The OAS Conclusions about the election integrity of the Bolivian election are correct

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A recent article in the Washington Post and an associated background paper questioned the findings of the Organization of American States (OAS) that election irregularities compromised the integrity of the Bolivian Presidential Election of October 20, 2019. This paper reviews the evidence used to support the critique of the OAS findings, presents the arguments made in a rebuttal that was also published in the Washington Post, conducts additional statistical analysis on voting patterns in the election and concludes that the OAS findings were correct. The main results of the additional statistical analysis conducted in the paper are:

a) the distributions of the margins for MAS ((Votes for MAS - Votes for CC)/ Total votes) changed significantly after the suspension of reporting of the votes for administrative units which supported CC before the suspension, but not for administrative units that supported MAS;
b) the changes in distributions were sufficiently large that, were it not for the change in the distributions of margins in the administrative units supporting CC before the suspension, a second round would have had to be held;
c) the conclusions made in the original Washington Post article that there was likely no fraud in the election are based on assertions made from the correlation and scatterplots of the final margin with the margin before the suspension and not from a precise statistical test. The conclusions in this paper are based on the appropriate and standard non-parametric Kolmogorov-Smirnov test for the equality of distributions.

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Introduction

A recent paper (Williams and Curiel, 2020b) and a February 27, 2020 article in the Washington Post (Curiel and Williams, 2020a) called into question the findings of the Organization of American States (OAS) that election irregularities compromised the integrity of the Bolivian Presidential Election of October 20, 2019. Curiel and Williams (2020a) conclude that,

> "*there does not seem to be a statistically significant difference in the margin before and after the halt of the preliminary vote. Instead, it is highly likely that Morales surpassed the 10-percentage-point margin in the first round*"

A rebuttal by Irfan Nooruddin, who had been commissioned by the OAS to analyze election results, was published by the Washington Post on March 10, 2020. Based on a review of all three documents and a new analysis of data from all *mesas* reporting votes, precincts (*recintos*), *localidades* and *municipios*, this paper shows that the Curiel and Williams analysis is flawed, and their conclusions are unwarranted. Nooruddin and the OAS were correct in calling into question the integrity of the Bolivian Presidential Election of October 20, 2019.

The trend in the margin for MAS-IPSP shows a distinct break after reporting of the vote is suspended

Williams and Curiel (2020b) make two assertions about the trends in the vote, namely:

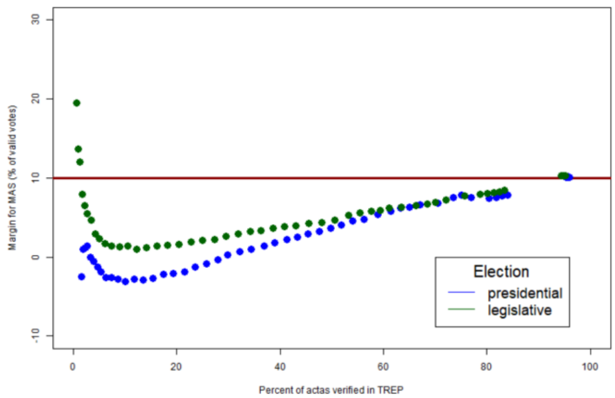
a) "*Following 20% of the vote, the trend in favor of MAS-IPSP is constant*"; and
b) "*Further, the results seen in the TREP are mirrored in the computo, which saw no interruption in the verification of vote totals*"

They base these assertions on two figures which plot the cumulative value of the margin over time. Figure 1 from Williams and Curiel (2020b) presents information on the cumulative value of the margin for MAS up to the stopping of the unofficial preliminary count *(TREP*) and then a few values after the *TREP* resumed. The figure indicates that there was a trend prior to the suspension of the *TREP*. The authors state that, following 20% of the vote, the trend in favor of MAS is constant. However, no statement can be made from this figure about whether the trend remains constant after the suspension, that is after 83.8 percent of the votes. There is a data gap and, though there may be the temptation to extrapolate a constant trend over the gap, there is no basis for doing so.

Figure 2 from Williams and Curiel (2020a) does provide information over the entire distribution of acts, using the official count. This figure plots the cumulative margin and, while a careful reader may detect a change in the slope of the cumulative measure, it does not appear that dramatic.

The use of the cumulative margin makes it difficult to detect a change in the trend. A much more revealing picture of what was happening to the vote is provided in the bottom graph from Nooruddin (2020) which shows the <u>average</u> polling level vote share, before the suspension, during the suspension of reporting, and after the resumption of reporting. The pattern of the average polling level share looks markedly (and suspiciously) different over the three periods.

Figure 1: The MAS-IPSP margin increased steadily through most of the quick count (TREP) as more tally sheets (actas) were verified



From Williams and Curiel (2020a).

Figure 2: The MAS-IPSP margin increased steadily through most of the official count (computo) as more tally sheets (actas) were verified

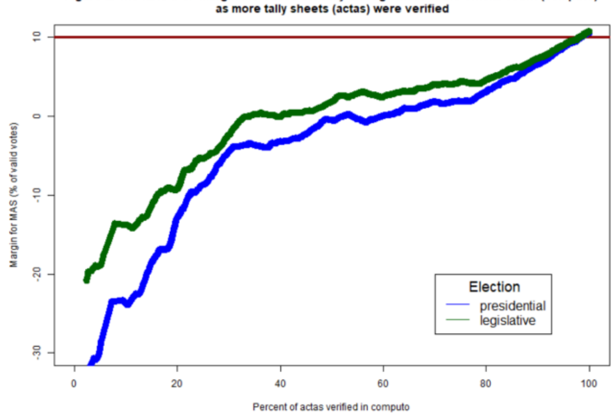
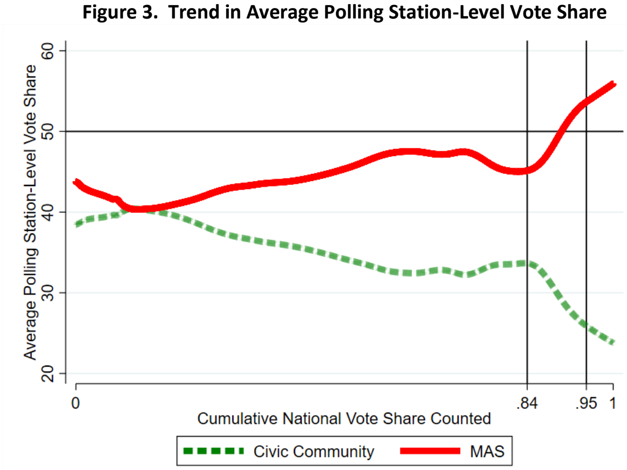


Figure 3. Trend in Average Polling Station-Level Vote Share



From Nooruddin (2020)

While looking at graphs of the average voting shares is illuminating, it is also useful to look at what was happening to the total number of votes over the different periods. Table 1 first reports on the percentage of votes reported for Evo Morales and Carlos Mesa.

Table 1. Percentage of votes at different points in the election counting process

Time period	Votes for Evo Morales	Votes for Carlos Mesa	Percentage point difference
Prior to suspending reporting of votes (83.76 percent of verified *actas*)	45.28%	38.16%	7.12
Immediately after lifting suspension of reporting of votes (95.05 percent of verified *actas*)	46.87%	36.73%	10.14
Final tally (100 percent of verified *actas*)	47.08%	36.51%	10.57

The change in the percentage point difference from 7.12 to 10.14 took place very rapidly, during a period which accounts for only 11.29 percent of the verified *actas*. To bring about that large a change, the margin for Morales during the time when the reporting was suspended had to be significantly larger than the average with 83.76 percent of the *actas* verified. This can be seen in the Table 2, which presents the total number of votes, based on the percentages reported in Table 1 and the total number of valid votes cast in the 2019 election of 6,137,671.

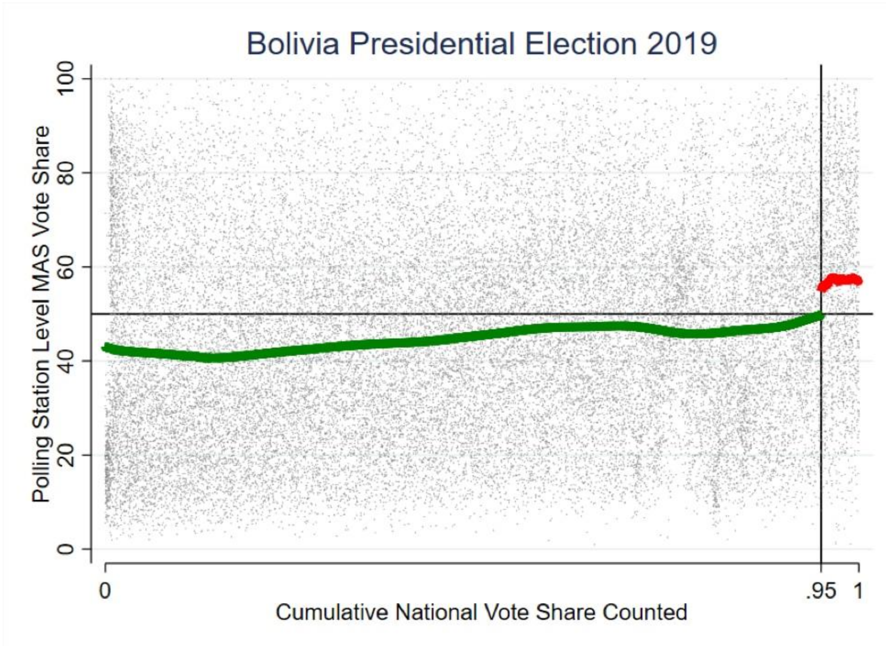
Table 2 reveals that the margin for Morales during the time that the reporting of the vote was suspended was over 2.5 times what it had been prior to the suspension. Moreover, the margin during the period when the margin was suspended was 43 percent higher than after the reporting had resumed. The OAS was correct in drawing attention to the significantly higher margin of votes for Evo that occurred during the time when the reporting of votes was suspended. This high margin was decisive. If the margin of 0.192 that was apparent between 95.05 and 100 percent of the verified *actas* had prevailed over the time when reporting was blacked out, the final margin would have been 0.0959 and the election would have had to go to a runoff.

Table 2. Absolute number of votes and margins at different points in the election counting process

Time period	Votes for Evo Morales (a)	Votes for Carlos Mesa (b)	Total Votes (including other candidates) (c)	Margin for Evo Morales (a – b)/c
Number of votes prior to suspending reporting of votes (83.76 percent of *actas* verified)	2,256,603	1,901,891	4,983,626	0.0712
Number of votes during time of suspension (between 83.76 and 95.05 percent of *actas* verified)	484,153	246,060	864,263	0.2755
Number of votes after time of suspension (between 95.05 and 100 percent of *actas* verified)	148,603	92,969	289,782	0.1920
Total votes	2,889,359	2,240,920	6,137,671	0.1056

Nooruddin (2020) points to the period after the 95.05 percent mark as the most suspicious period, as it reflects a discontinuity in the trend of the vote (see Figure 4). However, the margin for Evo Morales was considerably higher during the period when the reporting was suspended then it was after the suspension of reporting of the vote.[2]

Figure 4. Trend in Polling Station Level MAS Vote Share



The Curiel and Williams analysis of the 1477 precincts which reported votes before and after the cut-off is flawed

Williams and Curiel (2020b) argue that comparing the margins before the suspension with the margins after the suspension is misleading because the composition of the precincts changed. They decompose the sample into three groups:

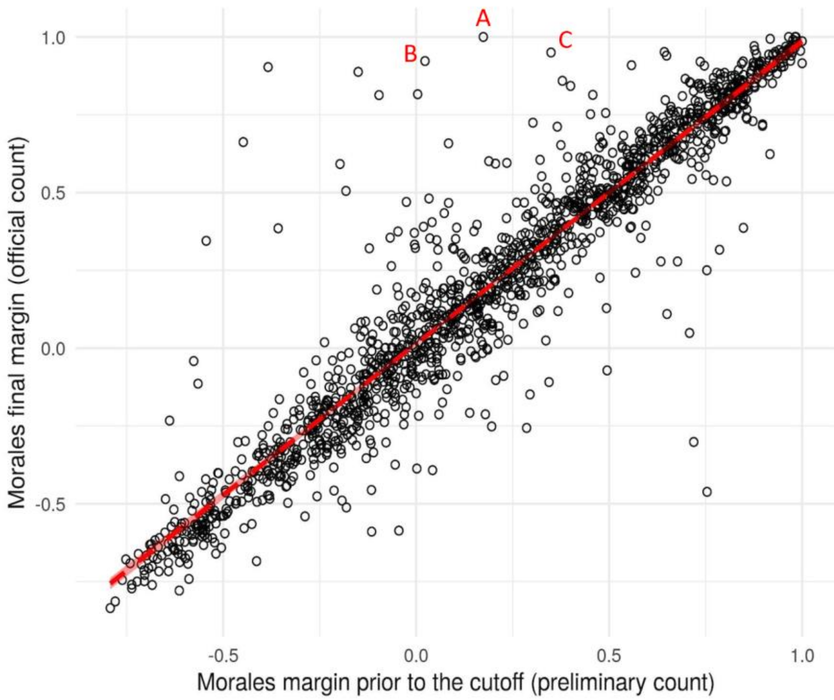
a) Those who had 100% of their results reported prior to the suspension (2805 precincts);
b) Those who reported some of their results reported prior to the suspension and some of their results during and after the suspension (1477 precincts); and
c) Those who had no results reported prior to the suspension (545 precincts).

Prior to the suspension, the data consist of contributions from groups (a) and (b). After the suspension, the data consist of contributions from groups (b) and (c). Given that the number of votes in group (b) is vastly more than those in group (c), they concentrate on analyzing the patterns in group (b), those voting in the 1477 precincts who had reported votes before and after the suspension. If the pattern of votes for these precincts is similar before and after the suspension, they argue that the large gains in the MAS margin apparent after the suspension are due to the change in the composition of the sample, with more pro-Mesa precincts included in the group (a) and more pro-Morales precincts included in the groups (b). There would be no evidence to support the view that the election was fraudulent.

[2] It is possible that the slightly different inference could be based on differences in the data sets. Except for the final row, the data are taken from the TREP in Table 2. The discontinuity apparent in Figure 4 is based on analysis of the Cómputo data.

The point that Williams and Curiel make about the potential importance of changes in the composition of the sample is valid and important. The evidence they use to argue that the change in composition completely accounts for the change in the pattern of voting before and after the cut-off is not at all convincing. The evidence that Williams and Curiel point to is a scatterplot of the Morales margin prior to the cut-off (TREP) and the final margin (official count), along with a finding of a correlation of 0.946 between the two margins. As a measure of the importance they attach to the scatterplot, it is the only graph reproduced in the Washington Post article. It is reproduced below, with some added highlighted points that did not appear in the original graph.

Figure 5. Morales final margin (official count) vs. margin prior to the cut-off (preliminary count)



There are two problems with this evidence. First, there are some suspicious data points. Consider point A with a value of approximately 0.2 as a value of Morales in first 84% of *TREP* and a value of 1 for the Morales final margin. If these values were from the same data source, this would be mathematically impossible because the two margins are not independent. The final margin for Morales is calculated as:

(Equation 1)

$$\frac{(Votes\ Morales\ at\ 84\% + Votes\ Morales\ between\ 84\%\ and\ 100\%) - (Votes\ Mesa\ at\ 84\% + Votes\ Mesa\ between\ 84\%\ and\ 100\%)}{(Total\ Votes\ Mesa, Morales\ and\ Other\ Candidates)}$$

A value of 1 for the Morales final margin means that Morales received all votes cast over the entire voting period. However, a value of 0.2 at the first 84% of *TREP* means that Mesa had received at least some votes over the initial period. As long as there were any votes collected at all in the precinct corresponding to point A in the first 84% of *TREP* (which must be the case to have a value of 0.2), the final margin value cannot be 1, again if the data points were from the same source. However, the *TREP* is not the same source as the official count. Point A could be a legitimate data point – provided all the votes recorded as being for Mesa at the time of the *TREP* were dropped at the time of the recording of the official vote. Not knowing the number of observations associated with point A, it is not clear whether this represented innocent errors in recording in *TREP,* an attempt to purge real votes for Mesa at the time of the official recording or an error by Williams and Curiel in the construction on the data set.

The nonindependence between the two margins makes Points B and C suspicious as well. At Point B, the value of the margin in the first 84% is 0, while the value at the final margin appears to be over 0.9. An example illustrates why this point is suspicious. If the total number of votes in the precinct is 1000 and 100 votes were collected at the time of the first 84% of *TREP*, the maximum value that the final margin could be is 0.9 (assuming all votes after the first 84% of *TREP* go to Morales). If 200 votes were collected at the time of the first 84% of *TREP*, the maximum value possible for the final margin is 0.8, for 300 votes the maximum value is 0.7, and so on. Of course, because the *TREP* is a different data source from the official source, what would otherwise be a mathematical impossibility could be a legitimate data point – provided that votes for Mesa were dropped between the *TREP* and the official count.

If the corrections made to the official count were simple recording errors, one might expect it to be as likely to adjust votes for Morales downwards as to adjust votes for Mesa downward. The data exist to compare the theoretical maximum increase in the margin between the intermediate and final calculation (assuming no correction in going from *TREP* to the official count) with the actual increase. One could also compare the number of times that the vote for Mesa was adjusted downwards in going from the TREP to the official vote (and the total votes reduced) with the number of times that the vote for Morales was adjusted downwards (and the total votes reduced).

If the number of votes in precincts with suspicious outcomes is large enough, this could sway the results of the election. The election is not won by winning a majority of mini-elections in 1477 precincts. The election is won on the basis of the aggregate vote totals.

The second problem is that looking at the scatterplot or the correlation coefficient of the two margins is not the correct way to test whether the distribution of the margins changed after the cut-off period from what it was prior to the cut-off. Williams and Curiel provide no test statistic or critical value to justify the statement that "*there does not seem to be a statistically significant difference in the margin before and after the halt of the preliminary vote*". One should not have to qualify a statement about a statistically significant difference with the added phrase, "*there does not seem to be*". A test either rejects the null hypothesis of equal distributions or fails to reject it. The authors add this phrase because they do not present a true statistical test. Instead, they point to the correlation coefficient and the pattern of the scatterplot and make their assertion.

There is a standard test that would allow for a definitive statement whether the differences are statistically significantly different or not and that is the Kolmogorov-Smirnov nonparametric test for the equality of two distributions[3]. Williams and Curiel have access to the detailed data on votes that would allow them to perform this test, but do not do so. The most sensitive test is to check for differences in the distributions of the margins before and after the cut-off, not differences of the margin before with the final margin (as from equation 1, they can be seen to be definitionally related).

[3] For more information on the Kolmogorov-Smirnov test, see https://en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_test

The distributions of margins changed after the cut-off for *recintos*, *localidades* and *municipios* that favored CC over MAS before the cut-off, but not those that favored MAS over CC before the cut-off

With access to the votes from individual *mesas* provided by the TSE, it is possible to construct the proper tests for equality of distributions and draw precise statistical conclusions. Note that the data from *mesas* are transmitted all at once. A *mesa* will either be reported before the cut-off or after the cut-off. No *mesa* will have part of its data reported before the cut-off and part afterwards. However, at the *recinto* level, it is possible to have: a) a set of *recinto*s with all their *mesas* reported before the cut-off; b) a set of *recinto*s with all their *mesas* reported after the cut-off; and c) a set of *recinto*s with some *mesas* reported before the cut-off and some *mesas* reported after. Table 3 presents a breakdown of the full data set from TSE data into the different sets.

Table 3. Key characteristics of 3 sets of *recintos*

Category	*recinto*s (a)	*mesas* before cut-off (b)	*mesas* after cut-off (c)	Votes MAS (d)	Votes CC (e)	Total Votes (f)	MAS Margin (d-e)/f
All votes before cut-off	3,182	12,449		971,892	775,875	2,066,192	0.0949
All votes after cut-off	618		1,122	104,519	24,358	154,166	0.5200
Votes before and after cutoff	1,496						
Votes before		16,520		1,386,260	1,175,695	3,089,174	0.0682
Votes after			4,464	426,688	264,992	828,139	0.1953
Subtotals		28,969	5,586				
Totals	5,296	34,555		2,889,359	2,240,920	6,137,671	0.1056

The number of *recinto*s, total number of *mesas*, votes for MAS, votes for CC, total votes and margin match those reported by the TSE. The number of *mesas* before the cut-off (28,969) differs by 4 from the number of *mesas* before the cut-off reported by TSE (28,973). This is a difference of 0.014% and will not change the results of the analysis. These numbers are somewhat different than the numbers reported by Williams and Curiel (2020b) and presented earlier in this paper in reporting on their results[4].

Table 3 reveals considerable differences in the MAS margin over the different groups, based on the sum of votes within each category. It is also possible to calculate the margins for the individual *recinto*s before and after the cut-off, as well as the final margin which incorporates the votes both before and after the cut-off. The calculations are as follows:

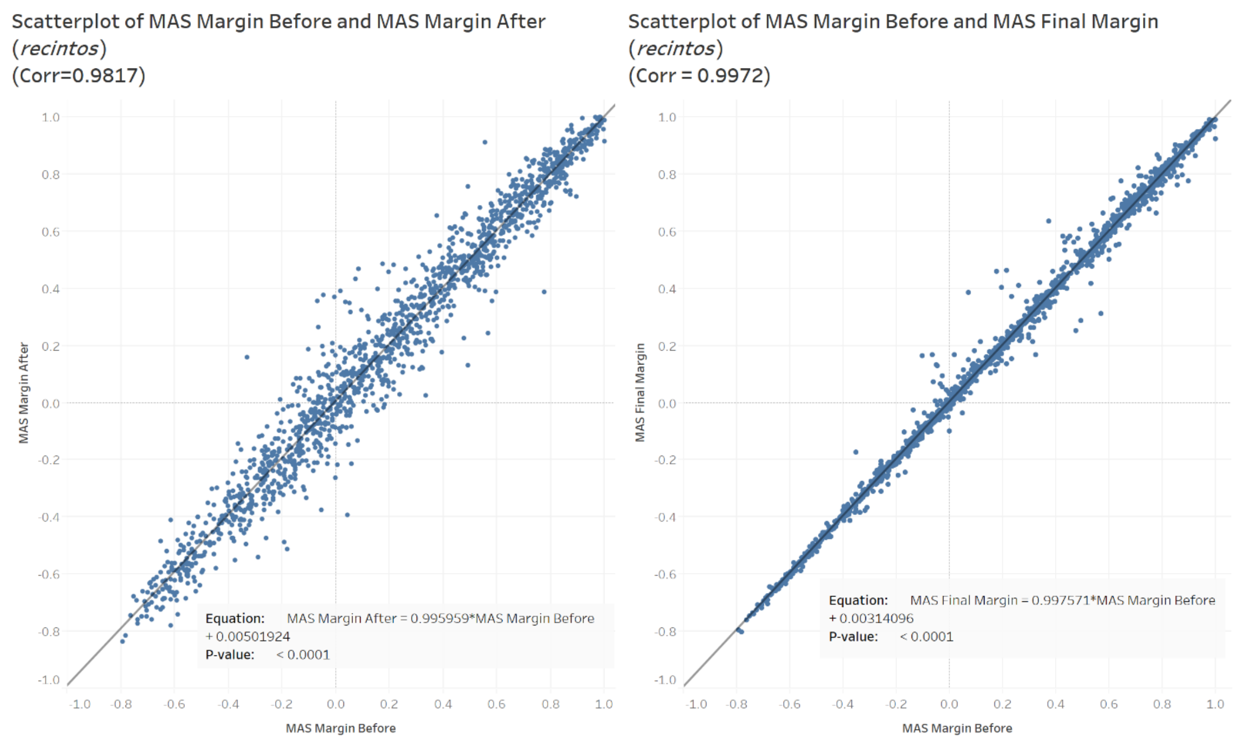
$$m^i_{before} = \frac{\left(MAS^i_{before} - CC^i_{before}\right)}{All\ Votes^i_{before}} \ ; \ m^i_{after} = \frac{\left(MAS^i_{after} - CC^i_{after}\right)}{All\ Votes^i_{after}}$$

$$m^i_{final} = \frac{\left(MAS^i_{before} + MAS^i_{after} - CC^i_{before} - CC^i_{after}\right)}{\left(All\ Votes^i_{before} + All\ Votes^i_{after}\right)}$$

[4] The total number of *recinto*s and total number of votes reported in Williams and Curiel (2020b) do not match the final values reported by TSE. The number of *recinto*s with mesas reported before and after the cut-off in Williams and Curiel (2020b) also differs slightly in what was calculated here (1,477 vs. 1,496). The differences are detailed in Annex A.

Figure 6 presents a scatterplot of the margin before the cut-off versus the margin after the cut-off on the left-hand side and a scatterplot of the margin before versus the final margin on the right-hand side. The tighter scatter on the right-hand side reflects the fact that the final margin is definitionally related to the margin before, a point made earlier. With the cut-off occurring after roughly 84 percent of the *mesas* had their votes recorded, the weight of the votes for MAS^i_{before}, CC^i_{before} and $All\ Votes^i_{before}$ in m^i_{final} is large.[5]

Figure 6. Scatterplots of MAS Margin Before and MAS Margin After and MAS Margin Before and MAS Final Margin (*recintos*)



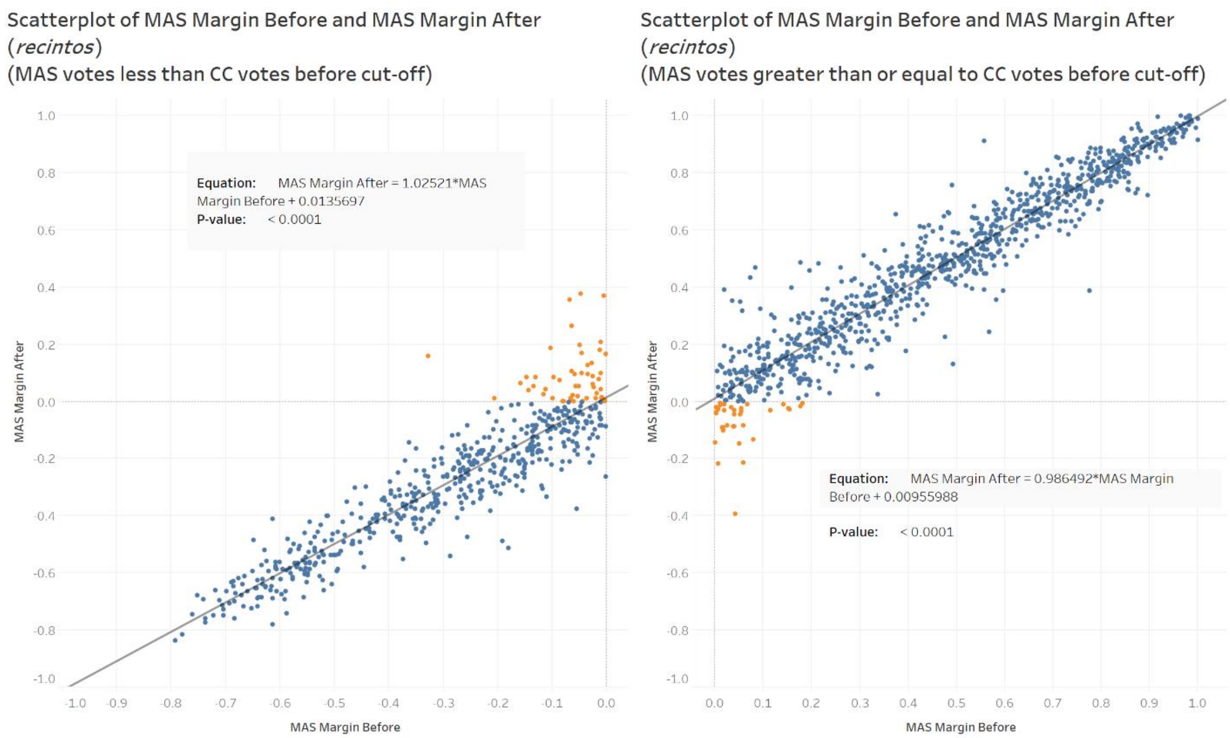
An important observation from the right-hand side plot is that there are no suspicious data points, as was the case with the scatterplot of Figure 5. There are no points equivalent to points A, B and C in the scatterplot in Figure 5. For point A to exist, there must be a positive number of votes for CC before the cut-off and zero votes for CC in the final computo tally. There were no observations that fulfilled those conditions. It is not clear how point A arose in the Williams and Curiel scatterplot.

Figure 7 presents the scatter plots of the margin before versus the margin for those *recinto*s with less votes for MAS than CC before the cut-off (on the left-hand side) and those *recinto*s with more votes for

[5] In a simulation (not reported), making random draws from the same distribution for the margin before a cut-off at 84 percent of the votes and the margin after yielded a correlation of 0.0296, but a correlation of 0.9641 between the margin before and the final margin. This difference was due to the definitional relationship between the margin before and the final margin and the fact that such a high percentage of the votes were collected before the cut-off. If the cut-off had occurred roughly after 30 percent of the votes had been cast, the correlation in the simulated sample between the margin before and the final margin drops to 0.4287.

MAS than CC before the cut-off (on the right-hand side). The equations of the trend lines are different in the two cases and there appear to be more *recinto*s that turned from supporting CC to supporting MAS after the cut-off than *recinto*s that turned from supporting MAS to supporting CC after the cut-off. However, here one runs into a limitation with the analysis that Williams and Curiel employ. It is not possible to make more definitive conclusions about the differences before and after the cut-off for *recinto*s supporting MAS and supporting CC based on a visual reading of the scatterplots. Instead, one must examine the distributions of margins and use the Kolmogorov-Smirnov test to determine whether one can reject the null hypothesis that the distributions of margins before and after are the same.

Figure 7. Scatterplots of MAS Margin Before and MAS Margin After (*recintos*)

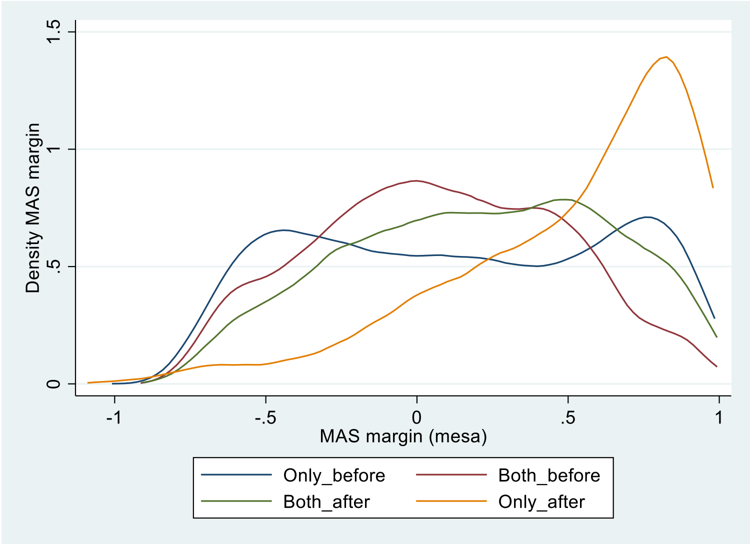


Prior to presenting the statistical tests of the equality of distributions, it is useful to have a visual representation of the distributions. Figure 8 presents kernel density[6] estimates of the distributions of the margins calculated at the mesa level for:

a) margins before for *recinto*s where all *mesas* were recorded prior to the cut-off;
b) margins before for *recinto*s where some *mesas* were recorded before and some after;
c) margins after for *recinto*s where some *mesas* were recorded before and some after;
c) margins after for *recinto*s where all *mesas* were recorded after the cut-off;

[6] Kernel density estimation is a non-parametric way to estimate the probability density function of a random variable. For more information, see https://en.wikipedia.org/wiki/Kernel_density_estimation.

Figure 8. Kernel density estimations for *mesas*



These distributions look considerably different, and the distribution that stands out is *Only_after*, where all mesas reported after the cut-off. The statistical tests confirm that they are different. Table 4 presents the results of Kolmogorov-Smirnov tests that reject the null hypothesis of equal distributions for each pair-wise comparison of the distributions in Figure 8.

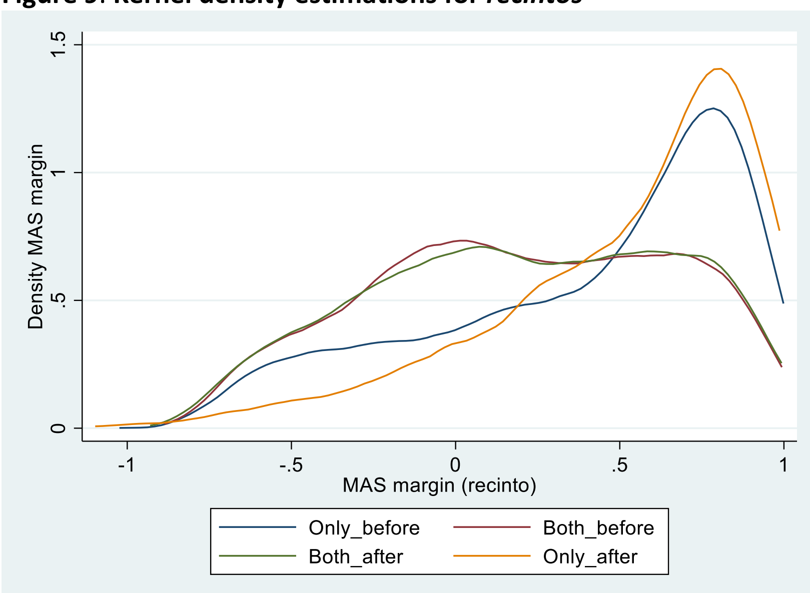
Table 4. Kolmogorov-Smirnov tests for *mesas*

	Sample	D	P value
Both after vs. Both before			
Both_after	4,464	0.0002	1.000
Both_before	16,520	-0.1316	0.000
Combined K-S		0.1316	0.000
Both before vs. Only before			
Both_before	16,520	0.1489	0.000
Only_before	12,449	-0.0574	0.000
Combined K-S		0.1489	0.000
Both after vs. Only after			
Both_after	4,464	0.3281	0.000
Only_after	1,122	-0.0053	0.950
Combined K-S		0.6485	0.000
Both after vs. Only before			
Both_after	4,464	0.0482	0.000
Only_before	12,449	-0.1126	0.000
Combined K-S		0.1126	0.000
Both before vs. Only after			
Both_before	16,520	0.4453	0.000
Only_after	1,122	-0.0043	0.963
Combined K-S		0.4453	0.000
Only after vs. Only before			
Only_after	1,122	0.0039	0.969
Only_before	12,449	-0.3338	0.000
Combined K-S		0.3338	0.000

By construction, there are no *mesas* in common across these different groups. As mentioned earlier, Williams and Curiel (2020b) make the valid point one should look not at the mesa level, but at the level of the *recinto* (one level of aggregation higher) and make comparisons of margins before and after the cut-off across groupings of the same *recinto*s.

Figure 9 presents distributions of the margins for the four sets of *recinto*s, where the unit of analysis is the *recinto* rather than the mesa. For those *recinto*s which have some of the *mesas* reported before the cut-off and some after, the estimated kernel density function for the margin before looks considerably closer to that of the margin after than it did in Figure 8. In Figure 9, the *recinto*s are the same for the margins identified as both_before and both_after. Again in Figure 9, the distributions of the margins only_before and only_after look different, but they are composed of different sets of *recinto*s.

Figure 9. **Kernel density estimations for *recintos***



To make the strongest possible comparison, we focus on the same *recinto*s and look at the distributions of both_before and both_after[7]. Table 5 presents some basic information about these *recinto*s, comparing the percentages from other countries and from Bolivia and the distribution across departments. One can note some differences in the distributions across departments compared to the full sample, but there are still *recinto*s represented across all departments. In addition, the departments with the largest number of *recinto*s in the full sample (La Paz and Santa Cruz) are also the departments with the largest number of *recinto*s in the sample of *recinto*s with votes recorded before and after the

[7] The results were done with and without the 225 *mesas* from Potosí whose computo results were lost in a fire and were replaced by photos of the results from *TREP*. This resulted in a reduction from 1,496 to 1,483 *recinto*s (a reduction of 1 percent) in the sample of *recinto*s with results both before and after the cut-off. The results did not change in any appreciable way.

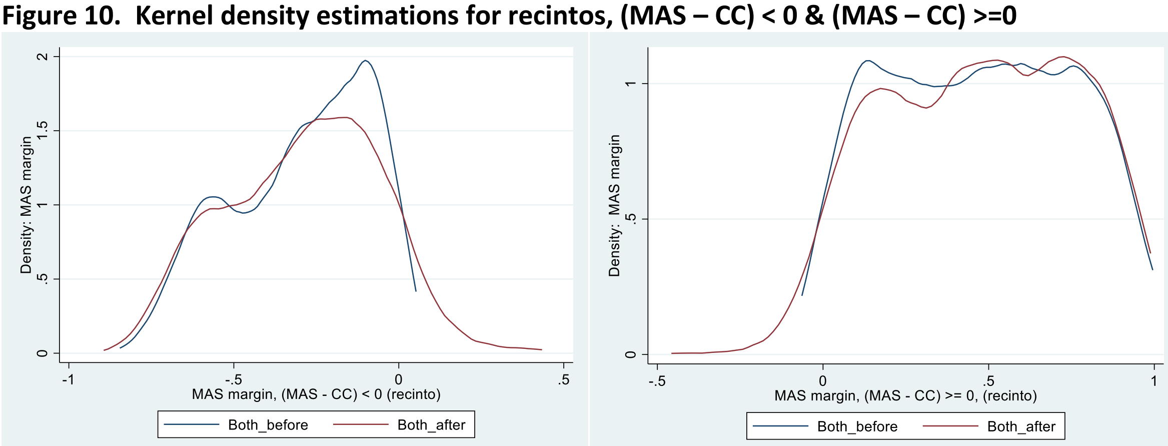
cut-off. The total number of votes recorded in these *recinto*s was 3,917,313, amounting to 63.82% of the total votes cast.

Table 5. Comparison of *recintos* with votes recorded before and after cut-off to the full sample

	Full sample		*recinto*s with votes recorded before and after cut-off	
	Number	Percentage	Number	Percentage
Other countries	165	3.12%	38	2.54%
Bolivia	5,131	96.88%	1,458	97.46%
Beni	270	5.26%	70	4.80%
Chuquisaca	435	8.48%	41	2.81%
Cochabamba	727	14.17%	307	21.06%
La Paz	1,143	22.28%	348	23.87%
Oruro	365	7.11%	68	4.66%
Pando	161	3.14%	18	1.23%
Potosí	680	13.25%	145	9.95%
Santa Cruz	1,015	19.78%	364	24.97%
Tarija	335	6.53%	97	6.65%
Votes before			3,089,174	
Votes after			828,139	
Total votes			3,917,313	
Pct of votes in *recinto*s with votes before and after cut-off to total votes cast				63.82%

Figure 10 presents kernel density estimates for the margins before and after the cut-off separately for those *recinto*s which supported CC before the cut-off as compared to *recinto*s which supported MAS before the cut-off.

Figure 10. Kernel density estimations for recintos, (MAS – CC) < 0 & (MAS – CC) >=0



The difference in the estimated kernel density functions before and after the cut-off look different for the *recintos* where (MAS – CC) < 0 as compared to the case where (MAS – CC) >= 0. Table 6, which presents the results of the Kolmogorov-Smirnov tests, confirms the differences. For those *recinto*s that supported CC before the cut-off, (MAS-CC) < 0, the combined K-S rejects the null hypothesis that the distribution of margins after is the same as that of the margins before, at a conventional significance level of 5%. For those *recintos* that supported MAS before the cut-off, (MAS – CC) >= 0, it is not possible to reject the null hypothesis of equal distributions before and after the cut-off.[8]

Table 6. Kolmogorov-Smirnov tests for *recintos*

Kolmogorov – Smirnov tests	All observations *recintos* (1,496 obs)	*recinto*s where (MAS – CC) < 0 before cut-off (538 obs)	*recinto*s where (MAS – CC) >= 0 before cut-off (957 obs)
Margin after			
D	0.0147	0.0353	0.0365
P value	0.724	0.511	0.278
Margin before			
D	-0.0227	-0.0967	-0.0355
P value	0.462	0.007	0.299
Combined K-S			
D	0.0227	0.0967	0.0365
P value	0.834	0.013	0.545

In addition to knowing whether there is a change in the distribution of the margins before and after the cut-off (from the results of the Kolmogorov-Smirnov tests), it is also possible to know how much of the change in the aggregate margins is due to a change in the distribution of the margins and how much is due to a change in the share of votes of individual *recintos* to the total votes before and after the cut-off. While the margins before and after the cut-off are calculated from the same set of *recintos*, the distribution of the relative share of votes of individual *recintos* to the total votes of all the recintos in this group, cast before and after the cut-off can differ. This additional information on the relative importance of the change in the distribution of margins to the change in the aggregate margins is obtained by employing a Oaxaca-type decomposition, a common technique used in labor economics and poverty analysis[9]. The procedure is as follows.

First note that the difference in the aggregate margins can be written as the weighted sum of margins of the individual *recinto*s after the cut-off minus the weighted sum of margins of the individual *recinto*s before the cut-off. The weights are the share of the votes in that *recinto* relative to the total votes, for votes before and after the cut-off respectively. Following the approach of the Oaxaca decomposition,

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[8] Annex B reports the results of a simulation on the group that recorded all their votes before cut-off. For this group, there should be less risk of voter manipulation than for the group with votes both before and after the cut-off. The simulation involved setting a counterfactual cut-off after 84 percent of the votes in that group had been recorded and testing whether there were any differences in margins before and after that counterfactual cut-off for *recintos* that favored CC and *recintos* that favored MAS. It was not possible to reject the null hypothesis of equal distributions of margins.

[9] The Oaxaca decomposition was introduced in Oaxaca (1973). For more information about the Oaxaca decomposition, see https://en.wikipedia.org/wiki/Blinder%E2%80%93Oaxaca_decomposition

add and subtract the sum of margins before the cut-off, weighted by the share of votes after the cut-off. Rearranging terms yields the difference in the aggregate margin (after – before) as the sum of the individual shares after the cut-off times the difference in individual margins (after – before) and the individual margin before the cut-off times the difference in the shares (after-before). Dividing by the difference in the aggregate margin gives the proportion of the change due to weighted differences in the margins and the proportion of the change due to weighted differences in the shares. The equations describing the steps are provided below.

(Equation 2)

$$m_{after} - m_{before} = \sum_i s^i_{after}\, m^i_{after} - \sum_i s^i_{before} m^i_{before}$$

$$= \sum_i s^i_{after}\, m^i_{after} + \sum_i s^i_{after}\, m^i_{before} - \sum_i s^i_{before} m^i_{before} - \sum_i s^i_{after}\, m^i_{before}$$

$$= \sum_i s^i_{after}\, (m^i_{after} - m^i_{before}) + \sum_i m^i_{before}\, (s^i_{after} - s^i_{before})$$

where:

$$m_{after} = \frac{(MAS_{after} - CC_{after})}{All\,Votes_{after}}\ ;\ m_{before} = \frac{(MAS_{before} - CC_{before})}{All\,Votes_{before}}\ ;$$

$$m^i_{after} = \frac{(MAS^i_{after} - CC^i_{after})}{All\,Votes^i_{after}}\ ;\ m^i_{before} = \frac{(MAS^i_{before} - CC^i_{before})}{All\,Votes^i_{before}}\ ;$$

$$s^i_{after} = \frac{Votes^i_{after}}{All\,Votes_{after}}\ ;\quad s^i_{before} = \frac{Votes^i_{before}}{All\,Votes_{before}}$$

Table 7 presents the results of these Oaxaca-type decompositions. It is worth noting that the sum of the weighted margins before and after match the values reported in Table 3 (as they should in accordance with equation 2). For the entire sample, as well as for those *recinto*s with greater support for MAS and those with greater support for CC, most of the change in the total margins is due to changes in the weighted change in the share of votes. However, in addition to the change in distributions being significant only for *recinto*s with more support for CC before the cut-off, the proportion of the change due to the weighted difference in margins is close to 3 times as great for *recintos* with more support for CC before the cut-off than for *recinto*s with more support for MAS before the cut-off.

Table 7. Oaxaca-type decompositions for *recintos*

	All observations *recintos* (1,496 obs)	*recinto*s where (MAS – CC) < 0 before cut-off (538 obs)	*recinto*s where (MAS – CC) >= 0 before cut-off (957 obs)
Sum of weighted margin before, weighted by share of votes before (a)	0.0682	-0.1292	0.1973
Sum of weighted margin after, weighted by share of votes after (b)	0.1953	-0.0960	0.2912
Δ margins (b–a)	0.1271	0.0332	0.0939
Sum of weighted difference in margins (margin after - margin before), weighted by share of votes after (c)	0.0055	0.0028	0.0027
Sum of weighted difference in shares (share of votes after – share of votes before), weighted by margin before (d)	0.1216	0.0304	0.0912
Proportion of Δ margins due to weighted difference in margins (c/(b-a))	4.33%	8.42%	2.92%
Proportion of Δ margins due to weighted difference in shares of votes (d/(b-a))	95.67%	91.58%	97.08%

The finding that there was a significant difference in the margins provides support to the position that there were compromises to the integrity of the Bolivian Presidential Election. However, it does not provide a complete picture of the extent of the compromise, because the votes of the 1,496 *recintos* constituted only 63.82 percent of the total votes.

To obtain a more complete pictgure of the extent of the compromise requires an attempt to understand what happened over the rest of the sample. It is important to note that one need not be limited to looking at a common set of *recintos* which had votes recorded both before and after the cut-off. It is possible to follow the same approach and construct a set of *localidades* (the next level of aggregation) with votes recorded both before and after the cut-off. This set will have the same *localidades* of the 1,496 *recinto*s with votes recorded before and after the cut-off. The 1,496 *recintos* with votes recorded both before and after the cut-off are in 678 *localidades*. These 678 *localidades* contain not only the 1,496 *recintos*, but additional *recintos* that are included in the set of *recintos* with all votes before and all votes afterwards. For analysis to be carried out at the level of *localidad*, it is necessary to add these *recintos* by merging the data. The 3,218 *recintos* with all votes before the cut-off are located in 2,477 *localidades*. Merging these *localidades* with the 678 yields 94 matched *localidades*. Thus, the votes recorded before the cut-off in these 94 *localidades* can be added to the data set for analysis done at the *localidad* level. The 618 *recintos* with all votes after the cut-off are located in 607 *localidades*. Merging these 607 *localidade*s with the 678 yields a match of 16 *localidades*. The votes recorded after the cut-off can be added to the data set when conducting analysis at the level of the *localidad*. The data set of *localidades* with votes recorded both before and after the cut-off consists of 678 *localidades*. With the additional *recintos* picked up from the matched *localidades*, the data set includes 84.57 percent of the total votes in the country. There are 2,383 *localidades* (2,477 – the matched 94) that only have votes recorded before the cut-off. There are 591 *localidades* (607 – the matched 16) that only have votes recorded after the cut-off. As a final check, an attempt was made to match the 2,383 *localidades* with only votes recorded before the cut-off with the 591 that only had votes recorded after the cut-off. No matches were found, so the final number of *localidades* with votes both before and after the cut-off remained at 678.

The 678 localidades which have votes recorded both before and after the cut-off are located in 275 municipios (both in Bolivia and internationally). The 2,383 localidades with votes only before the cut-off are located in 417 municipios. Out of these 417 municipios, 249 were also in the set of 275 municipios and 168 were not. Thus, the 249 contributed additional votes before the cut-off to the totals recorded in the 275 municipios obtained from collapsing the results of the 678 localidades. The 591 localidades with only votes after the cut-off are located in 210 municipios. Of these, 158 were also in the set of 275 municipios and 52 were not. Thus, the 158 contributed additional votes after the cut-off to the totals recorded in the 275 municipios. Finally, there were 40 municipios which appeared in both the 168 of the 417 with only votes before that had not been matched and in the 52 with only votes after that had not been matched. These 40 municipios are added to the 275 which gives a total of 315 that contained votes both before and after the cut-off. These 40 municipios are dropped from the groups of only before and only after, leaving a final total of 128 municipios (of which 46 were in Bolivia) that only had votes before and 12 municipios (of which 1 was in Bolivia) that only had votes after. The 315 municipios with votes both before and after the cut-off constitute 69.23% of the municipios in the sample (in Bolivia and in other countries), but account for 97.70% of the votes recorded in the TSE.

Table 8 presents some additional information on the full sample of *localidades* and *municipios* and the sample of *localidades* and *municipios* with votes recorded before the cut-off and after.

Table 8. Comparison of *localidades* and *municipios* with votes recorded before and after cut-off to the full sample

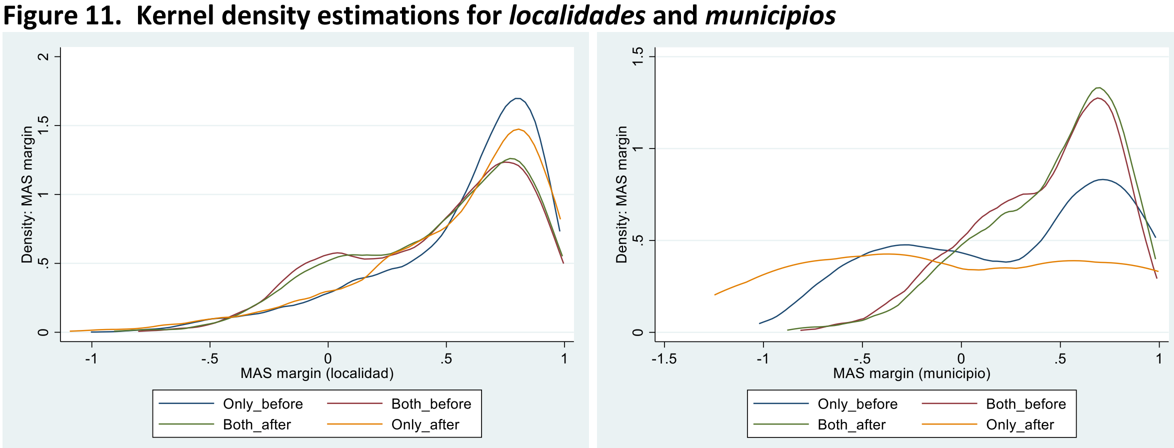
	Full sample (*Localidades*)		Votes before and after cut-off (*Localidades*)		Full sample (*Municipios*)		Votes before and after cut-off (*Municipios*)	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Other countries	125	3.32%	24	3.54%	115	25.27%	22	6.98%
Bolivia	3,637	96.68%	654	96.46%	340	74.73%	293	93.02%
Beni	196	5.39%	21	3.21%	19	5.59%	18	6.14%
Chuquisaca	355	9.76%	27	4.13%	29	8.53%	25	8.53%
Cochabamba	457	12.57%	146	22.32%	48	14.12%	42	14.33%
La Paz	805	22.13%	173	26.45%	87	25.59%	78	26.62%
Oruro	263	7.23%	24	3.67%	35	10.29%	19	6.48%
Pando	136	3.74%	9	1.38%	15	4.41%	13	4.44%
Potosí	588	16.17%	87	13.30%	40	11.76%	38	12.97%
Santa Cruz	575	15.81%	123	18.81%	56	16.47%	50	17.06%
Tarija	262	7.20%	44	6.73%	11	3.24%	10	3.41%
Votes before			4,339,913				5,018,122	
Votes after			850,549				978,131	
Votes before & after			5,190,462				5,996,253	
Pct of votes before & after out of total votes cast				84.57%				97.70%

As with *Recinto*s, four different distributions can be identified for *localidades* and *municipios*:

a) margins before where all votes were recorded prior to the cut-off;
b) margins before where some votes were recorded before and some after the cut-off;
c) margins after where some votes were recorded before and some after the cut-off;
d) margins after where all votes were recorded after the cut-off;

Analogously to Figure 9, Figure 11 presents kernel density estimates for these four different distributions, for *localidades* on the left-hand side and *municipios* on the right hand side. With the higher level of aggregation, the kernel density estimations for *localidades* look more similar to each other than was the case for *recintos*, seen in Figure 9. The kernel density estimations for Only_before and Only_after for municipios look somewhat different from the Both_before and Both_after, largely due to the small number of observations in the Only_before and Only_after (128 municipios in the case of Only_before and 12 in the case of Only_after).

Figure 11. Kernel density estimations for *localidades* and *municipios*



One can then perform the same Kolmogorov-Smirnov tests for the equality of distribution of margins before and after the cut-off for *localidades* and *municipios* supporting CC before the cut-off and supporting MAS before the cut-off and carry out the same Oaxaca-type decompositions. Again, it is useful to first examine plots of the distributions. Figure 12 plots the kernel density estimations for the relevant cases.[10] A comparison of the plots for those with (MAS – CC) < 0 and (MAS – CC) > 0 reveals a similar pattern for *localidades* and *municipios*, with a greater rightward shift of the density function (towards a greater margin for MAS) after the cut-off for *municipios* as opposed to *localidades*. The pattern for *localidades* can be different from that of *municipios*, because (as indicated in Table 8) the percent of votes covered in the analysis using *municipios* is greater than that using *localidades* (97.70% vs. 84.57%). The analysis based at the level of *localidades* and *municipios* both cover a higher percentage of votes than in the analysis based at the level of *recintos* (63.82%).

[10] Annex C presents scatterplots of the margin before versus margin after and the margin before versus final margin for all *localidades* and *municipios* with votes both before and after the cut-off. Scatterplots of the margin before and margin after are also presented separately for groups with (MAS – CC) < 0 and (MAS – CC) >=0 for both *localidades* and *municipios*.

Figure 12. Kernel density estimations for *localidades* and *municipios*
(MAS – CC) < 0 & (MAS – CC) >= 0

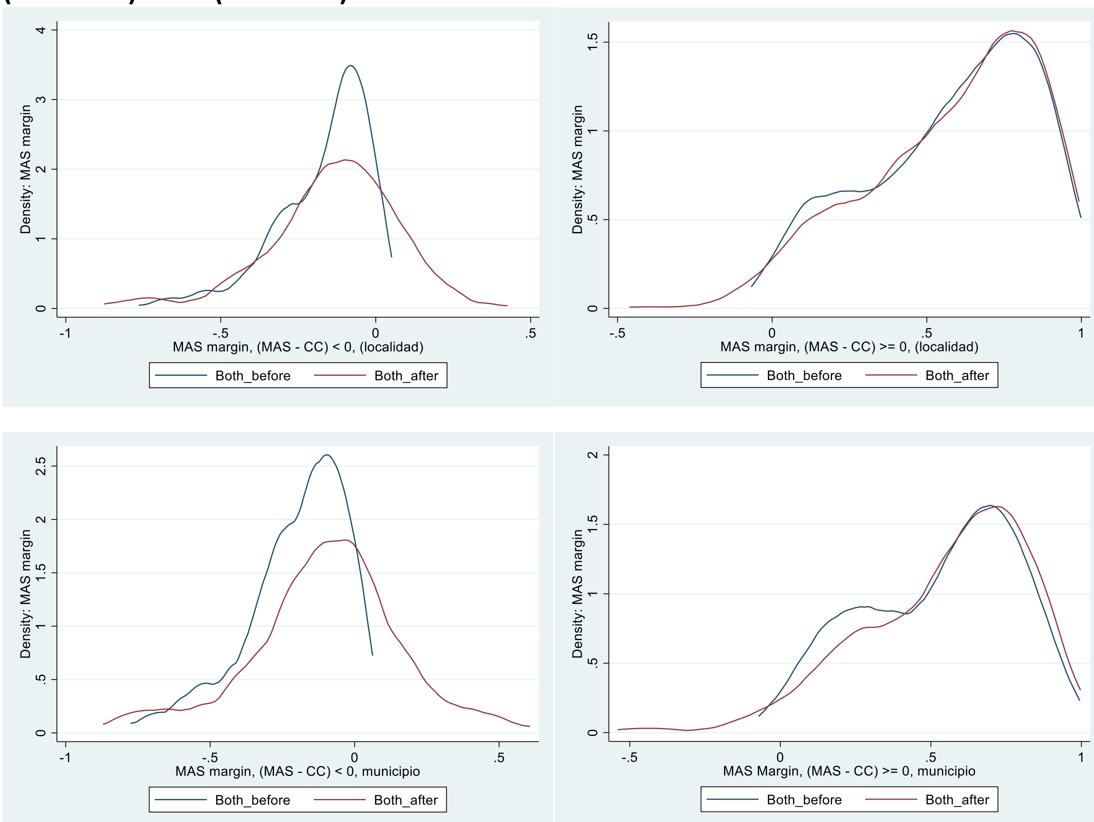


Table 9 presents the results of Kolmogorov-Smirnov tests for *localidades* and *municipios*. The pattern is exactly the same as with *recintos*, only stronger. As was the case for *recintos*, it is possible to reject the null hypothesis that the distribution of the margin after the cut-off is the same as before the cut-off for those *recintos* that supported CC before the cut-off, (MAS – CC) < 0, but not for those *recintos* that supported MAS before the cut-off. For *localidades* and *municipios*, the rejection of the null hypothesis is supported at the 1% significance level, a stronger result than was the case with *recintos*.

Table 9. Kolmogorov-Smirnov tests for *localidades* and *municipios*

	All observations		(MAS – CC) < 0 before cut-off		(MAS – CC) >= 0 before cut-off	
	Localidades (678 obs)	*Municipios* (315 obs)	*Localidades* (111 obs)	*Municipios* (48 obs)	*Localidades* (567 obs)	*Municipios* (267 obs)
Margin after						
D	0.0147	0.0095	0.0631	0.0417	0.0265	0.0375
P value	0.863	0.972	0.643	0.920	0.672	0.688
Margin before						
D	-0.0265	-0.0603	-0.2703	-0.3333	-0.0265	-0.0712
P value	0.620	0.318	0.000	0.005	0.672	0.259
Combined K-S						
D	0.0265	0.0603	0.2703	0.3333	0.0265	0.0712
P value	0.971	0.615	0.001	0.010	0.989	0.508

Table 10 presents the results of a Oaxaca-type decomposition for *localidades* and *municipios*. Compared to similar decompositions at the recinto level presented in Table 7, the change in the margin for *localidades* and *municipios* with (MAS – CC) < 0 is larger and the proportion of the change due to the weighted difference in margins is considerably larger. Part of the explanation of the increase in the relative importance of the difference in margins at a higher level of aggregation is that the dispersion of voting shares around the mean is lower for *localidades* and *municipios* than it is for *recintos*. This is apparent in the last two columns of Table 11 which present the coefficient of variation (a measure of dispersion around the mean) for the voting shares before the cut-off and after the cut-off. There is a big change in the coefficient of variation before and after the cut-off for *recintos* with (MAS – CC) < 0, with a much lower average value of the coefficient of variation and a much smaller difference in the coefficient of variation before and after the cut-off for *localidades* and *municipios*. The change in the coefficient of variation before and after the cut-off for *localidades* and *municipios* with (MAS – CC) >= 0 is larger than the change in the coefficient before and after the cut-off for *localidades* and *municipios* with (MAS – CC) < 0. This may partly explain why the proportion of the total change due to the weighted difference in shares is higher for those with (MAS – CC) > = 0.

Table 10. Oaxaca-type decompositions for *localidades* and *municipios*

	All observations		(MAS – CC) < 0 before cut-off		(MAS – CC) >= 0 before cut-off	
	Localidades (678 obs)	*Municipios* (315 obs)	*Localidades* (111 obs)	*Municipios* (48 obs)	*Localidades* (567 obs)	*Municipios* (267 obs)
Sum of weighted margin before, weighted by share of votes before (a)	0.0042	0.0743	-0.1258	-0.1024	0.1299	0.1767
Sum of weighted margin after, weighted by share of votes after (b)	0.1918	0.2462	-0.0410	-0.0275	0.2328	0.2737
Δ margins (b–a)	0.1876	0.1720	0.0848	0.0749	0.1028	0.0970
Sum of difference in margins (margin after - margin before), weighted by share of votes after (c)	0.0659	0.0755	0.0606	0.0562	0.0054	0.0193
Sum of difference in shares (share of votes after – share of votes before), weighted by margin before (d)	0.1216	0.0965	0.0242	0.0187	0.0975	0.0778
Proportion of Δ margins due to weighted difference in margins (c/(b–a))	35.16%	43.89%	71.48%	75.01%	5.22%	19.84%
Proportion of Δ margins due to weighted difference in shares of votes (d/(b-a))	64.84%	56.11%	28.52%	24.99%	94.78%	80.16%

Table 11. Weighted margins, Oaxaca-type decompositions and Coefficients of Variation for voting shares

	Weighted margin before	Weighted margin after	Change due to weighted difference in margins	Change due to weighted difference in voting share	Coefficient of variation of voting share before	Coefficient of variation of voting share after
All observations						
recintos	0.0682	0.1952	4.33%	95.67%	1.0669	0.8090
localidades	0.0042	0.1918	35.16%	64.84%	0.1379	0.1937
municipios	0.0743	0.2462	43.89%	56.11%	0.2324	0.3177
(MAS – CC) < 0						
recintos	-0.1292	-0.0960	8.42%	91.58%	1.5100	0.8786
localidades	-0.1258	-0.0410	71.48%	28.52%	0.2529	0.2715
municipios	-0.1024	-0.0275	75.01%	24.99%	0.3824	0.4088
(MAS – CC) >= 0						
recintos	0.1973	0.2912	2.92%	97.08%	0.8911	0.7802
localidades	0.1299	0.2328	5.22%	94.78%	0.1151	0.2489
municipios	0.1767	0.2737	19.84%	80.16%	0.2425	0.4188

The information from the Kolmogorov-Smirnov tests and the Oaxaca-type decompositions now provide the building blocks to construct a counterfactual estimate of what the aggregate margin would have been had the margins for those municipios supporting CC before the cut-off not changed. If the margins for (MAS – CC) < 0 had not changed, the aggregate value of -0.0275 would have been lower by 0.0562, yielding a value of -0.0837. This would have resulted in a national margin of 0.0967, instead of 0.1056. A second round would have had to be held.

Table 12. Counterfactual estimates indicate that a second round would have had to be held were it not for the change in distributions of margins before and after the cut-off for *municipios* supporting CC before the cut-off

	Share of votes in total votes (a)	Aggregate Margin for groups of municipios (b)	Counterfactual Aggregate Margin for Both After (if there had been no weighted difference in margins for (MAS – CC) < 0) (c)	Share of votes x Aggregate Margin (a x b)	Share of votes x Counter-factual Aggregate Margin (a x c)
Only before	2.24%	0.2467	0.2467	0.0056	0.0056
Only after	0.06%	0.2422	0.2422	0.0001	0.0001
Both before	81.76%	0.0743	0.0743	0.0607	0.0607
Both after	15.94%	0.2462	0.1900	0.0392	0.0303
(MAS – CC) < 0		-0.0275	-0.0837		
(MAS – CC) >= 0		0.2737	0.2737		
Total	100.00%			0.1056	0.0967

It is important to note that the Oaxaca-type decomposition suggested that for the *municipios* that favored MAS before the cut-off, close to 20 percent of the increase in the aggregate margin before the cut-off as compared to after the cut-off was due to a weighted change in the margins. This was not factored into the counterfactual estimate, because the Kolmogorov-Smirnov test indicated that one could not reject the null hypothesis that the distributions were equal. Only where there was support for the proposition that the change in distribution was significant, was the change incorporated into the counterfactual estimate. For the *municipios* supporting CC before the cut-off, the change in the distribution of margins before and after the cut-off was significant.

It is also important to note that the calculation of the margins before and after the cut-off involves exactly the same *municipios*. There are no changes in composition of *municipios*. Thus, whatever socioeconomic and political differences existed for the calculation of the margins before the cut-off remained the same for after the cut-off. There can be differences in the share of votes made by each *municipio* before and after the cut-off, but this is taken care of by the Oaxaca decomposition. The counterfactual estimate only took account of the difference due to the weighted difference in margins. It did not make an adjustment for the weighted difference in voting shares, which in the case of the group (MAS – CC) < 0 amounted to 25 percent of the total change.

Conclusions

The analyses presented in Curiel and Williams (2020a) and Williams and Curiel (2020b) are flawed. The figures from Nooruddin (2020) demonstrate that the change in the Morales vote margins before and after the suspension of recording of votes is considerably greater than suggested by the figures from Williams and Curiel. The core of their argument that "*there does not seem to be a statistically significant difference in the margin before and after the halt of the preliminary vote*" is based on assertions made from the correlation and scatterplots of the final margin with the margin before the halt and not from a precise statistical test. They also did not examine suspicious observations in their scatterplot, which can only exist if a *recinto* with a positive number of votes for CC before the cut-off is reported as having zero votes for CC in the final cómputo.

Using detailed data on votes from the Tribunal Supremo Electoral (TSE), this paper uses an appropriate and standard non-parametric statistical test, the Kolmogorov-Smirnov test, to test the null hypothesis that there are no differences in the distributions of margins before and after the cut-off. This is done separately for those *recintos*, *localidades* and *municipios* that favored CC before the cutoff and those that favored MAS before the cut-off. The results were consistent across all levels of aggregation of the administrative units - *recintos*, *localidades* and *municipios*. The statistical tests rejected the hypothesis that the distribution of the margins was the same before and after the cut-off for administrative units that favored CC, but not for administrative units that favored MAS.

A Oaxaca-type decomposition was carried out that also highlighted differences between the administrative units that supported CC before the cut-off and the administrative units that supported MAS before the cut-off. The Oaxaca decomposition allows the total change in margins over all *recintos*, *localidades* or *municipios* to be divided into a part that is due to a weighted difference in the margins (weighted by the voting share after) and a weighted difference in voting shares (weighted by the margin before). In all cases, the relative importance of the weighted difference in margins was higher for the administrative units that supported CC before the cut-off than for the administrative units that supported MAS before the cut-off.

The analysis of the TSE also allowed one to conclude that the suspicious data points in the Williams and Curiel are likely due to an artefact in the way they constructed their data sets. The analysis of the TSE data indicated that there was no *recinto* which had positive votes for CC before the cut-off and zero votes for CC in the final cómputo.

A simulation was carried out on a set of *recintos* that should have less risk of vote manipulation than the full TSE sample. A counterfactual cut-off that mimicked the effect of the actual cut-off was made. The same statistical test was applied for this group and no significant differences in the margins before and after the counterfactual cut-off were found for recintos that favored CC and that favored MAS before the counterfactual cut-off. Using the full sample and the actual cut-off, where there is a suspicion of voter manipulation, statistical tests show that there is a significant difference in the distribution of margins before and after the cut-off where CC was supported, but not where MAS was supported.

Using information from the Kolmogorov-Smirnov tests and the Oaxaca-type decompositions allows for a counterfactual estimate of the extent of the influence of the change in the distribution of margins before and after the cut-off for municipios that favored CC before the cut-off. The counterfactual estimates suggest that a second round would have had to be held were it not for the change in distributions of margins before and after the cut-off for *municipios* supporting CC before the cut-off.

Thus, in contrast to the analysis of Williams and Curie, the analysis presented in this paper and in Nooruddin (2020) and the irregularities in voting procedures found by the OAS investigative team indicate that Nooruddin and the OAS were correct to question the integrity of the 2019 Bolivian General Election.

ANNEX A

Information on the samples used in the analysis and the sample used by Williams and Curiel

Bolivia's Organo Electoral Plurinacional (OEP) provides information on the final results of the 2019 General Elections for all candidates. The results are presented below.

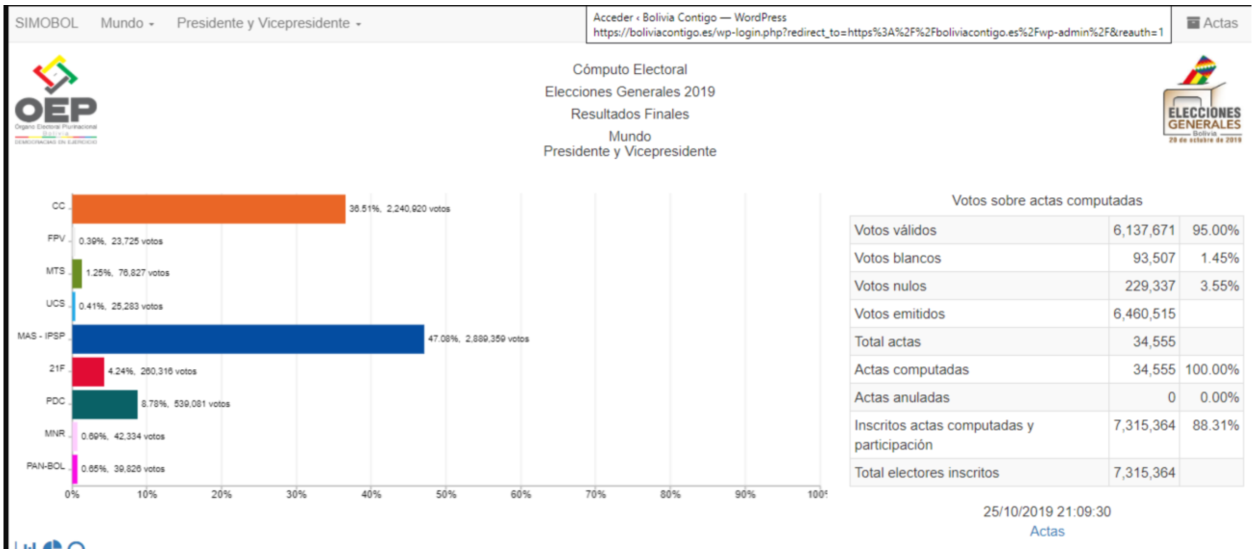


Table 3 from the body of the paper is reproduced below.

Table A1. Key characteristics of 3 sets of *recintos*

Category	*Recintos* (a)	*Mesas* before cut-off (b)	*Mesas* after cut-off (c)	Votes MAS (d)	Votes CC (e)	Total Votes (f)	Margin (d-e)/f
All votes before cut-off	3,182	12,449		971,892	775,875	2,066,192	0.0949
All votes after cut-off	618		1,122	104,519	24,358	154,166	0.5200
Votes before and after cutoff	1,496						
Votes before		16,520		1,386,260	1,175,695	3,089,174	0.0682
Votes after			4,464	426,688	264,992	828,139	0.1953
Subtotals		28,969	5,586				
Totals	5,296	34,555		2,889,359	2,240,920	6,137,671	0.1056

The key characteristics for the equivalent 3 sets of *recintos* from Williams and Curiel (2020b) are provided in Table A2. This is drawn from footnote 6 of their paper, which is reproduced after the table. Not as much detail was provided on the 3 sets of *recintos* by Williams and Curiel as is provided in Table A1.

Table A2. Key characteristics from 3 sets of *recintos* from Williams and Curiel (2020b)

Category	*Recintos* (a)	*Mesas* before cut-off (b)	*Mesas* after cut-off (c)	Votes MAS (d)	Votes CC (e)	Total Votes (f)	Margin (d-e)/f
All votes before cut-off	2,805					1,922,419	0.0885
All votes after cut-off	545					136,286	
Votes before and after cutoff	1,477						
Votes before						3,230,560	0.0729
Votes after						845,560	0.2012
Subtotals							
Totals	4,827					6,134,825	

Footnote 6 from Williams and Curiel (2020b) provides information on four categories of precincts. Information contained in this footnote is used in the construction of the synthetic sample to generate numbers of total votes, votes before the cut-off and margins before and after the cut-off. Details on how this information was used is provided in the annotated STATA program in Annex B.

Footnote 6 from Williams and Curiel, 2020b

[6] This is the arithmetic for solving the required margin to surpass 10 percentage points:
all reported before cutoff (margin = .08853, votes = 1922419)
unfinished before cutoff (margin = .07288, votes = 3230560)
unfinished after cutoff (margin = .201184, unfinished after votes = 845560)
all reported after cutoff (margin = ?, votes = 136286)

$$\frac{(a \times a_n + b \times b_n + c \times c_n + d \times d_n)}{(total\ vote)} = margin$$

$$a = \frac{(total\ vote \times .10 - b \times b_n - c \times c_n - d \times d_n)}{a_n}$$

$$a = \frac{(6134825 \times .10 - .08853 \times 1922419 - .07288 \times 3230560 - .201184 \times 845560)}{136286}$$

$$a = 0.2768113$$

The total number of votes in the four categories sums to 6,134,825, which does not match exactly the total number of votes reported by OEP, but is very close (99.95% of the official value).

ANNEX B

When a simulated cut-off at 84 percent of the vote was applied to *recintos* with all votes before the actual cut-off, no suspicious results were found

To provide additional information that could allow the reader to judge whether OAS was correct to question the integrity of the 2019 Bolivian Presidential Election, a simulated cut-off was applied to the set of *recintos* which had all of their mesas report votes before the cut-off. The simulated cut-off was set at the point when 84 percent of the mesas were collected for *recintos* reporting all their votes before the actual cut-off, thereby mimicking the same cut-off that occurred with the full sample. If no manipulation of votes took place before the actual cut-off, then there should be no difference in the distribution of margins before and after the simulated cut-off for recintos with (MAS – CC) < 0 and (MAS – CC) >= 0. That is, indeed, what was found.

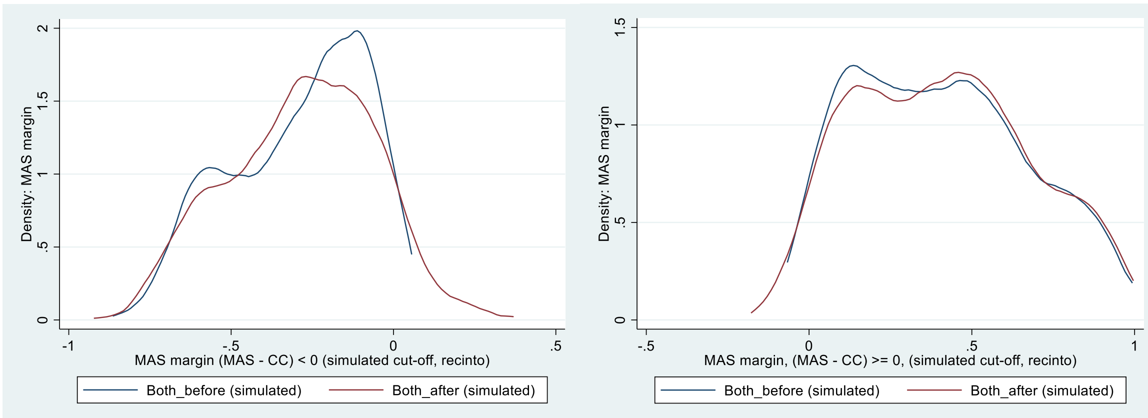
Table B.1 compares some aspects of the sample created by applying the simulated cut-off to the full sample of all *recintos*. There were 904 *recintos* that had mesas reporting votes both before and after the simulated cut-off. This amounts to 17.07% of all *recintos*, but 40.81 percent of all votes.

Table B.1. Comparison of *recintos* with votes recorded before and after simulated cut-off to the full sample

	Full sample		*Recinto*s with votes before actual cut-off and votes before and after simulated cut-off	
	Number	Percentage	Number	Percentage
Other countries	165	3.12%	16	1.77%
Bolivia	5,131	96.88%	888	98.23%
Beni	270	5.26%	53	5.97%
Chuquisaca	435	8.48%	14	1.58%
Cochabamba	727	14.17%	199	22.41%
La Paz	1,143	22.28%	190	21.40%
Oruro	365	7.11%	41	4.62%
Pando	161	3.14%	10	1.13%
Potosí	680	13.25%	71	8.0%
Santa Cruz	1,015	19.78%	252	28.38%
Tarija	335	6.53%	58	6.53%
Votes before			1,996,474	
Votes after			508,205	
Total votes			2,504,679	
Pct of votes in *Recinto*s with votes before actual cut-off and before and after simulated cut-off to total votes cast				40.81%

Figure B.1 presents the kernel density estimations of the distributions of margins before and after the simulated cut-off for recintos with (MAS – CC) < 0 before the simulated cut-off and (MAS – CC) >= 0 before the simulated cut-off.

Figure B.1.



The critical information in this simulation is provided in Table B.2. Based on the Kolmogorov-Smirnov tests, it is not possible to reject the null hypothesis that the distribution of the margins before and after the synthetic cut-off is the same, both for *recintos* favoring CC and for *recintos* favoring MAS.

Table B.2. Kolmogorov-Smirnov tests for *recintos* with votes recorded both before and after simulated cut-off

Kolmogorov – Smirnov tests Simulated cut-off *Recintos*	Simulated cut-off All observations (904 obs)	Simulated cut-off: *Recintos* where (MAS – CC) < 0 before cut-off (374 obs)	Simulated cut-off: *Recintos* where (MAS – CC) >= 0 before cut-off (530 obs)
Margin after			
D	0.0188	0.0455	0.0377
P value	0.726	0.0462	0.470
Margin before			
D	-0.0232	-0.0829	-0.0302
P value	0.614	0.077	0.617
Combined K-S			
D	0.0232	0.0829	0.0377
P value	0.968	0.153	0.845

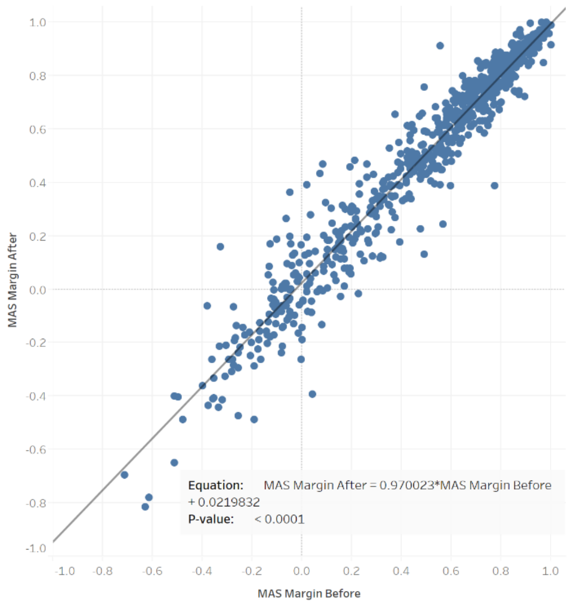
Table vB.3 presents results of the Oaxaca-type decomposition. The change in the aggregate margin for those *recintos* where (MAS – CC) < 0 is not as large as it was for the case where the actual cut-off was applied (as indicated in Table 7 in the body of the paper). In addition to there being no significant difference in the distribution of margins for this group, the proportion of the total change (0.0089) due to

Table B.3. **Oaxaca- type decompositions tests for *recintos* with votes recorded both before and after simulated cut-off**

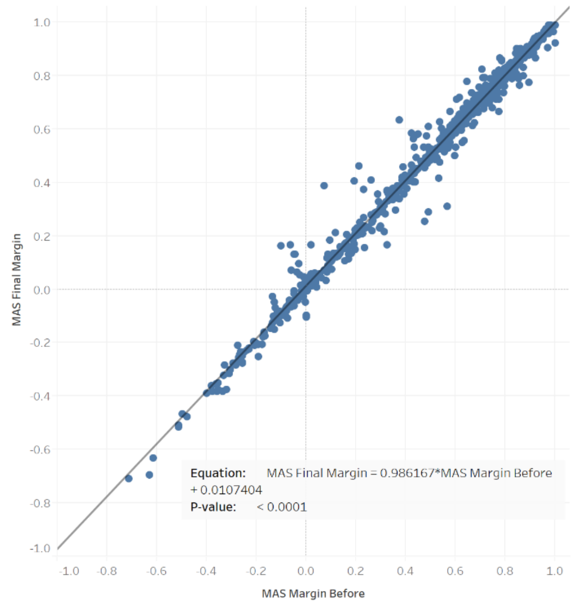
	Simulated cut-off *recintos* All observations (904 obs)	Simulated cut-off: *Recinto*s where (MAS – CC) < 0 before cut-off (374 obs)	Simulated cut-off: *Recinto*s where (MAS – CC) >= 0 before cut-off (530 obs)
Sum of weighted margin before, weighted by share of votes before (a)	0.0623	-0.1256	0.1299
Sum of weighted margin after, weighted by share of votes after (b)	0.0958	-0.1167	0.2328
Δ margins (b–a)	0.0336	0.0089	0.1028
Sum of difference in margins (margin after - margin before), weighted by share of votes after (c)	0.0063	0.0022	0.0054
Sum of difference in shares (share of votes after – share of votes before), weighted by margin before (d)	0.0272	0.0067	0.0975
Proportion of Δ margins due to weighted difference in margins (c/(b–a))	18.84%	24.85%	16.66%
Proportion of Δ margins due to weighted difference in shares of votes (d/(b-a))	81.16%	75.15%	83.34%

ANNEX C

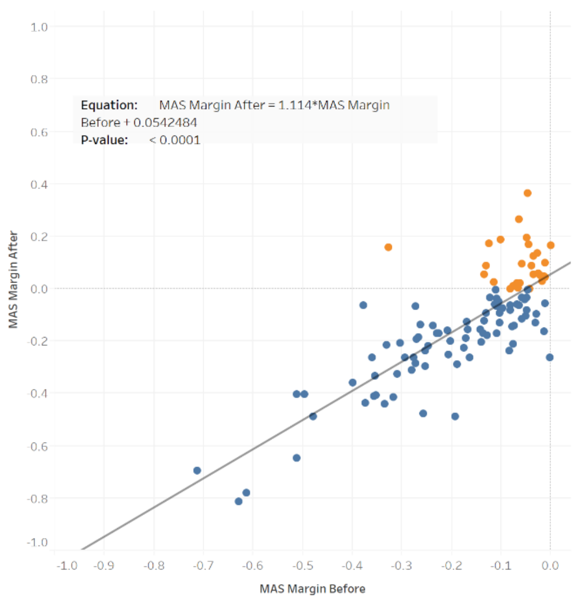
Scatterplot of MAS Margin Before and MAS Margin After
(*localidades*)
(Corr 0.9677)



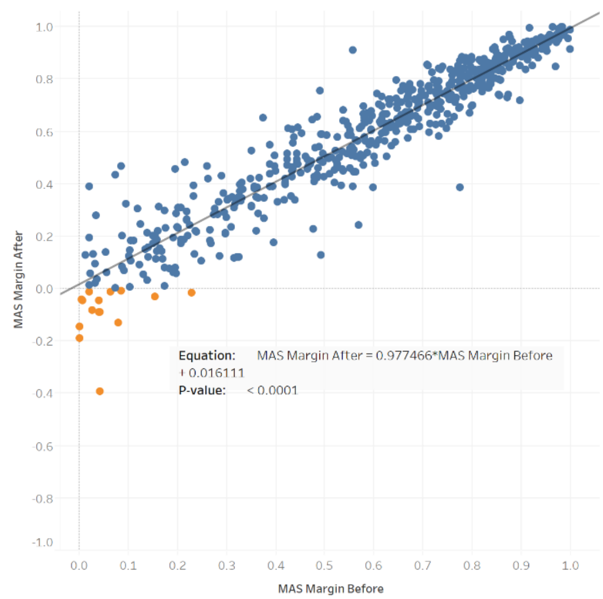
Scatterplot of MAS Margin Before and MAS Final Margin
(*localidades*)
(Corr 0.9927)



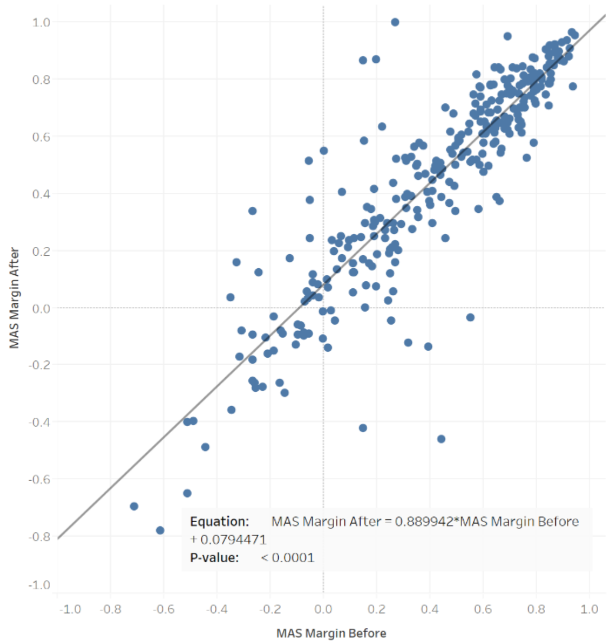
Scatterplot of MAS Margin Before and MAS Margin After
(*localidades*)
(Corr 0.8025)
(MAS votes less than CC votes before cut-off)



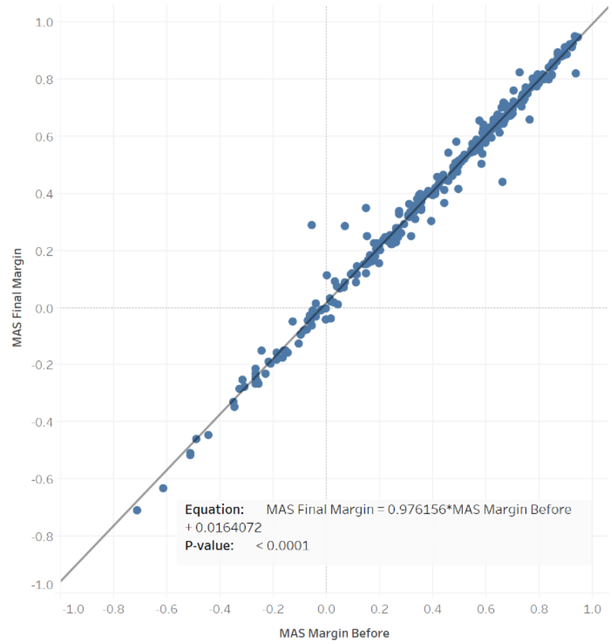
Scatterplot of MAS Margin Before and MAS Margin After
(*localidades*)
(Corr 0.9479)
(MAS votes greater than or equal to CC votes before cut-off)



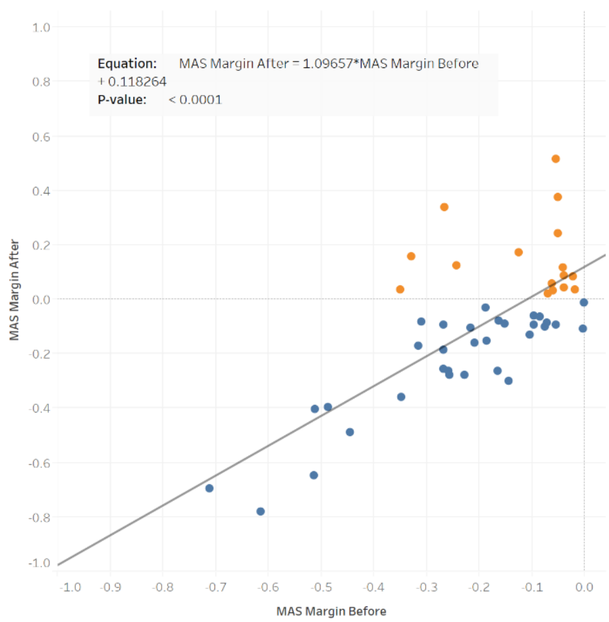
Scatterplot of MAS Margin Before and MAS Margin After
(*municipios*)
(Corr 0.8925)



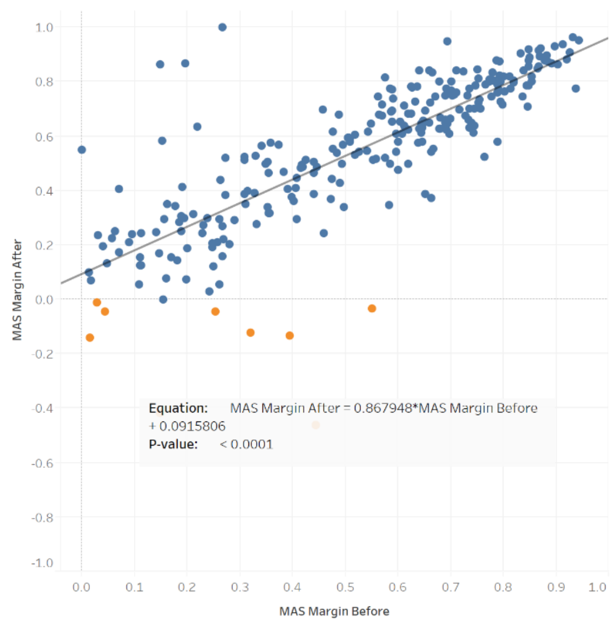
Scatterplot of MAS Margin Before and MAS Final Margin
(*municipios*)
(Corr 0.9940)



Scatterplot of MAS Margin Before and MAS Margin After
(*municipios*)
(Corr 0.7247)
(MAS votes less than CC votes before cut-off)



Scatterplot of MAS Margin Before and MAS Margin After
(*municipios*)
(Corr 0.8113)
(MAS votes greater than or equal to CC votes before cut-off)



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