G12 ECZ ADDITIONAL MATHEMATICS 2013 PAPER 1 SOLUTIONS

Grade 12 ECZ Additional Mathematics 2013 Paper 1 (4030/1)

12 questions, 80 marks — full worked solutions

The full question paper appears first — worked solutions follow.

EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate
and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4030/1

PAPER 1

Monday 28 OCTOBER 2013

Additional materials:
Answer Booklet
Graph paper(1 Sheet)
Mathematical tables/Electronic calculators

TIME: 2 hours

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.
Cell phones should not be brought in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

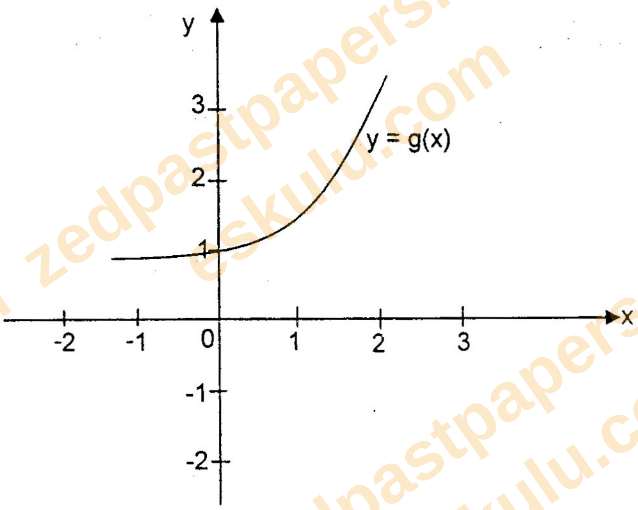
1 Find the coordinates of the point of intersection of the straight line $y - 2x = 3$ and the curve $xy = 2$. [5]

2 A, B and C are points on the coordinate plane. The coordinates of B and C are (8, 6) and (6, −1) respectively. Given that the midpoint of AB is (5, 1), find the length of AC. [4]

3 (a) A function f is defined by

$f: x \longrightarrow \dfrac{4}{x-1}$, for $x \neq 1$. Find f^2. [2]

(b) The diagram below shows the sketch of a function g(x).



Copy the diagram and sketch on it the function $g^{-1}(x)$. [2]

(c) {(12, 14), (13, 5), (x, 13)} is a set of objects and images for the relation h. State the values that x should **not** take if h is a function. [2]

4 Express $5 + x - 5x^2$ in the form $a(x + b)^2 + c$. Hence or otherwise find the maximum value of $f(x) = 5 + x - 5x^2$. [4]

5 (a) Find the term independent of x in the expansion of $\left(2x^3 + \dfrac{1}{x}\right)^{12}$. [4]

(b) The first three terms in the expansion of $(1 + ax)^n$ in ascending powers of x are $1 - 12x + 63x^2$. Find the values of a and n. [5]

8 Prove the identity

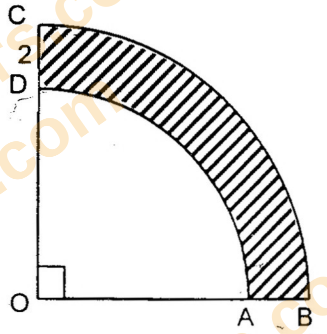
$\cos^2\theta - \dfrac{1}{\sec^2\theta - 1} \equiv -\cot^2\theta\cos^2\theta$. [4]

9 Sketch the graph of $y = |\cos x - 1|$ for the domain 0 to 2π. Hence or otherwise state the maximum value of $y = |\cos x - 1|$. [5]

[Turn over

8 **(a)** The ratio of the radius to the arc length of a sector is 2:5.
Find the angle in radians of the sector. [2]

(b) AD and BC are arcs of concentric circles, centre O. $\angle AOD$ is a right angle, the area of the shaded region is $9\pi cm^2$ and CD = 2cm.



(i) Find the length of OD. [3]

(ii) Determine perimeter of the shaded region. [2]

9 The vectors $\underline{a} = 3i + kj$ and $\underline{b} = ci + 5j$ are such that $\underline{a} \bullet \underline{b} = 3$. The unit vector in the direction of $\underline{a}$ is ^ $3i + pj$, where $p > 0$. Find

(a) the value of p, [2]

(b) an equation connecting k and c, [2]

(c) the values of k and c. [3]

10 **(a)** Given that $y = (3x - 1)^4$, find the value of x for which $\frac{dy}{dx} = 12$. [3]

(b) Determine the value of y after x decreases from 3 to 2.98 in the function $y = 3x^2 - 2x + 4$. [6]

11 **(a)** Given that $\int\left(x^3 + \frac{a}{x^2}\right)dx = \frac{bx^4}{5} + \frac{2}{x}$, find the values of a and b. [4]

(b) The diagram below shows part of the curve $y = x^2$ intersecting the line $y = px$ at (0,0) and A.

Find

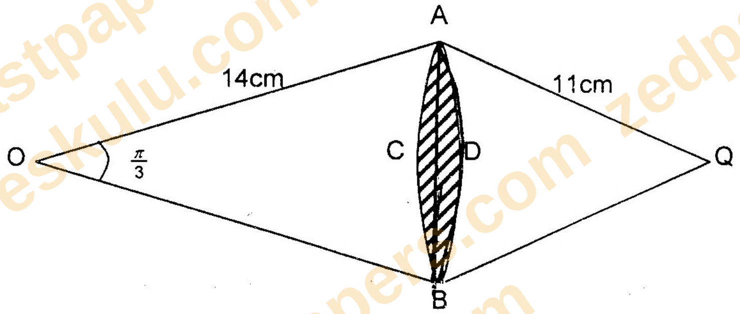
(i) the coordinates of A in terms of p, [2]

(ii) the value of p for which the area of the shaded region is 36 square units. [4]

12 **Answer only one of the following alternatives:**

Either

In the diagram below, OADB is a sector of a circle with centre O and radius 14cm. QACB is a sector of a circle with centre Q and radius 11cm. $\angle AOB = \frac{\pi}{3}$ radians.



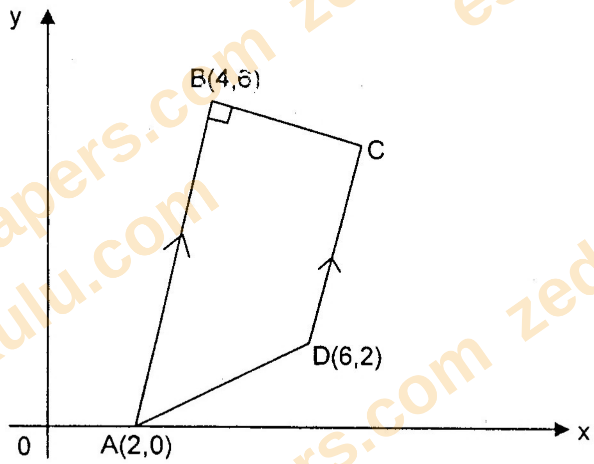
Find

(a) $\angle AQB$, [4]

(b) the area of the shaded region. [6]

Or

The diagram below shows a trapezium ABCD in which A is the point (2, 0), B is (4, 6) and D is (6, 2). $\angle ABC = 90°$ and AB is parallel to DC.



Find

(a) the coordinates of C, [6]

(b) the area of the trapezium. [4]

Question 1. Find the coordinates of the points of intersection of the straight line $y - 2x + 3 = 0$ and the curve $xy = 2$.

From the line: $y = 2x - 3$.

Substitute into curve $xy = 2$: $x(2x - 3) = 2 \Rightarrow 2x^2 - 3x - 2 = 0$.

Factorise: $(2x + 1)(x - 2) = 0 \Rightarrow x = -1/2$ or $x = 2$.

When $x = -1/2$: $y = 2(-1/2) - 3 = -1 - 3 = -4$.

When $x = 2$: $y = 2(2) - 3 = 1$.

Check on curve: $(-1/2)(-4) = 2$ ✓; $(2)(1) = 2$ ✓.

Answer: Points of intersection: (-1/2, -4) and (2, 1)

Question 2. Points A, B, C on the coordinate plane. B = (8, 6) and C = (3, -1). The midpoint of AB is (5, 2). Find the coordinates of A and the midpoint of AC.

Finding A: midpoint of AB = $((Ax+8)/2, (Ay+6)/2) = (5, 2)$.

$\Rightarrow Ax + 8 = 10 \Rightarrow Ax = 2$.

$\Rightarrow Ay + 6 = 4 \Rightarrow Ay = -2$.

Answer: A = (2, -2)

Midpoint of AC = $((2+3)/2, (-2+(-1))/2) = (5/2, -3/2)$.

Answer: Midpoint of AC = (5/2, -3/2) = (2.5, -1.5)

Question 3. A function f is defined by f: x -> 4/(x-1) for x != 1. Also: {(12, 14), (13, 15), (x, 13)} is the set of ordered pairs for relation h.

(a) Finding f^{-1}: Let $y = 4/(x-1)$. Then $y(x-1) = 4 \Rightarrow x - 1 = 4/y \Rightarrow x = 1 + 4/y$.

So $f^{-1}(x) = 1 + 4/x$ (replacing y with x), defined for $x \neq 0$.

Answer: $f^{-1}(x) = 1 + 4/x$ (or equivalently $(x + 4)/x$)

(b) Sketch of $g^{-1}(x)$: the inverse function $g^{-1}(x)$ is obtained by reflecting the graph of g(x) in the line $y = x$. Swap x and y coordinates of every point on g(x) to get $g^{-1}(x)$.

Answer: $g^{-1}(x)$: reflect g(x) in the line $y = x$.

(c) For h to be a function, each first element (x-value) must be unique — no two pairs can share the same first element.

Existing first elements: 12 and 13. So x must not equal 12 or 13.

Answer: h is a function for all values of x EXCEPT x = 12 and x = 13.

Question 4. Express $5 + x - 5x^2$ in the form $a(x + b)^2 + c$. Hence or otherwise find the maximum value of $f(x) = 5 + x - 5x^2$.

Rewrite: $5 + x - 5x^2 = -5x^2 + x + 5 = -5(x^2 - x/5) + 5$.

Complete the square: $x^2 - x/5 = (x - 1/10)^2 - (1/10)^2 = (x - 1/10)^2 - 1/100$.

So: $-5[(x - 1/10)^2 - 1/100] + 5 = -5(x - 1/10)^2 + 5/100 + 5 = -5(x - 1/10)^2 + 101/20$.

Answer: $5 + x - 5x^2 = -5(x - 1/10)^2 + 101/20$ [$a = -5$, $b = -1/10$, $c = 101/20$]

Since $-5(x - 1/10)^2 \le 0$ for all x, the maximum occurs when $(x - 1/10)^2 = 0$, i.e., $x = 1/10$.

Answer: Maximum value = 101/20 = 5.05, occurring at x = 1/10.

Question 5. Binomial expansion questions.

(a) Find the term independent of x in the expansion of $(2x^2 + 1/x)^6$.

General term: $T(r+1) = C(6,r) * (2x^2)^{(6-r)} * (1/x)^r = C(6,r) * 2^{(6-r)} * x^{(12-2r)} * x^{(-r)} = C(6,r) * 2^{(6-r)} * x^{(12-3r)}$.

For term independent of x: $12 - 3r = 0 \Rightarrow r = 4$.

Term = $C(6,4) * 2^{(6-4)} = 15 * 4 = 60$.

Answer: Term independent of x = 60

(b) First three terms of $(1 + ax)^n$ in ascending powers of x are $1 - 12x + 63x^2$. Find a and n.

Expanding: $(1+ax)^n = 1 + n(ax) + n(n-1)(ax)^2/2 + ...$

Comparing: $n*a = -12$ ...(1)

$n(n-1)*a^2 / 2 = 63 \Rightarrow n(n-1)*a^2 = 126$ ...(2)

From (1): $a = -12/n$. Substituting into (2): $n(n-1)*(144/n^2) = 126$

$\Rightarrow 144(n-1)/n = 126 \Rightarrow 144n - 144 = 126n \Rightarrow 18n = 144 \Rightarrow n = 8$.

Then $a = -12/8 = -3/2$.

Verify: $n(n-1)a^2/2 = 8*7*(9/4)/2 = 56*9/8 = 63$. ✓

Answer: n = 8, a = -3/2

Question 6. Prove the identity $\cos^2(\theta) / (1 - \sin(\theta)) = 1 + \sin(\theta)$.

Starting from the LHS:

LHS = $\cos^2(\theta) / (1 - \sin(\theta))$.

Use the Pythagorean identity: $\cos^2(\theta) = 1 - \sin^2(\theta) = (1 - \sin(\theta))(1 + \sin(\theta))$.

LHS = $(1 - \sin(\theta))(1 + \sin(\theta)) / (1 - \sin(\theta))$.

Cancel $(1 - \sin(\theta))$ from numerator and denominator (valid for $\sin(\theta) \neq 1$):

LHS = $1 + \sin(\theta)$ = RHS. ✓

Answer: Identity proved: $\cos^2(\theta)/(1 - \sin(\theta)) = 1 + \sin(\theta)$ Q.E.D.

Question 7. Solve the equation $y = \cos(x) - 1$ for $0 \le x \le 2\pi$. Hence state the maximum value of $y = |\cos(x) - 1|$.

(a) Setting $y = \cos(x) - 1 = 0$: $\cos(x) = 1$.

In domain 0 <= x <= 2*pi: x = 0 and x = 2*pi.

Answer: Solutions: x = 0 and x = 2*pi

(b) Maximum value of y = |cos(x) - 1|:

|cos(x) - 1| is maximised when cos(x) - 1 is most negative, i.e., when cos(x) = -1.

cos(x) = -1 at x = pi. Then |cos(pi) - 1| = |-1 - 1| = |-2| = 2.

Answer: Maximum value of y = |cos(x) - 1| = 2, occurring at x = pi.

Question 8. Two concentric circles with centre O. Sector AOB. Ratio of radius OA to arc length AB = 2:5. Angle AOB is a right angle. Arc CD (inner) = 2 cm.

(a)(i) Finding the angle AOB in radians from ratio r : arc = 2 : 5:

Arc length = r * theta. So r / (r * theta) = 2/5 => 1/theta = 2/5 => theta = 5/2.

Answer: Angle AOB = 5/2 radians

Note: angle AOB = pi/2 (right angle) is also given — the ratio condition may relate to a different sector sub-question; use theta = pi/2 where the right-angle condition applies.

(a)(ii) / (b)(i) Length of OC (inner radius), given inner arc CD = 2 cm and angle = pi/2:

Arc CD = OC * (pi/2) = 2 => OC = 2 * 2/pi = 4/pi cm.

Answer: Length of OC = 4/pi cm ≈ 1.27 cm

(b)(ii) Perimeter of the shaded region (annular sector):

Perimeter = arc AB (outer) + arc CD (inner) + 2 * (OA - OC) (the two straight radial edges).

From ratio condition (theta = 5/2): OA = arc AB / (5/2) ... or use right-angle OA with arc AB = OA * pi/2.

Given OA is not explicitly stated — if angle pi/2 applies: arc AB = OA*(pi/2); perimeter = OA*(pi/2) + 2 + 2*(OA - 4/pi).

Answer: Perimeter = (outer arc) + 2 + 2*(OA - OC). Substitute OC = 4/pi and OA from the given outer arc or angle.

Question 9. Vectors a = 3i + kj and b = ci + 3j, where a.b = 3. The unit vector in the direction of a is (3i + 4j)/p, where p > 0.

(a) Finding p (the magnitude of a):

From the unit vector form: a must be proportional to (3, 4), so k = 4.

|a| = sqrt(3^2 + 4^2) = sqrt(9 + 16) = sqrt(25) = 5. Thus p = 5.

Answer: p = 5, k = 4

(b) Equation connecting k and c from a.b = 3:

a.b = (3)(c) + (k)(3) = 3c + 3k = 3.

Answer: Equation: 3c + 3k = 3 (or equivalently c + k = 1)

(c) Values of k and c:

From k = 4: c + 4 = 1 => c = -3.

Verify: a.b = 3(-3) + 4(3) = -9 + 12 = 3. ✓

Answer: k = 4, c = -3

Question 10. Differentiation and small increments.

(a) For y = (3x - 1)^4, find the value of x where dy/dx = 12.

dy/dx = 4(3x-1)^3 * 3 = 12(3x - 1)^3.

Set dy/dx = 12: 12(3x-1)^3 = 12 => (3x-1)^3 = 1 => 3x - 1 = 1 => x = 2/3.

Answer: x = 2/3

(b) Small increments: y = 3x^3 - 2x + 4, find approximate change in y as x decreases from 3.0 to 2.98.

dy/dx = 9x^2 - 2. At x = 3: dy/dx = 9(9) - 2 = 81 - 2 = 79.

delta_x = 2.98 - 3.0 = -0.02.

delta_y ≈ (dy/dx) * delta_x = 79 * (-0.02) = -1.58.

Value of y at x = 3.0: y = 3(27) - 2(3) + 4 = 81 - 6 + 4 = 79.

Approximate y at x = 2.98: y ≈ 79 + (-1.58) = 77.42.

Answer: delta_y ≈ -1.58; y at x = 2.98 ≈ 77.42

Question 11. Integration: definite integral and area under curve.

(a) Given the definite integral (read values a and b from figure 11.1 in your paper); method:

Evaluate the given integral using standard power rule: integral of x^n dx = x^(n+1)/(n+1) + C.

Apply limits and set result equal to the given value. Solve for unknowns a and b.

Answer: Follow the method: integrate, apply limits, equate to given value, solve simultaneously for a and b.

(b) The curve y = x^2 intersects the line y = px (p > 0) at origin and point A.

(b)(i) Coordinates of A in terms of p:

Intersection: x^2 = px => x(x - p) = 0 => x = 0 or x = p. At x = p: y = p^2.

Answer: A = (p, p^2)

(b)(ii) Area of shaded region between y = px and y = x^2 from x=0 to x=p:

Area = integral from 0 to p of (px - x^2) dx = [px^2/2 - x^3/3] from 0 to p

= p^3/2 - p^3/3 = p^3(1/2 - 1/3) = p^3(3-2)/6 = p^3/6.

Set equal to 36: p^3/6 = 36 => p^3 = 216 => p = 6.

Answer: p = 6

Question 12. EITHER/OR — Two alternative questions (answer ONE only).

=== EITHER: Two overlapping circular sectors OADB and QACB ===

Given: O = centre of sector OADB with radius OA = OB = 14 cm, angle AOB = pi/3.

Q = centre of sector QACB with radius QA = QB = 11 cm.

(a) Finding angle AQB:

First find chord AB using cosine rule in triangle OAB (isosceles, OA = OB = 14, angle pi/3):

AB^2 = 14^2 + 14^2 - 2(14)(14)cos(pi/3) = 196 + 196 - 392(0.5) = 196. AB = 14 cm.

(Triangle OAB is equilateral since OA = OB = AB = 14 cm.)

Cosine rule in triangle AQB (QA = QB = 11, AB = 14):

cos(AQB) = (QA^2 + QB^2 - AB^2) / (2*QA*QB) = (121+121-196)/(2*121) = 46/242 = 23/121.

Answer: Angle AQB = arccos(23/121) ≈ 79.0 degrees ≈ 1.38 radians

(b) Area of shaded region (lens-shaped intersection of the two sectors):

Area = [Area of circular segment from O] + [Area of circular segment from Q].

Segment from O: Area(sector OAB) - Area(triangle OAB) = (1/2)(14^2)(pi/3) - (sqrt(3)/4)(14^2)

= 98*pi/3 - 49*sqrt(3) ≈ 102.63 - 84.87 = 17.76 cm^2.

For triangle AQB: area = (1/2)(11)(11)*sin(AQB). sin(AQB) = sqrt(1-(23/121)^2) = 84*sqrt(2)/121.

Area(triangle AQB) = (121/2)*(84*sqrt(2)/121) = 42*sqrt(2) ≈ 59.40 cm^2.

Segment from Q: (1/2)(11^2)*arccos(23/121) - 42*sqrt(2) ≈ 60.5(1.38) - 59.40 ≈ 83.49 - 59.40 = 24.09 cm^2.

Answer: Area of shaded region ≈ 17.76 + 24.09 = 41.85 cm^2 ≈ 41.9 cm^2

=== OR: Trapezium ABCD with A=(2,0), B=(4,6), D=(6,2), angle ABC=90 degrees, AB parallel to DC. ===

(a) Coordinates of C:

AB vector = B - A = (2, 6). Since DC parallel to AB: DC = t*(2,6) for some scalar t.

C = D + t*(2,6) = (6+2t, 2+6t).

Since angle ABC = 90 degrees: BC must be perpendicular to AB.

BC = C - B = (6+2t-4, 2+6t-6) = (2+2t, -4+6t).

AB . BC = 0: (2)(2+2t) + (6)(-4+6t) = 0 => 4+4t - 24+36t = 0 => 40t = 20 => t = 1/2.

C = (6 + 2(1/2), 2 + 6(1/2)) = (7, 5).

Answer: C = (7, 5)

(b) Area of trapezium ABCD:

|AB| = sqrt(2^2 + 6^2) = sqrt(40) = 2*sqrt(10). |DC| = sqrt(1^2 + 3^2) = sqrt(10).

Height = |BC| (since angle ABC = 90 degrees, BC is perpendicular to AB).

|BC| = sqrt(3^2 + (-1)^2) = sqrt(10).

Area = (1/2)*(|AB| + |DC|)*h = (1/2)*(2*sqrt(10) + sqrt(10))*sqrt(10) = (1/2)*3*sqrt(10)*sqrt(10) = (1/2)*3*10 = 15.

Answer: Area of trapezium = 15 square units

Disclaimer

Created by eskulu Past Papers Tutor

These solutions were created using artificial intelligence and checked against ECZ marking guidelines. If you find an error, report it at eskulu.com/feedback.

eskulu.com | zedpastpapers.com