O is $5\mathbf{i} - 3\mathbf{j}$. Given that $\overrightarrow{AB} = 3\mathbf{i} - 5\mathbf{j}$, find angle AOB. [4]

**10** **(a)** Given that $y = \dfrac{3}{\sqrt{2x^2 + 5}}$, find an expression for $\dfrac{dy}{dx}$. [3]

**(b)** Given that $y = \dfrac{x - 2}{2x + 3}$ where $x \neq -1.5$, find the approximate increase in $y$ as $x$ increases from 2 to 2.35. [6]

**11** **(a)** The gradient function of a curve is $2x + \dfrac{3}{\sqrt{x}}$. Given that the curve passes through the point (4, 19), find the equation of the curve. [5]

**(b)** The diagram below shows the curves $y = x^2$ and $y^2 = x$ intersecting at O and A.




Find

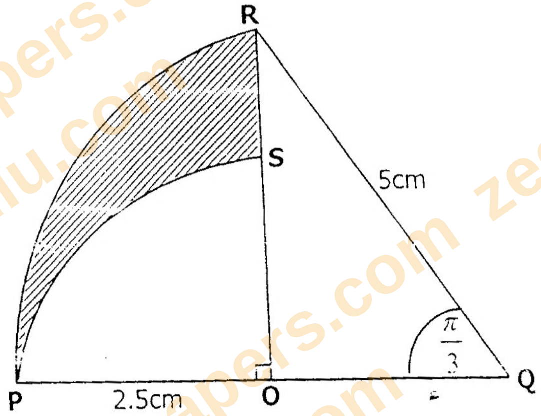
**(i)** the coordinates of A, [3]

**(ii)** the area of the shaded region. [2]

**12 Answer only one of the following alternatives:**

**Either**

In the diagram below, PR is an arc of a circle with centre Q and radius 5cm. PS is an arc of another circle with centre O and radius 2.5cm. OSR and POQ are straight lines. Angle OQR $= \frac{\pi}{3}$ radians.



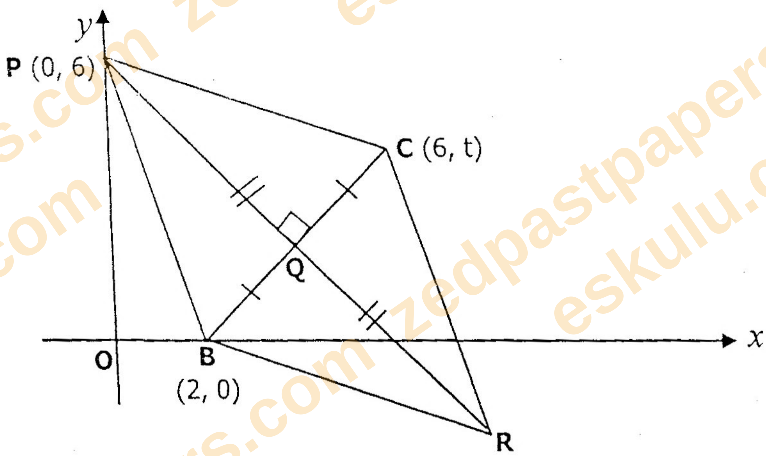

Find

**(a)** the area of the shaded region, [6]

**(b)** the perimeter of the shaded region. [4]

**Or**

In the diagram below, BPCR is a rhombus. P is (0, 6), C is (6, $t$) and B is (2, 0). The lines PR and BC bisect at Q, PR = 2BC and PR $= 8\sqrt{2}$.




Find

**(a)** the value of $t$ and the coordinates of R, [5]

**(b)** the equation of PR, [3]

**(c)** the area of the rhombus. [2]

## Question 1. Solve simultaneously: $3x^2 + xy - 2y^2 = 27$ and $y = -x + 2$.

Substitute $y = -x + 2$ into the quadratic equation.

Note: $3x^2 + xy - 2y^2 = (3x - 2y)(x + y)$ (factorised form).

With $y = -x + 2$: $x + y = x + (-x+2) = 2$.

And $3x - 2y = 3x - 2(-x+2) = 3x + 2x - 4 = 5x - 4$.

So $(5x - 4)(2) = 27 \Rightarrow 10x - 8 = 27 \Rightarrow 10x = 35 \Rightarrow x = 7/2 = 3.5$.

Then $y = -7/2 + 2 = -3/2 = -1.5$.

Verify: $3(7/2)^2 + (7/2)(-3/2) - 2(-3/2)^2 = 3(49/4) - 21/4 - 2(9/4) = 147/4 - 21/4 - 18/4 = 108/4 = 27$. ✓

**Answer: $x = 7/2$, $y = -3/2$ (i.e. $x = 3.5$, $y = -1.5$)**

## Question 2. The line $2y - x + 8 = 0$ is the perpendicular bisector of line segment AB, where A = (2, -2). Find the coordinates of the midpoint of AB and the coordinates of B.

Perpendicular bisector slope: from $2y - x + 8 = 0 \Rightarrow y = x/2 - 4$. Slope = 1/2.

Slope of AB (perpendicular to bisector) = $-1/(1/2) = -2$.

Line AB through A(2, -2): $y + 2 = -2(x - 2) \Rightarrow y = -2x + 2$.

Midpoint M = intersection of AB and bisector: $-2x + 2 = x/2 - 4 \Rightarrow -5x/2 = -6 \Rightarrow x = 12/5$.

$y = -2(12/5) + 2 = -24/5 + 10/5 = -14/5$.

**Answer: Midpoint M = (12/5, -14/5)**

B = 2M - A = $(2(12/5) - 2, 2(-14/5) - (-2)) = (24/5 - 10/5, -28/5 + 10/5) = (14/5, -18/5)$.

**Answer: B = (14/5, -18/5)**

## Question 3. Functions $f(x) = x^2 - 2x - 24$ and $g(x) = 8/(x + 5)$.

(a) Range of $f(x) = x^2 - 2x - 24$:

Complete the square: $f(x) = (x - 1)^2 - 1 - 24 = (x - 1)^2 - 25$.

Minimum value = -25, occurring at x = 1. f has no upper bound.

**Answer: Range of f: $f(x) \geq -25$**

(b) Sketching $g(x) = 8/(x+5)$ and $g^{-1}(x)$ on the same axes:

$g(x)$: rectangular hyperbola with vertical asymptote $x = -5$ and horizontal asymptote $y = 0$.

$g^{-1}$: let $y = 8/(x+5)$. Then $y(x+5) = 8 \Rightarrow x = 8/y - 5$. So $g^{-1}(x) = 8/x - 5$.

$g^{-1}(x)$: vertical asymptote $x = 0$, horizontal asymptote $y = -5$.

**Answer: $g^{-1}(x) = 8/x - 5$. Sketch: $g(x)$ reflected in line $y = x$ gives $g^{-1}(x)$.**

## Question 4. The line $y = 3 - 4x$ is a tangent to the curve $y = x^2 + kx + 3$. Find k.

For the line to be tangent: substitute $y = 3 - 4x$ into $y = x^2 + kx + 3$.

$3 - 4x = x^2 + kx + 3 \Rightarrow x^2 + (k+4)x = 0 \Rightarrow x(x + k + 4) = 0$.

For tangency (single contact point): the two roots $x = 0$ and $x = -(k+4)$ must coincide.

So $k + 4 = 0 \Rightarrow k = -4$. (Both roots become $x = 0$ — a double root.)

Verify: curve $y = x^2 - 4x + 3$. At $x = 0$: $y = 3$. Line at $x = 0$: $y = 3$. ✓

$dy/dx = 2x - 4$. At $x = 0$: slope $= -4 =$ slope of line. ✓

**Answer: $k = -4$**

## Question 5. Find the middle term in the expansion of $(2x + 1/(2x))^8$.

$(2x + 1/(2x))^8$ has $n = 8$, so there are 9 terms. Middle term is $T_5$ ($r = 4$).

$T_{r+1} = C(n,r) \cdot (2x)^{n-r} \cdot (1/(2x))^r$.

$T_5 = C(8,4) \cdot (2x)^4 \cdot (1/(2x))^4 = 70 \cdot 16x^4 \cdot (1/(16x^4)) = 70 \cdot 1 = 70$.

**Answer: Middle term = 70 (a constant, independent of x)**

## Question 6. Expand $(1 + a - a^2)^3$ in ascending powers of a up to the term containing $a^3$.

Let $u = a - a^2$. Then $(1 + u)^3 = 1 + 3u + 3u^2 + u^3$.

$u = a - a^2$.

$3u = 3a - 3a^2$.

$3u^2 = 3(a - a^2)^2 = 3(a^2 - 2a^3 + a^4) = 3a^2 - 6a^3 + O(a^4)$.

$u^3 = (a - a^2)^3 = a^3 + O(a^4)$.

Sum up to $a^3$: $1 + (3a - 3a^2) + (3a^2 - 6a^3) + a^3$

$= 1 + 3a + (-3 + 3)a^2 + (-6 + 1)a^3$

$= 1 + 3a + 0 \cdot a^2 - 5a^3$.

**Answer: $(1 + a - a^2)^3 = 1 + 3a - 5a^3$ (up to term in $a^3$)**

## Question 7. Prove the identity: $1 / (\sec(\theta) - \tan(\theta)) = \sec(\theta) + \tan(\theta)$.

Multiply numerator and denominator of LHS by $(\sec\theta + \tan\theta)$:

LHS $= (\sec(\theta) + \tan(\theta)) / [(\sec(\theta) - \tan(\theta))(\sec(\theta) + \tan(\theta))]$.

Denominator $= \sec^2(\theta) - \tan^2(\theta) = 1$ (standard Pythagorean identity).

LHS $= (\sec(\theta) + \tan(\theta)) / 1 = \sec(\theta) + \tan(\theta) =$ RHS. ✓

**Answer: Identity proved. Q.E.D.**

## Question 8. Sketch y = 3|sin x| for 0 <= x <= 2*pi and find all solutions to 3|sin x| = 1.5.

Graph: y = 3|sin x| repeats every pi; peaks at x = pi/2 and x = 3*pi/2 with value 3; zeros at 0, pi, 2*pi.

The graph has two arches (both above x-axis) each with peak 3.

Solving 3|sin x| = 1.5: |sin x| = 0.5.

sin x = 0.5 gives x = pi/6 and x = 5*pi/6 (in [0, 2*pi]).

sin x = -0.5 gives |sin x| = 0.5 as well: x = 7*pi/6 and x = 11*pi/6.

**Answer: Solutions: x = pi/6, 5*pi/6, 7*pi/6, 11*pi/6 (approximately 0.52, 2.62, 3.67, 5.76 radians)**

## Question 9. Vectors OA and OB.

(a) OA has magnitude 15 and same direction as (24, 7) or as column vector [24; 7].

||(24, 7)|| = sqrt(24^2 + 7^2) = sqrt(576 + 49) = sqrt(625) = 25.

Unit vector = (1/25)(24, 7). OA = 15 * (1/25)(24, 7) = (15/25)(24, 7) = (3/5)(24, 7).

OA = (72/5, 21/5).

**Answer: OA as column vector = [72/5; 21/5] (or [14.4; 4.2])**

(b) OB = 5i - 7j, AB = 3i - 5j. OA = OB - AB = (5-3)i + (-7+5)j = 2i - 2j.

**Answer: Position vector of A: OA = 2i - 2j**

Angle AOB: OA.OB = (2)(5) + (-2)(-7) = 10 + 14 = 24.

|OA| = sqrt(4+4) = 2*sqrt(2). |OB| = sqrt(25+49) = sqrt(74).

cos(AOB) = 24 / (2*sqrt(2) * sqrt(74)) = 12 / sqrt(148) = 6 / sqrt(37).

**Answer: Angle AOB = arccos(6/sqrt(37)) ≈ 9.5 degrees**

## Question 10. Differentiation and small increments.

(a) y = 1/(2x^2 - 1) = (2x^2 - 1)^(-1). Find dy/dx.

dy/dx = -1(2x^2 - 1)^(-2) * 4x = -4x / (2x^2 - 1)^2.

**Answer: dy/dx = -4x / (2x^2 - 1)^2**

(b) y = (x - 2) / (2*sqrt(x)). Find approximate increase in y as x increases from 2 to 2.35 (delta_x = 0.35).

Use quotient rule: let u = x-2, v = 2x^(1/2). du/dx = 1, dv/dx = x^(-1/2).

dy/dx = (v - u * dv/dx / du/dx) ... more cleanly:

dy/dx = [2*sqrt(x) * 1 - (x-2) * x^(-1/2)] / (2*sqrt(x))^2

= [2x - (x-2)] / (4x * x^(1/2)) = (x + 2) / (4 * x^(3/2)).

At x = 2: dy/dx = 4 / (4 * 2^(3/2)) = 1 / (2*sqrt(2)) = sqrt(2)/4.

delta_y ≈ (dy/dx) * delta_x = (sqrt(2)/4) * 0.35 = 0.35*sqrt(2)/4 ≈ 0.124.

**Answer: Approximate increase in y ≈ 0.124**

## Question 11. Integration — gradient function and area between curves.

(a) Gradient function of curve: dy/dx = 2x + 3/sqrt(x) = 2x + 3x^(-1/2).

Integrate: y = x^2 + 3*(2x^(1/2)) + C = x^2 + 6*sqrt(x) + C.

Curve passes through (4, 19): 16 + 6(2) + C = 19 => 28 + C = 19 => C = -9.

**Answer: Equation of curve: y = x^2 + 6*sqrt(x) - 9**

(b) Curves y = x^4 and y^2 = x (so y = sqrt(x) = x^(1/2)) intersect at O and A.

(b)(i) At intersection: x^(1/2) = x^4 => x^4 - x^(1/2) = 0 => x^(1/2)(x^(7/2) - 1) = 0.

x = 0 (point O) or x^(7/2) = 1 => x = 1. At x = 1: y = 1. So A = (1, 1).

**Answer: A = (1, 1)**

(b)(ii) Area of shaded region between y = x^(1/2) (upper) and y = x^4 (lower), x from 0 to 1:

Area = integral from 0 to 1 of (x^(1/2) - x^4) dx = [2x^(3/2)/3 - x^5/5] from 0 to 1

= 2/3 - 1/5 = 10/15 - 3/15 = 7/15.

**Answer: Area = 7/15 square units**

## Question 12. EITHER/OR — Two alternative questions (answer ONE only).

=== EITHER: Two concentric circular arcs. PR centred at Q radius 5cm, PS centred at O radius 2.5cm. OQR = pi/3. ===

Geometry: Q at origin, O directly above at distance QO = 2.5 cm (since QP = 5, OP = 2.5, POQ collinear).

P = (0, 5), O = (0, 2.5), Q = (0, 0).

Angle OQR = pi/3 (angle at Q between QO and QR directions).

R = Q + 5*(sin(pi/3), cos(pi/3)) = (5*sqrt(3)/2, 5/2). S = O + 2.5*(sin(pi/2), cos(pi/2)) = (2.5, 2.5).

(a) Area of shaded region = Area(sector QPR, angle pi/3, r=5) - Area(sector OPS, angle pi/2, r=2.5) - Area(triangle QOS).

Area(sector QPR) = (1/2)(25)(pi/3) = 25*pi/6.

Area(sector OPS) = (1/2)(6.25)(pi/2) = 25*pi/16.

Area(triangle QOS) = (1/2)(2.5)(2.5) = 25/8 (right angle at O, legs OQ=2.5 and OS=2.5).

Shaded = 25*pi/6 - 25*pi/16 - 25/8 = 25*pi*(8-3)/48 - 25/8 = 125*pi/48 - 25/8.

**Answer: Area of shaded region = (125*pi - 150)/48 = 25(5*pi - 6)/48 ≈ 5.06 cm^2**

(b) Perimeter of shaded region = Arc PR + Arc PS + Straight line SR.

Arc PR = 5*(pi/3) = 5*pi/3.

Arc PS = 2.5*(pi/2) = 5*pi/4.

SR = 5*sqrt(3)/2 - 2.5 = 5(sqrt(3)-1)/2.

Perimeter = 5*pi/3 + 5*pi/4 + 5(sqrt(3)-1)/2 = 5*pi*(4+3)/12 + 5(sqrt(3)-1)/2 = 35*pi/12 + 5(sqrt(3)-1)/2.

**Answer: Perimeter = 35*pi/12 + 5(sqrt(3)-1)/2 ≈ 11.0 cm**

=== OR: Rhombus BPCR. P=(0,6), B=(2,0), C=(6,t), PR and BC bisect at Q, PR=2BC, PR=8*sqrt(2). ===

(a) Finding t and R:

PR = 8*sqrt(2). BC = PR/2 = 4*sqrt(2).

BC = sqrt((6-2)^2 + t^2) = sqrt(16+t^2) = 4*sqrt(2) => 16 + t^2 = 32 => t^2 = 16 => t = 4.

**Answer: t = 4**

Midpoint of BC = Q = ((2+6)/2, (0+4)/2) = (4, 2).

Q is also midpoint of PR: (0+x_R)/2 = 4 => x_R = 8. (6+y_R)/2 = 2 => y_R = -2.

**Answer: R = (8, -2)**

(b) Equation of PR: P=(0,6), R=(8,-2). Slope = (-2-6)/(8-0) = -8/8 = -1.

y - 6 = -1(x - 0) => y = -x + 6.

**Answer: Equation of PR: y = -x + 6 (or x + y = 6)**

(c) Area of rhombus = (1/2) * d1 * d2 where d1 = PR = 8*sqrt(2), d2 = BC = 4*sqrt(2).

Area = (1/2)(8*sqrt(2))(4*sqrt(2)) = (1/2)(32)(2) = 32.

**Answer: Area of rhombus = 32 square units**

## Disclaimer

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