



VOLTAGE STABILITY USING RECTANGULAR STATIC MODAL ANALYSIS FOR POWER SYSTEMS IN THE PRESENCE OF DISTRIBUTED GENERATION

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ABSTRACT

Static modal analysis has been a study tool used to identify critical buses or areas of an electrical system related to voltage stability. In this article, the methodology proposed is based on the rectangular coordinate representation of the nodal voltage phasors. With the expansion of Distributed Generation (DG) in Brazil and around the world, it is essential to evaluate the stability levels of electrical power systems. The aim of this proposal was therefore to use simulations to evaluate the voltage stability criterion, considering the insertion of DG into the system. Rectangular modal analysis was applied to evaluate the behavior of the system and identify the advantages and disadvantages that the allocation of a DG can bring to the network in terms of voltage stability. To implement the methodology, tests were carried out for rectangular modal analysis on a fictitious, simple distribution network with a radial topology, comparing the performance prior to the installation of a renewable source and its performance after insertion. The results obtained show that DG has a positive impact on the system, with an increase in the voltage stability margin, especially when the DG is allocated to the load buses.

Keywords: Static Modal Analysis; Distributed Generation; Voltage Stability; Rectangular Power Flow.

1 INTRODUCTION

The Electrical Power System (EPS) is an interconnected set of components designed to generate, transmit, and distribute electrical energy. Composed of generators, transmission lines, substations, and distribution networks, an EPS aims to reliably deliver electricity to consumers. Over time, the EPS has been operating under increasingly stressed conditions due to the rise in global energy consumption, making the need to maintain acceptable levels of voltage stability a key concern (Gao et al., 1992).

Voltage stability can be understood as the ability of an electrical system to maintain acceptable voltage levels for all buses in the network following a disturbance (Kopcak, 2003). A system enters a state of instability when load demand increases as a result of a disturbance, or when significant changes in the system's topology cause a progressive and uncontrollable voltage drop (Kundur, 1994). Ensuring appropriate voltage stability margins not only safeguards the operational reliability of equipment but also contributes to the overall quality of the energy supplied to consumers (Sena and Santos, 2024).

Among the accepted methodologies for assessing voltage stability, static modal analysis stands out. This method allows for the determination of critical buses, identifying locations with the greatest deficiencies and enabling reinforcement measures against instability. In this work, static modal analysis in rectangular coordinates is employed, which involves evaluating the critical eigenvalues and eigenvectors associated with the reduced Jacobian matrix derived from the Rectangular Power Flow



(RPF) solution at the singularity point. Successive demand increments are simulated to stress the system, resulting in different power flow solutions. From these solutions, the buses that may contribute to voltage collapse can be identified and classified (Sena and Santos, 2024). Another important assessment method for voltage stability is the PV curve approach, which enables the determination of the VSM. The PV curve represents the distance from the system's base operating point to its maximum loading point, and its derivation follows a process similar to modal analysis.

The expansion of DG in Brazil and worldwide has raised not only potential advantages but also concerns regarding system security criteria. This has brought to the forefront discussions about the benefits and drawbacks of integrating DG into the EPS, with voltage stability being a particularly relevant and debated aspect (Abreu, 2017). This issue has become especially important in distribution systems due to the increasing penetration of renewable energy sources. Consequently, evaluating voltage stability with DG integration emerges as a major challenge, requiring the application of adapted or alternative methods to traditional analyses.

This article investigates the behavior of voltage stability in electrical power systems considering the presence of distributed generation, identifying the system's critical buses and analyzing the behavior of the VSM. Critical buses are evaluated before and after the allocation of DG units in the network, assessing the potential advantages and disadvantages that DG may introduce for the system's critical areas.

2 METHODOLOGY

2.1 Voltage Stability and Voltage Stability Margin

Understanding voltage stability limits poses a challenge for power system operation. Therefore, the use of analysis methodologies is important to indicate the system's operating margin, representing its maximum loading capability and stability limit. In addition to methods capable of identifying critical areas—those prone to voltage collapse—stability analysis can also be applied to determine corrective measures that ensure acceptable security levels. Voltage stability studies are carried out using either dynamic or static analyses. Static analysis, used in this study, are associated with small disturbances, which allow for more simplified and fluid modeling, mostly addressing phenomena with a slow dynamic response (Sena and Santos, 2024). Even when limited to this kind of response, static methods also enable the definition of stability margins.

Among static methods, modal analysis stands out. This method indicates situations where stability is lost, allowing the identification of corrective and/or preventive actions to improve operational security. It involves calculating the critical eigenvalues and eigenvectors associated with the system's state matrix, which is obtained by solving the power flow equations.

Another widely used static method is the one that employs PV curves to assess the voltage stability margin. With this method, the loading levels at which the system can maintain stable operation can be identified (Kopcak, 2003). The PV curve represents the distance, in MW or percentage, from the system's base operating point to the point of maximum loading, commonly referred to as the “nose” of the curve. The PV curve is obtained through successive power flow solutions, considering an incremental increase in load and generation. It can be determined either for the system as a whole or for specific areas.



These analyses are widely used by agents in the electrical sector because they allow for defining the maximum demand capacity that the studied system can accommodate. The point commonly referred to as the “nose” of the curve defines the maximum loading—the maximum demand—that the system can withstand; at this point, the system’s Jacobian matrix becomes singular. Thus, the Voltage Stability Margin (VSM) can be defined as the maximum additional load that the system under study can support, measured as the difference between the base operating point and the point of voltage instability.

2.2 Distributed Generation

Distributed Generation (DG) is defined as a source of electrical energy connected directly to the distribution network (Vivenza and Gomes, 2022). DG can operate using various renewable energy sources. In this study, DG is understood as electrical energy generated at or near the point of consumption, mostly from renewable sources such as solar, wind, or hydropower. At its simplest, DG refers to a generation source located close to the load. In this study, DG is represented by the insertion of generation at the bus identified as critical—i.e., the bus prone to voltage collapse—to evaluate its effect on system voltage stability and whether this type of decentralized generation offers benefits or drawbacks to this parameter.

2.3 Rectangular Power Flow (RPF)

The power flow problem (PFP) is one of the most common analyses performed in electrical power systems (EPS), as its solution is fundamental to broader studies such as optimization, short-circuit analysis, contingency analysis, control, and network stability (Brigatto, 2023). Given the topology and the operating conditions of the studied system, solving the PFP enables the determination of bus voltage states as well as the powers generated, transmitted, and dissipated across the network (Brigatto, 2023). The conventional model of the PFP is based on a set of nonlinear algebraic equations, whose solution is obtained through iterative methods developed for these equations and their unknowns. In the basic PFP formulation, the state variables are the complex bus voltages (magnitude and phase angle) expressed in polar coordinates, but the formulation can also be reformulated in rectangular coordinates (Brigatto, 2023).

Based on the formulations presented in Brigatto (2023), the net active power injection P_k and the net reactive power injection Q_k are calculated as:

$$P_k = (e_k^2 + f_k^2)G_{kk} + e_k \sum_{m \in \Omega_k} (e_m G_{km} - f_m B_{km}) + f_k \sum_{m \in \Omega_k} (e_m B_{km} + f_m G_{km})$$

$$Q_k = -(e_k^2 + f_k^2)B_{kk} - e_k \sum_{m \in \Omega_k} (e_m B_{km} + f_m G_{km}) + f_k \sum_{m \in \Omega_k} (e_m G_{km} - f_m B_{km})$$

These are the basic equations of the rectangular coordinate power flow problem. From this formulation, similar to the polar approach, the Newton-Raphson Jacobian matrix is decomposed into four submatrices. Thus, the Jacobian [J] is expressed as:

$$[J] = -[H \quad N; M \quad L]$$

Elements of [H] and [N], based on partial derivatives of P_k :

$$[H] = \begin{cases} H_{kk} = \partial P_k / \partial e_k = 2e_k G_{kk} + \sum_{m \in \Omega_k} (e_m G_{km} - f_m B_{km}) \\ H_{km} = \frac{\partial P_k}{\partial e_m} = e_k G_{km} + f_k B_{km} \end{cases}$$



$$[N] = \begin{cases} N_{kk} = \frac{\partial P_k}{\partial f_k} = 2f_k G_{kk} + \sum_m \in \Omega_k (e_m B_{km} + f_m G_{km}) \\ N_{km} = \partial P_k / \partial f_m = f_k G_{km} - e_k B_{km} \end{cases}$$

For PQ buses, [M] and [L] are based on partial derivatives of Q_k :

$$[M] = \begin{cases} M_{kk} = \partial Q_k / \partial e_k = -2e_k B_{kk} - \sum_m \in \Omega_k (e_m B_{km} + f_m G_{km}) \\ M_{km} = \partial Q_k / \partial e_m = f_k G_{km} - e_k B_{km} \end{cases}$$

$$[L] = \begin{cases} L_{kk} = \partial Q_k / \partial f_k = -2f_k B_{kk} - \sum_m \in \Omega_k (e_m B_{km} - f_m G_{km}) \\ L_{km} = \partial Q_k / \partial f_m = -e_k G_{km} - f_k B_{km} \end{cases}$$

For PV buses, [M] and [L] are based on derivatives of V^2 :

$$M_{kk} = \partial(V^2) / \partial e_k = 2e_k; \quad M_{km} = 0$$

$$L_{kk} = \partial(V^2) / \partial f_k = 2f_k; \quad L_{km} = 0$$

Thus, the Jacobian matrix elements for the Rectangular Power Flow are defined, enabling, in this work, the analysis of eigenvalues and eigenvectors associated with this matrix (Sena and Santos, 2024).

2.4 Static Rectangular Modal Analysis

The static rectangular modal analysis allows for the identification of the system's critical areas, complementing voltage stability studies by indicating the weakest buses and the most effective reinforcement strategies (Sena and Santos, 2024). This method relies on the eigenvalues and eigenvectors of the Jacobian matrix. The eigenvalues indicate the proximity of the system to instability. In rectangular analysis, the smallest negative eigenvalue (closest to zero) represents the system's critical mode. The left and right eigenvectors associated with the critical eigenvalues are computed. From them, the participation factors (PF) of load buses (PQ type) are determined, providing insights into instability mechanisms. The PF of bus k in mode i is given by: $PF_{ik} = u_{ik} v_{ik}$, Where: u_{ik} is the right eigenvector of bus k associated with λ_i , and v_{ik} is the left eigenvector.

According to previous studies Sena and Santos (2024), PF can often be approximated by the right eigenvector, simplifying computations. The bus with the largest PF is identified as the system's critical bus. Therefore, with the rectangular PFP formulation, the system is stressed by gradually increasing load until maximum loading, where the Jacobian becomes singular. In this neighborhood, the smallest eigenvalue indicates the critical mode, and its eigenvectors provide the classification of buses contributing to instability risk. The bus with the highest magnitude participation factor corresponds to the critical bus Sena and Santos (2024).

2.4 Static Modal Analysis Algorithm in Rectangular Coordinates

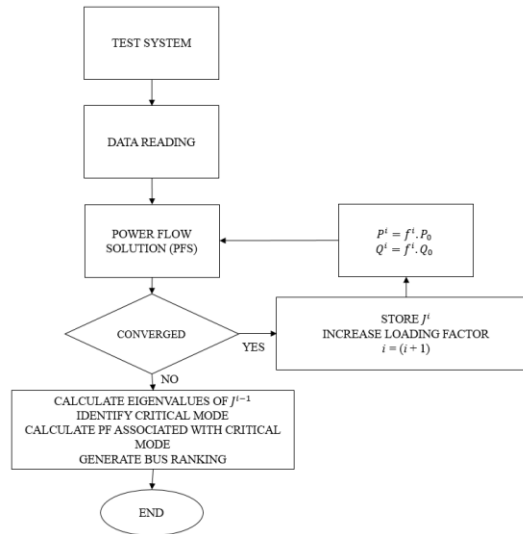
According to studies conducted by Sena and Santos (2024), the procedure for evaluating voltage stability can be summarized in the flowchart shown in Figure 1. The flowchart presents the algorithm for static modal analysis in rectangular coordinates, which begins with stressing the electrical system, allowing for the derivation of the PV curve and the calculation of eigenvalues and eigenvectors of the Jacobian matrix from the last solution of the Rectangular Power Flow (RPF).

If the step size of the loading factor increment is small, the stored Jacobian matrix will be close to the 'nose' of the PV curve or near singularity. This enables precise identification of the critical eigenvalue mode and its associated eigenvectors, which are then used to classify the system's buses. The non-convergence of the RPF represents the stopping



criterion of the algorithm, as it indicates the singularity point of the Jacobian matrix. The iterations of the algorithm vary depending on the selected step size and the analyzed system, since each system exhibits a different stability margin. For the test conducted in this study, a loading factor increment of 1×10^{-3} pu was adopted.

Figure 1. Flowchart of Static Modal Analysis applied to the RPF

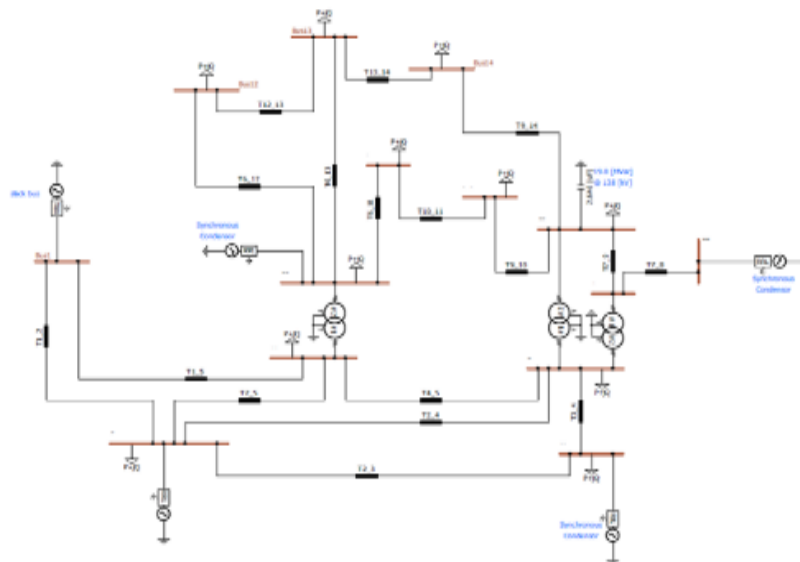


3 TEST AND RESULTS ANALYSIS

3.1 IEEE 14-Bus Test System

The test system selected corresponds to the 14-bus distribution system, as presented in Lima (2019). The topology of the system is represented in Figure 2.

Figure 2. Topology of the 14-bus test distribution system





The voltage stability analysis of the test distribution system was carried out using the proposed methodology. The case studies were performed in two stages: the first stage evaluated the voltage stability and voltage stability margin without the insertion of DG in the network; the second stage considered the insertion of DG at the system's load buses, with penetration levels of 1MW, 2MW, and 3MW.

3.2 System without DG

Following the proposed methodology, the voltage stability margin was initially obtained through the PV curve, as shown in Figure 3. According to Figure 3, the obtained margin was 298.50%, representing the maximum loading supported by the system. After obtaining the PV curve and performing Static Modal Analysis (SMA), the eigenvalue in the critical mode and the associated eigenvectors were determined, allowing for the classification of the buses. Figure 4 shows the bus classification according to participation factors.

Figure 3. PV Curve – System without DG

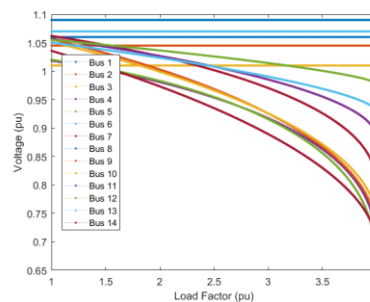
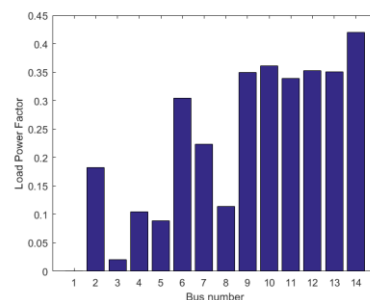


Figure 4. Bus classification – System without DG



Thus, it was identified that the system's critical bus, i.e., the bus most likely to experience voltage collapse, is bus number 14.

3.3 System with DG

In this second analysis, an additional generation of 1MW, 2MW, and 3MW relative to the total system load was considered, supplied by DG. The results of the stability margin are presented in Table 1.

Table 1. SMA results with DG insertion



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	2	3	4	5	6	7	8	9	10	11	12	13	14
1MW	298,70%	299,50%	299,80%	299,60%	300,40%	300,40%	300,40%	300,70%	300,80%	300,60%	300,50%	300,70%	301,40%
2MW	299,00%	300,60%	301,10%	300,60%	302,20%	302,20%	302,20%	302,90%	303,10%	302,70%	302,60%	302,80%	304,20%
3MW	299,20%	301,60%	302,40%	301,70%	304,10%	304,10%	304,10%	305,20%	305,40%	304,80%	304,50%	305%	307%

It can be seen that the insertion of DG into the system had positive impacts. The bus that benefited most from DG allocation was bus 14. The bus identified as the most likely to experience voltage collapse remained bus 14, regardless of the DG penetration level considered.

4 CONCLUSION

This research aimed to demonstrate the behavior of voltage stability in a distribution system when considering the insertion of Distributed Generation, by applying static modal analysis in rectangular coordinates.

The main motivation for this study was the increasing integration of DG into distribution systems in Brazil and worldwide. Tests were conducted on a fictitious IEEE 14 bus system, consisting of one reference bus, four PV buses, nine PQ buses, twenty branches, eleven loads (259 MW and 75 MVar), and one shunt capacitor (19 MVar). The results highlighted improvements in system behavior with the application of DG at load buses, showing an increase in the Voltage Stability Margin (VSM), which indicates a benefit to the system.

Overall, when analyzing the insertion of DG into the system, it was possible to observe an improvement in system stability. The connection of generation sources directly at the load proved beneficial to voltage stability, reinforcing the advantages of DG integration in distribution systems.

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