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Object-oriented data structure for adaptive FEM-SPIM coupling in physically nonlinear analysis

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Abstract. This paper presents the data structure developed for a coupled adaptive refinement strategy using Finite Element Method (FEM) and Smoothed Point Interpolation Methods (SPIMs) in physically nonlinear analysis. The solution of elastic degradation constitutive models adopted for this type of problem requires highly refined meshes that, when applied to the entire model, may lead to high computational cost. To overcome this, in the proposed approach, the domain starts with a coarse mesh of FEM and is only refined with a SPIM method in regions that can be automatically detected by different parameters. These regions generally exhibit higher stress concentrations and crack propagation. The implementations were carried out in the INSANE (INteractive Structural ANalysis Environment) system, an open-source software developed by the Department of Structural Engineering at the Federal University of Minas Gerais. The software follows the Object-Oriented Programming (OOP) paradigm using the Java language. The data structure is presented through UML diagrams and flowcharts describing the solution of the problem and the adaptive strategy. The phase-field constitutive model was considered, along with different refinement criteria.

Keywords: Object-Oriented Programming, Adaptive Strategy, Phase-Field Modeling.

1 Introduction

The Finite Element Method (FEM) is a well-established and computationally efficient numerical tool, but its dependence on a mesh can be a considerable drawback in simulations involving domains with complex forms or excessive distortion. On the other hand, Smoothed Point Interpolation Methods (SPIMs) offer more flexibility in these scenarios by constructing approximation functions from a cloud of nodes, avoiding mesh related issues.

The coupling between FEM and meshfree methods allows us to take the advantages of both methods, with the FEM being initially applied to the entire domain and then replaced by a refined SPIM mesh only in the regions where the crack propagation occurs. This substitution is performed automatically through adaptive mesh refinement strategies, which aim to minimize the problem of high computational cost associated with complex problems where the crack propagation path is unknown and the mesh needs to be highly refined. The use of coupled methods and adaptive meshes results in an optimized way to solve nonlinear analysis problems.

In Novelli [1], an adaptive FEM-SPIM coupling strategy is proposed for the phase-field modelling of fracture. The data structure implemented for the adaptive strategy is presented in this article. The implementations were performed in the INSANE (INteractive Structural Analysis Environment) system and will be presented through UML (Unified Modeling Language) diagrams and flowcharts.

2 Phase-field modeling

The phase-field is a variational approach to fracture based on Griffith's theory, where the variable ϕ, called phase-field, represents the crack geometry diffusely. This model is based on the total energy functional (E_t) shown in Eq. 1.

$$E_t = \int_\Omega \psi(\varepsilon(\mathbf{u}), \phi)\mathrm{d}V + \int_B G_c\gamma(\phi, \nabla\phi)\mathrm{d}V - \int_\Omega \mathbf{b}\cdot\mathbf{u}\mathrm{d}V - \int_{\partial\Omega} \mathbf{t}\cdot\mathbf{u}\mathrm{d}A, \tag{1}$$

where ψ is the energy density, ε represents the strain tensor, $\mathbf{u}$ is the displacement vector, G_c is Griffith's critical energy release rate, γ is the crack surface density function, $\mathbf{b}$ and $\mathbf{t}$ are the body and surface forces respectively.

The strain energy is related to the value of the phase-field through an energy degradation function, defined as $g(\phi)$. The fracture energy is governed by Griffith's critical energy release rate and the fracture surface density function $\gamma(\phi, \nabla\phi)$. For brittle and quasi-brittle materials, Wu [2] proposed:

$$\gamma(\phi, \nabla\phi) = \frac{1}{c_0}\left[\frac{1}{l_0}\alpha(\phi) + l_0|\nabla\phi|^2\right], \tag{2}$$

where $\alpha(\phi)$ is the geometric crack function and l_0 is the width of the diffuse region parameter.

Wu [2] also proposed a general formulation for the fracture surface geometric crack function (Eq. 3), which includes a numerical parameter ξ that varies between 0 and 2.

$$\alpha(\phi) = \xi\phi + (1-\xi)\phi^2 \in [0,1] \quad \forall\phi \in [0,1]. \tag{3}$$

3 Adaptive coupled FEM-SPIM

The adaptive FEM-SPIM coupling approach begins with an initial domain discretized by a coarse finite elements mesh. During the solution process, regions of interest are automatically detected, such as those with high stress concentrations or propagating cracks, and replaced with a refined SPIM discretization. This detection is based on various criteria, including the equivalent strain thresholds, or strain energy limits. In phase-field models, the criteria include the value of the phase-field variable itself or its combination with the strain energy history (Shao criteria).

Direct coupling between the FEM and SPIM domains is performed, obviating the need for transition elements. This is possible because the Point Interpolation Method (PIM) shape functions used in the SPIM domain possess the Kronecker delta property.

In the refinement strategy, new nodes are inserted into the domain, after which a new local triangulation is generated to refine the discretization. In this process, original nodes are never removed, as they store the crucial information required for subsequent triangulations, such as internal and field variables.

The entire adaptive process is integrated into the non-linear incremental-iterative procedure, such that checks for model modification occur after the convergence of a load step. If the discretization is modified, the analysis remains within the same load step, performing additional iterations to restore equilibrium before proceeding to the next increment.

4 Data structure

4.1 INSANE's numerical core

The numerical core of INSANE is represented by the UML diagram shown in Fig. 1. The four fundamental structures that compose the software are the abstract classes `Model` and `Solution`, and the interfaces `Assembler` and `Persistence`. The communication between them occurs through the Observer-Observable design pattern, where all changes in the `Model` and `Solution` classes (observables) are automatically passed on to `Persistence` (observer).

Input data provided to the software is read by the `Persistence` interface, which communicates with the `Model` and `Solution` classes. These classes are responsible for characterizing the model and the solution parameters of the problem, respectively. Once the model and solution information has been set, the `Assembler` interface builds the matrices and vectors required for the `Solution` class to solve the system of equations defined by Eq. 4.

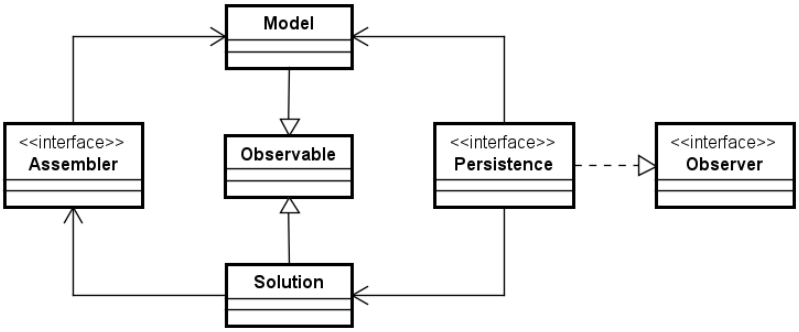


Figure 1. Numerical core of INSANE.

$$A \cdot \ddot{x} + B \cdot \dot{x} + C \cdot x = d, \tag{4}$$

where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are the systems coefficient matrices; $\mathbf{d}$ is the vector of independent terms; $\mathbf{x}$ is the vector of nodal dual parameters; $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the first and second time derivatives of $\mathbf{x}$. In static analysis, the process is simplified, resulting in Eq. 5.

$$C \cdot x = d \tag{5}$$

To incorporate the ability to perform adaptive analyses with FEM-SPIM coupling into the software, several changes and additions were made to the classes discussed in the following sections.

4.2 Persistence

The `Persistence` interface (Fig. 2) is responsible for processing input data, read from an XML file, and for generating output files. To support the representation of coupled models, Saliba [3] implemented the `CoupledMeshfreeFemModelPersistenceAsXml` class. Subsequently, Novelli [1] extended this framework to accommodate phase-field models. The contributions included enabling the reading of new data related to the parameters for the meshfree method, such as the type of smoothing domain, shape function and node selection schemes, as well as the parameters required for the adaptive analysis, including refinement criteria and levels.

The contributions included enabling the reading of new data such as steps, degenerations, and materials, as well as incorporating the phase-field variable into the `DiscreteModelPersistenceAsXml` class.

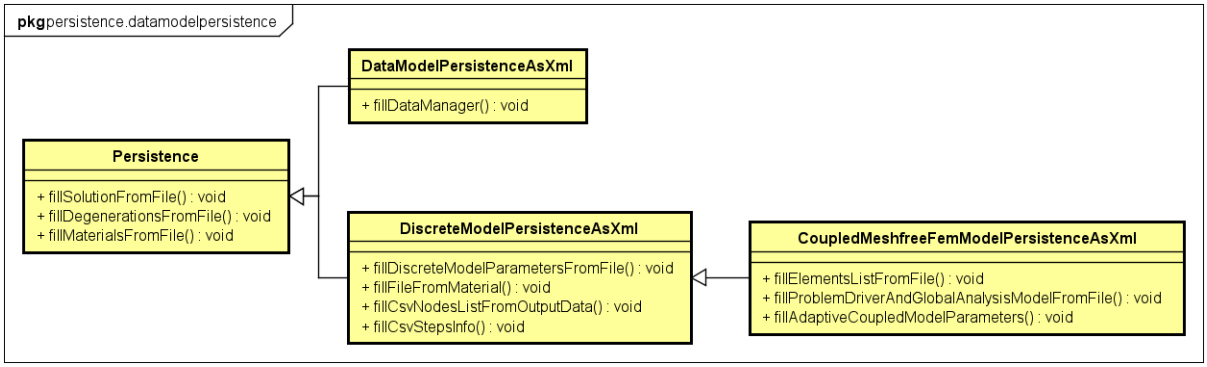


Figure 2. Modified classes from Persistence.

4.3 Model

The abstract class `Model` (Fig. 3) represents the discrete model, whose structure depends on the numerical method used. From it, classes with specific implementations are derived, such as: `FemModel`, which implements the model for the finite element method; `MeshfreeModel`, intended for meshfree methods with standard Gauss integration; `SmoothedPimMeshfreeModel`, which corresponds to the model for PIM meshfree methods with

smoothed shape functions; and `CoupledSmoothedPimMeshfreeFemModel`, which represents the coupled FEM-SPIM model. This latter class is central to the overall implementation. It was originally developed by Saliba [3] for constitutive damage models and later modified by Novelli [1] for the phase-field implementation.

To implement the phase-field model, Novelli [1] introduced modifications, such as adding a specific calculation in the `computedShapeFunction()` operation. Furthermore, new attributes and refinement criteria were added to the coupled model class. The `updateCoupledSmoothedModel()` operation is the responsible for checking and executing these updates or refinements in the model.

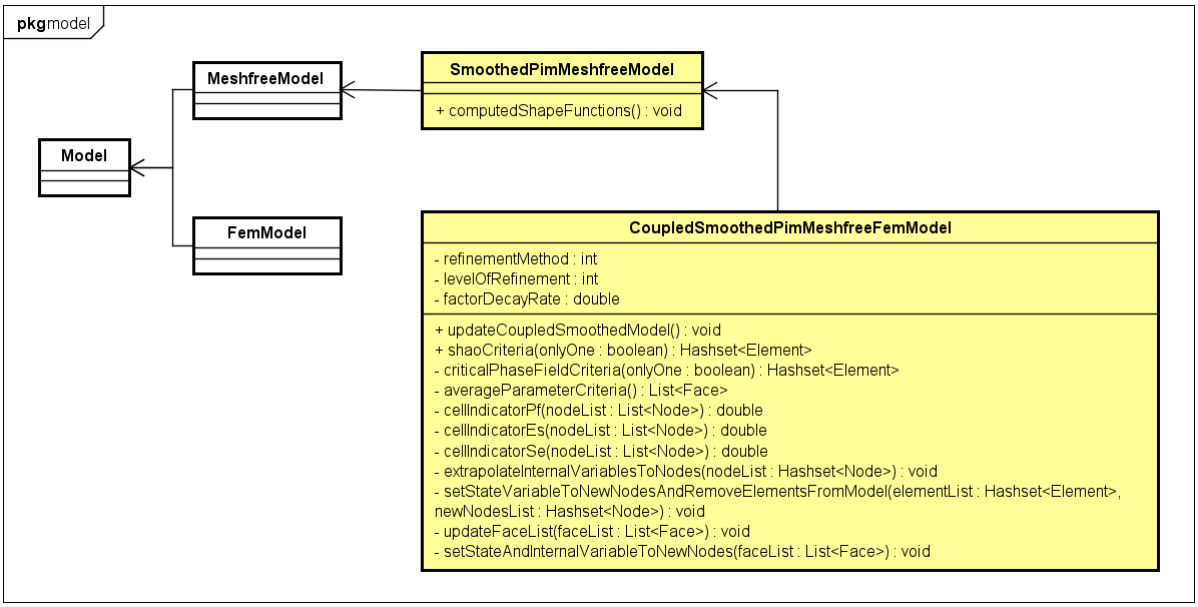


Figure 3. Modified classes from Model.

Other modifications were made by Novelli [1] to the classes related to the adaptive strategy. The `CoupledMeshfreeFemDataManager` class, along with its subclasses `CoupledMeshfreeFemModification` and `CoupledMeshfreeFemRefinement`, is responsible for modifying the model. The main changes included the addition of the phase-field variable to the nodes and a refinement level to control the process. Additionally, the `PimModelGenerator` class was also adjusted to add the phase-field's degree of freedom to the newly created nodes.

4.4 Assembler

The `Assembler` interface is responsible for assembling the global matrices and vectors required for the system solution. Specific classes that implement `Assembler` specialize this assembly process for each numerical method. Saliba [3] created new classes to handle the coupled FEM-SPIM methods, while Novelli [1] contributed with new classes that extend the meshfree method and contain new functionalities for the phase-field problem, as shown in Fig. 4.

The new classes introduced by Novelli [1] provide assemblers for the phase-field model, including both standard and sparse implementations. The classes `PhaseFieldMeshfreeAssembler` and `PhaseFieldMeshfreeAssemblerSparse` are responsible for phase-field operations in standard meshfree methods. Similarly, `PhaseFielSmoothedMeshfreeAssembler` and `PhaseFieldSmoothedMeshfreeAssemblerSparse` handle these operations for smoothed meshfree methods. The classes `PhaseFieldCoupledSmoothedMeshfreeFemAssembler` and `PhaseFieldCoupledSmoothedMeshfreeFemAssemblerSparse` serve as the assemblers for the coupled methods.

4.5 Solution

The abstract class `Solution` is responsible for solving the system of equations (Equation 4) assembled by the `Assembler` interface. Each implementing class specializes the solution procedure for different numerical methods. For non-linear static analyses, the `StaticEquilibriumPath` class manages the incremental-

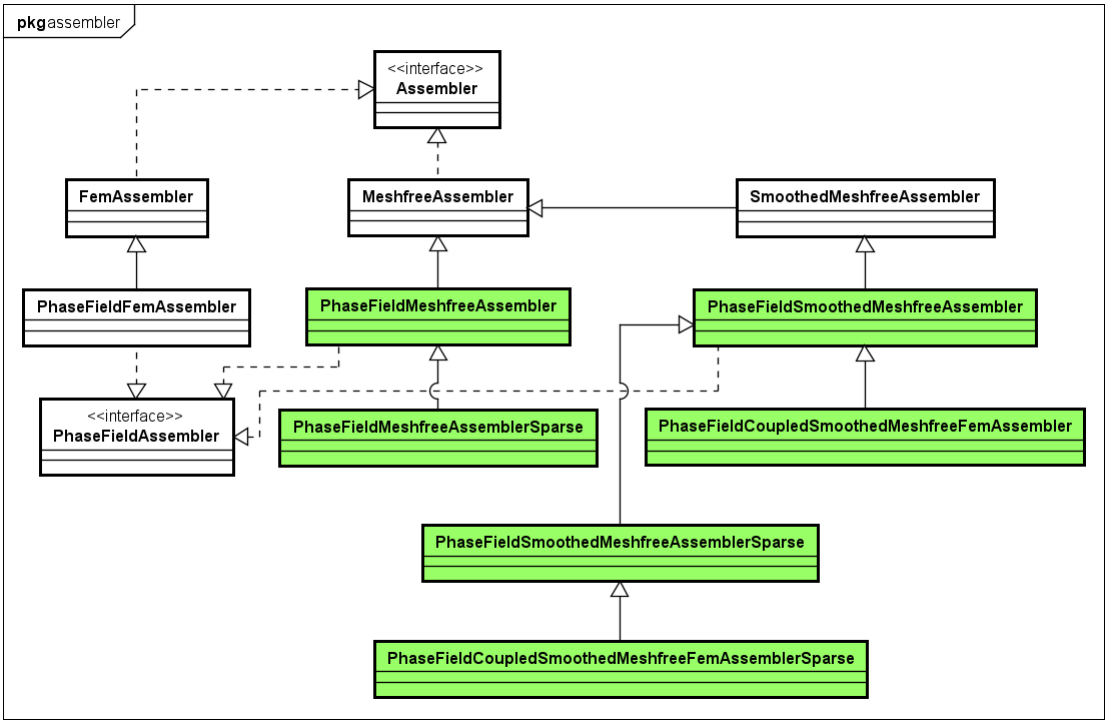


Figure 4. Modified classes from Assembler.

iterative process, utilizing an instance of a `Step` implementation to solve each load step.

To incorporate the adaptive strategy into the `Solution` framework, Saliba [3] developed the `StandardNewtonRaphsonCoupled` class. This was necessary because the standard Newton-Raphson implementation (`StandardNewtonRaphson`) is unsuitable for problems where the discrete model may undergo changes during the analysis. In his work, the mesh could be replaced (switched from FEM to SPIM) or refined (for SPIM) at the end of each step. In Novelli's [1] implementation, however, both mesh replacement and refinement can occur within a single step.

The adaptive strategy framework was extended by Novelli [1] for phase-field models (Fig. 5) as well as to allow for the insertion of multiple refinement levels. In this context, a check for model modification is performed only after the global convergence of a step is achieved, which includes the converge of both displacement and phase-field variables. If a modification is executed, additional iterations are performed within the same step to update and re-evaluate the convergence of the system.

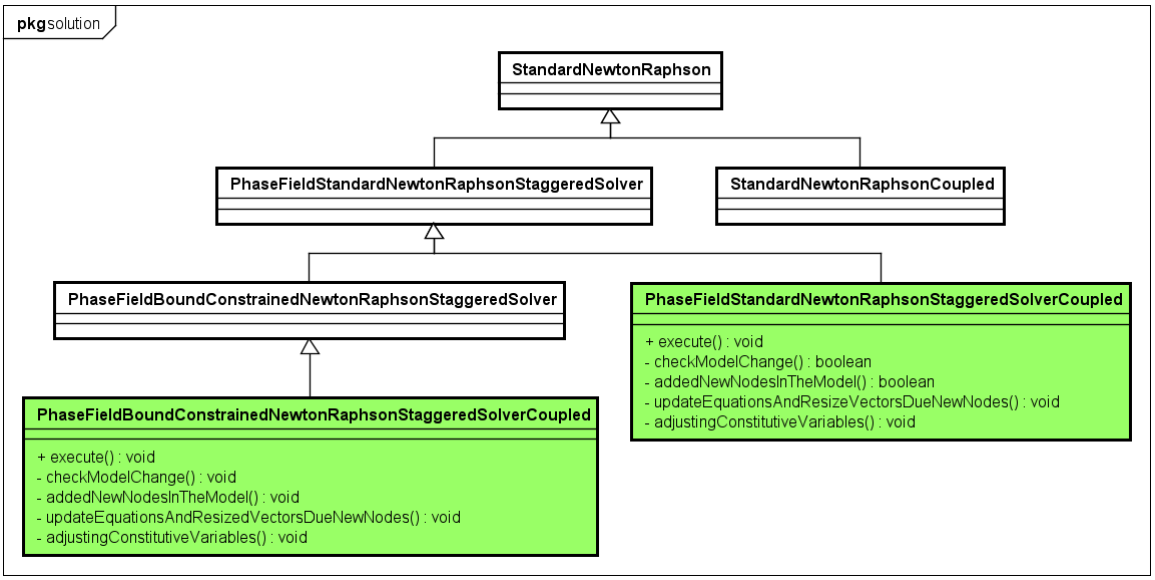


Figure 5. Modified classes from Solution.

5 Numerical simulations

This section presents numerical simulations of an L-shaped panel (Fig. 6 a) subjected to a vertical force, P, proposed by Winkler [4]. The material properties adopted for the analysis are: Young's modulus $E = 25.85kN/mm^2$, Poisson's ratio $\nu = 0.18$, critical energy release rate $G_c = 9.0x10^{-5}kN/mm$, $ft = 2.7x10^{-3}kN/mm^2$ and length scale parameter $l_0 = 10mm$. For the phase-field the Cornelissen energy degradation function was considered, with parameters $p = 2$, $a_2 = 1.3868$ and $a_3 = 0.6567$, and a crack geometry function with $\xi = 2$.

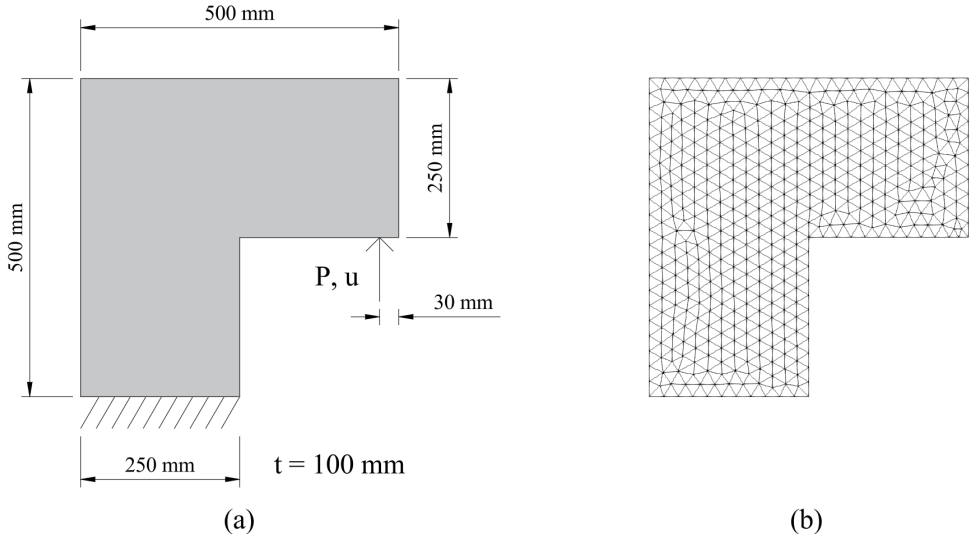


Figure 6. L-shaped panel: (a) Geometry, loading and boundary conditions, (b) Initial mesh.

The panel was subjected to simulations using Cell-Based T4, Edge-Based T3, and Edge-Based T36 adaptive schemes. The analyses started from the initial mesh shown in Fig. 6 b. The adaptive strategy parameters were set as follows: Shao's criterion was used to replace FEM elements with SPIM discretization, using a critical factor of 0.5 and a critical phase-field value of 0.05. The replacement scale was set to a factor of 2.0. The SPIM region was discretized using cell-based smoothing domains and RPIMp shape functions with an exponential radial basis (c=1). Subsequent adaptive refinement was triggered by the phase-field criterion, with maximum refinement level of 2 and a critical factor of 0.4.

The simulations, conducted under displacement control with increments of $\Delta u = 2.5 \times 10^{-3}$ mm, yielded equilibrium paths in good agreement with the reference numerical results from Wu [5] (Fig.7). The final crack patterns and the refined meshes, presented in Fig.8, are consistent with the expected behaviour for the test.

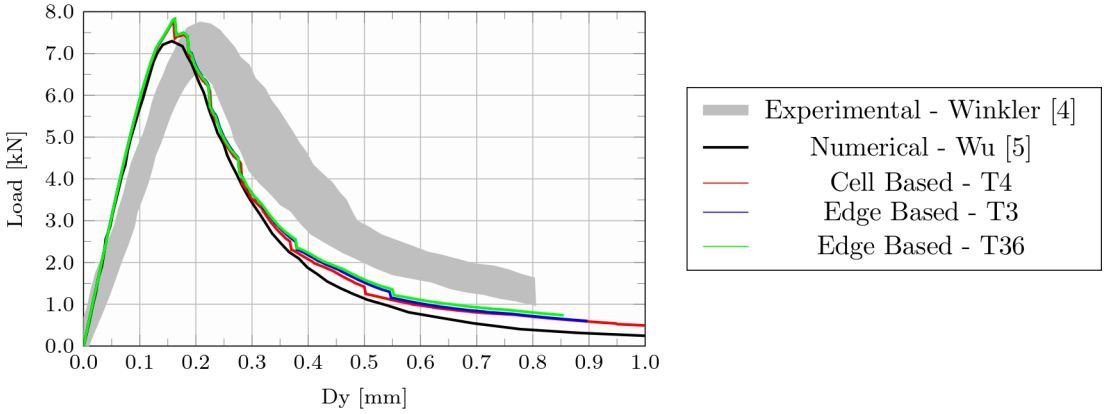
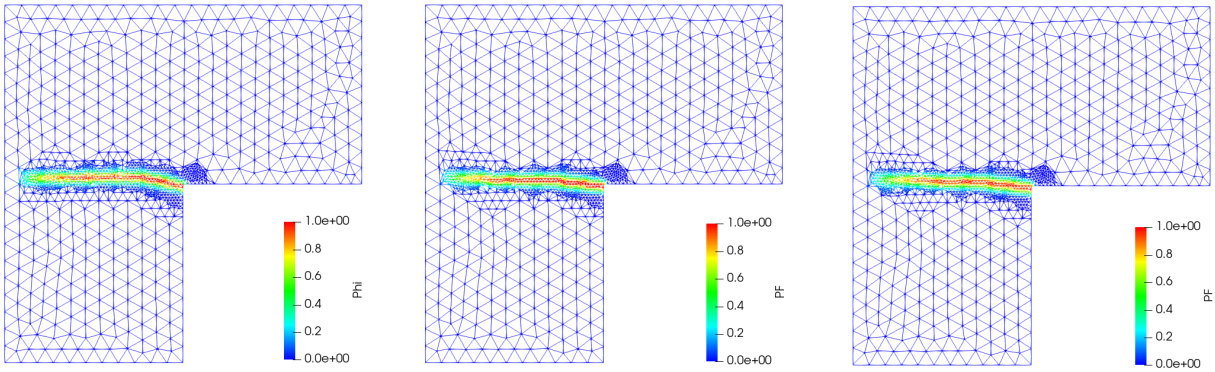


Figure 7. Equilibrium path.



(a) (b) (c)

Figure 8. Final mesh and phase-field contour plot for: (a) Cell Based T4, (b) Edge Based T3, (c) Edge Based T36.

6 Conclusions

An object-oriented data structure was developed and implemented for the adaptive coupling between the FEM and SPIM methods in the INSANE system. The object-oriented approach demonstrated to be effective and extensible, allowing the base structure, originally developed for damage models, to be successfully adapted for the analysis of fracture via phase-field models. The implementation supports multiple criteria for the substitution of elements and for the refinement of the mesh, conferring flexibility to the analysis strategy. The numerical simulation of an L-shaped panel with partially brittle behaviour validated the computational approach, with the equilibrium path and the final fracture pattern presenting good agreement with the reference results. The adaptive process, which starts from a coarse mesh in FEM and only refines locally with SPIM in the region of interest, was validated as a computationally efficient strategy for physically non-linear analyses.

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