XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING

Numerical simulation using nonlinear finite elements and machine learning for the evaluation of failure probability of reinforced concrete beams

Oswald Casaverde[1], Leandro L. Silva[1]

[1]*Dept. of Structural Engineering, University of Minas Gerais*
Antônio Carlos Avenue, 6627,31270-901, Minas Gerais/Belo Horizonte, Brazil
ocasaverde937@ufmg.br, leandro@dees.ufmg.br

Abstract. Reinforced concrete beams are critical structural elements subjected to bending and shear. Accurate prediction of their load-bearing capacity ensures safe and cost-effective designs. This study combines finite element analysis (FEA) and genetic programming (GP) to derive a predictive equation for the ultimate strength of reinforced concrete beams. A nonlinear FEA model was developed in Abaqus, evaluating the beam's capacity under varying parameters (compressive strength, yield stress, reinforcement area, and width). Simulation results were used to train a GP model, which generated a symbolic equation for strength prediction. This equation was then integrated into a structural reliability framework to compute the beam's probability of failure. Results demonstrate that the proposed approach achieves reliability indices exceeding thresholds set by international standards, validating its applicability for probabilistic design. The synergy of GP and FEA offers a computationally efficient alternative to traditional methods, particularly for complex nonlinear behaviors.

Keywords: Nonlinear finite element method, machine learning, reinforced concrete beam, structural reliability

1 Introduction

Reinforced concrete (RC) beams are essential structural components in civil engineering, designed to withstand bending moments and shear forces arising from gravitational and dynamic loads. Their structural performance is directly related to the safety, serviceability, and durability of buildings and infrastructure. The accurate prediction of their load-bearing capacity is therefore fundamental for the development of safer and more economical designs. However, due to the nonlinear behavior of concrete and steel reinforcement, this prediction remains a challenging task, especially when accounting for complex phenomena such as cracking, crushing, tension stiffening, and nonlinear bond-slip effects between steel and concrete [1].

Over the past decades, the use of nonlinear finite element analysis (NLFEA) has advanced significantly, providing engineers with powerful tools to simulate the mechanical response of RC beams with high accuracy [2–4]. These methods allow the incorporation of realistic constitutive models for concrete and reinforcement, such as the Concrete Damage Plasticity (CDP) model, enabling the analysis of different failure modes under varying loading and boundary conditions. Nevertheless, the computational cost and expertise required for NLFEA remain significant challenges in routine engineering practice.

In parallel, the field of artificial intelligence (AI) has experienced remarkable growth, with machine learning (ML) techniques proving effective in recognizing patterns and deriving predictive relationships from large datasets [5–7]. In structural engineering, these methods have been successfully applied to estimate material properties, predict structural capacity, and assess reliability without the need for exhaustive numerical simulations. Among these techniques, genetic programming (GP) has shown particular promise in generating symbolic regression models that capture the relationship between multiple influencing parameters and structural performance indicators [8, 9].

The integration of NLFEA and ML offers a promising hybrid approach: detailed numerical simulations can

be used to generate high-quality datasets that feed machine learning models capable of providing rapid predictions of beam capacity and reliability metrics. This synergy not only accelerates the design process but also enhances probabilistic safety assessments by quantifying the failure probability of RC beams under different conditions.

In this context, the present study aims to simulate the nonlinear behavior of RC beams using Abaqus with the CDP model, considering variations in compressive strength, yield stress, reinforcement area, and beam width. The generated numerical database is then used to train a genetic programming model to predict the beam's ultimate strength, which is subsequently applied in structural reliability analyses to estimate the probability of failure.

2 Calibration of nonlinear response of reinforced concrete beam

2.1 Experimental test

The experimental test was taken from the study conducted by Alcívar et al. [10], corresponding to a doubly reinforced concrete beam with a cross-section height of 400 mm, a width of 250 mm, and a total length of 3900 mm, with a clear span of 3500 mm.

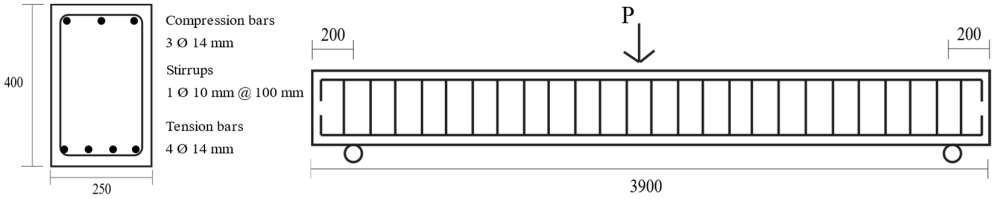


Figure 1. Beam geometry and reinforcing bar arrangement. Adapted from Alcívar et al. [10].

The reinforcement of the beam had a yield strength $f_y = 420$ MPa, arranged with four 14 mm bars (615.75 mm^2) at the bottom to resist tensile forces, and three 14 mm bars (461.81 mm^2) at the top to resist compressive forces. For transverse reinforcement, 10 mm diameter stirrups were used, spaced at 100 mm along the entire length of the beam. The modulus of elasticity of steel Es is 205939.65 MPa. The concrete of the beam had a compressive strength $f_c = 34.5$ MPa.

2.2 Numerical model – load and displacement curve

The numerical model was developed using Abaqus software. The concrete was modeled with CPS4R elements (4-node bilinear plane stress quadrilateral, reduced integration), while the reinforcement employed T2D2 elements (2-node linear 2D truss). The reinforcing bars were embedded in the concrete to simulate their bond interaction.

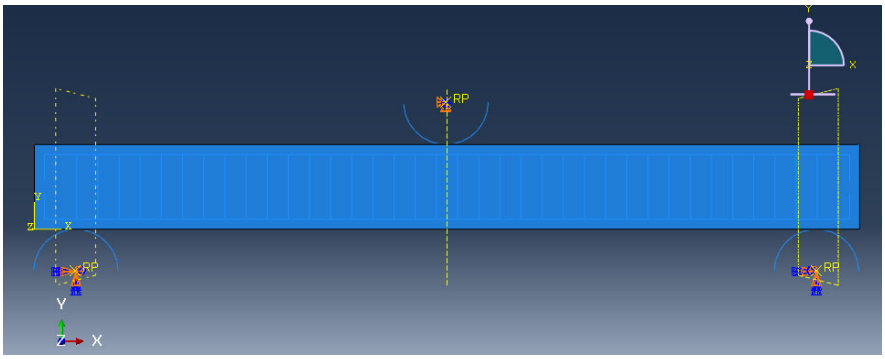


Figure 2. Numerical model in Abaqus

The finite element model was calibrated by comparing the load-displacement curves from the numerical model and the experimental test (see Fig. 3).

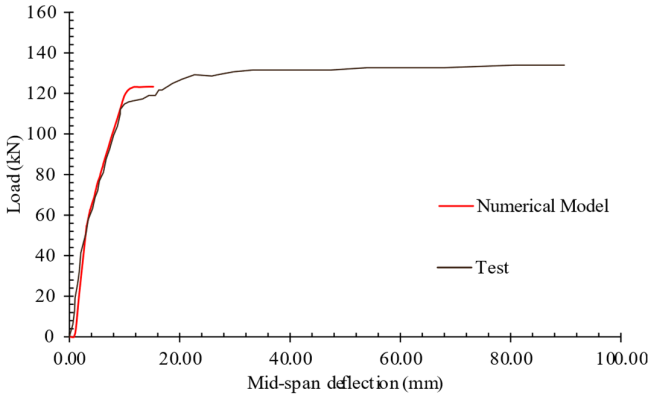


Figure 3. Load vs Mid-span deflection

As shown in Fig. 3, the finite element results are very close to the experimental model, showing strong similarity in the elastic branch. The ultimate load of the experimental model is 136.36 kN, while the ultimate load of the finite element model is 123.71 kN.

3 Predictive equation using genetic programming

With the calibrated model, a sensitivity analysis was performed. The number of simulations was determined using typical values of the main mechanical parameters employed in construction practice (see Tab. 1).

Table 1. Parameters analyzed

Concrete Compressive Strength fc (MPa)	Reinforcement area (mm^2)	Yield Stress fy (MPa)	Width (mm)
25	500	250	120
30	1000	420	200
35	1500	500	250
40	2000		

Using the interaction between Abaqus and the Python programming language, 144 simulations were carried out, serving as the database for the application of genetic programming. By employing the gplearn Python library for genetic programming, the following equation was generated to predict the capacity of the analyzed beam.

$$BC = \frac{A_s f_y b + 2A_s f_y f_c - 0.467 A_s b - 0.934 A_s f_c + 3 f_c^3 b + 3 f_c^4}{A_s f_y - 0.467 A_s + 3 f_c^2 b + 3 f_c^3} \quad (1)$$

In eq. (1), BC represents the beam capacity, As is the area of reinforcement bars, fc is the compressive strength of concrete, fy is the yield stress, and b is the beam width. The genetic programming model achieved an R^2 score of 0.64 on the test dataset, demonstrating satisfactory generalization capability for this complex structural reliability problem. While higher R^2 values approaching 0.90 could be attained by increasing the model complexity, the resulting symbolic expressions became excessively complex and impractical for reliability analysis. Specifically, these more complex equations proved unsuitable for reliability analysis using the First-Order Reliability Method (FORM) due to computational limitations in gradient calculations and convergence requirements.

4 Reliability analysis

With the generated equation, it is possible to perform a probabilistic analysis to calculate the probability of failure, or equivalently, the reliability index.

4.1 The structural reliability approach

Reliability analysis begins by defining a limit state, a performance criterion to determine if a structure is satisfactory [11]. This criterion is expressed mathematically as a limit state function, g(X), which defines the failure surface. For strength-related failures, this function is typically derived from structural mechanics, as shown in eq. (2).

$$g(X) = R(X) - S(X) \tag{2}$$

where R and S are the random variables that represent the resistance and the load effect, respectively, and X represents the vector containing all random variables such as those related to geometrical, material, load and workmanship parameters.

To perform a reliability analysis, the resistance (R) and the load (S) components of the limit state function (g=R-S) must be determined. The reliability problem can be expressed as the calculation of the probability of failure, pf, defined as pf = P[g(X)<0]. The reliability problem with n-dimensional vector X of random variables involved, and it can be expressed as follows:

$$p_f = P[R(X) \leq S(X)] = P[g(X) \leq 0] = \int \dots \int_{g(X) \leq 0} f_X(X) dX \tag{3}$$

Here $f_X(X)$ is the joint probability density function (PDF) for the n-dimensional vector X of basic variables. Even for the case of g=R-S, which considers only two random variables, these integrals are difficult to evaluate, in general [11]. Therefore, the probability of failure is calculated indirectly using other approach, in this study the reliability index (β) is used. The reliability index is related to the probability of failure by eq. (4).

$$p_f = \Phi(-\beta) \tag{4}$$

Calculated β are compared to target reliability indices (β_T) from design codes, which vary based on the consequence of failure and economic factors. Table 2 shows recommended target reliabilities in the guidelines for the development of limit states design in Canada [12], Table 3 shows the recommended values by the Joint Committee on Structural Safety [13].

Table 2. Target Reliabilities Indices from CSA S408 (2011) for 30-year (50-year)

Safety Class	Gradual Failure	Sudden Failure
Not Serious	2.5 (2.3)	3.0 (2.8)
Serious (normal buildings)	3.5 (3.4)	4.0 (3.9)
Very serious	4.0 (3.9)	4.5 (4.4)

Table 3. Target Reliabilities Indices from JCSS (2001a) for 50-year

Relative cost for enhancing the structural reliability	Minor failure consequences	Average failure consequences	Major failure consequences
Large	1.7	2.0	2.6
Medium	2.6	3.2	3.5
Small	3.2	3.5	3.8

4.2 Limit state function for gravity loads

The limit state function (g(X)) for reliability analysis can be re-cast as shown below:

$$g(X) = P_n - (P_D + P_L) \tag{5}$$

Here P_n is the capacity of the beam, and P_D and P_L are the dead and live load, respectively. The resistance part of the limit state function was calculated using the programming genetic technique. For the load part the optimal design approach is used.

4.3 Optimal design approach

The optimal design is defined as the most economical, yet still safe scenario, where the designer chooses the properties of the structural element and materials in such a way that the factored load is equal to the factored resistance [14].

Therefore, the amplification factors (for dead and live load) and the strength reduction factor of the material establish in the Brazilian concrete standard need to be used.

Table 4. Partial factors for actions according to NBR8681 [15]

Type of structure	Partial factor of permanent action (γ_g)	Partial factor of variable action (γ_q)
Large bridges	1.30	1.50
Type 1 buildings and bridges in general	1.35	1.50
Type 2 buildings	1.40	1.40

Table 5. Partial factors for strength according to NBR6118 [16]

Material	Partial factor for resistance (γ_R)
Concrete	1.40
Reinforcing steel	1.15
Structural steel	1.10

In this study and to be in agreement also with NBR6118 (Design of concrete structures), partial factors of permanent and variable action were taken as 1.40. The strength partial factors for concrete and reinforcing steel are 1.40 and 1.15 respectively, which imply strength reduction coefficients of 0.71 and 0.87 for these materials. In this study, due to the complexity of the beam capacity prediction equation, a single global strength reduction factor of 0.70 was adopted.

Using the optimal design approach the nominal dead and live load can be determined:

$$P_{D,n} = \frac{P_{fr}}{1.40 + 1.40\alpha} \tag{6}$$

$$P_{L,n} = \frac{P_{fr}}{1.40 + 1.40\alpha} \tag{7}$$

Where $P_{D,n}$ and $P_{L,n}$ are the nominal dead and live loads, $P_{fr}$is the factored load, and α is the live-to-dead load ratio that is taken equal to 1.00 in this research.

4.4 Analysis method

The first order reliability method (FORM) was used to assess the safety levels of the analyzed beam.

4.5 Statistical information for resistance

In this study, the statistical information for the load has been adopted from Santiago et al. [18]. The statistical information for the random variables used to calculate the resistance is summarized in Tab. 6.

Table 6. Statistical information for resistance

Random variable	Bias	COV	Distribution
C20	1.30	0.20	Normal
C30	1.22	0.15	Normal
C40	1.16	0.11	Normal
fy	1.22	0.04	Normal
b	1.00	0.010	Normal

4.6 Statistical information for loading

In this study, the statistical information for the load has been adopted from Bartlett et al. [19] and Galambos et al. [20]. Table 7 shows the statistical characteristics for loads.

Table 7. Statistical information for loads

Load	Bias	COV	Distribution
Dead Load	1.05	0.10	Normal
Live Load	1.00	0.25	Gumbel

5 Results

Figure 4(a) shows the effect of material compressive strength (varied from 25 to 40 MPa) on the reliability index for different reinforcement areas. Similarly, Figure 4(b) presents the effect of yield stress (varied from 250 to 500 MPa) on the reliability index, with the concrete compressive strength and beam width held constant at 35 MPa and 250 mm, respectively.

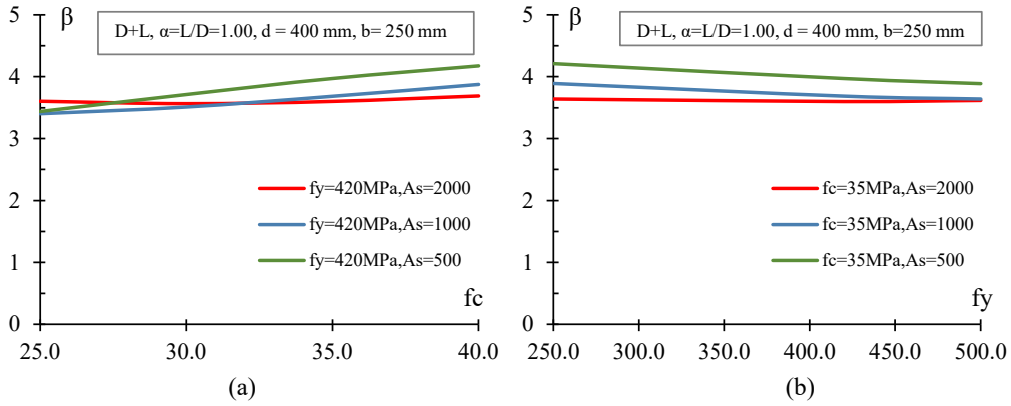


Figure 4. (a) Effect of the change in the concrete compressive strength on reliability index; (b) Effect of the change in yield stress on reliability index

As shown in Fig. 4(a), as the concrete compressive strength increases, the reliability indices also increase,

ranging from a minimum value of 3.44 to a maximum value of 4.17. Fig. 4(b) shows that, as the yield strength increases, the reliability indices tend to decrease, with values ranging from 3.62 to 4.21. The reliability indices from this analysis are compared to established targets: $\beta_T = 3.4$ for gradual failure and 3.9 for sudden failure per CSA S408, and $\beta_T = 3.2$ for medium-cost buildings per JCSS. The beam's calculated indices exceed these recommended values, indicating an adequate safety level for the load and material factors used in this study.

6 Conclusions

This study shows a relevant application of a machine learning technique—specifically genetic programming—in the field of structural reliability and its integration with the finite element method. The results indicate that the analyzed beam exhibits an acceptable probability of failure, as the calculated structural reliability indices exceed the threshold values recommended by international standards. Further research should be conducted to address more complex problems, such as considering the nonlinear response of structural systems and assessing the feasibility of developing analytical equations for estimating their probabilities of failure.

Acknowledgements. The authors gratefully acknowledge the financial support provided by the Brazilian agencies FAPEMIG, CAPES, and GCUB international mobility program.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] F. J. Vecchio, M. P. Collins. "The modified compression-field theory for reinforced concrete elements subjected to shear". *ACI Journal*, vol. 83, n. 2, pp. 219–231, 1986.
[2] ABAQUS Documentation, Dassault Systems, Providence, RI, USA, 2023.
[3] A. S. Genikomsou and M. A. Polak. "Finite element analysis of punching shear of concrete slabs using damaged plasticity model in ABAQUS,". *Engineering Structures*, vol. 98, pp. 38–48, 2015.
[4] V. Birtel and P. Mark. "Parameterised finite element modelling of RC beam shear failure," *ABAQUS Users' Conference*, 2006.
[5] J. Ghaboussi, J.H. Garrett, X. Wu. "Knowledge-based modeling of material behavior with neural networks". *Journal of Engineering Mechanics*, vol. 117, n. 1, pp. 132–153, 1991.
[6] A. C. C. Dantas, G.F. Medeiros, F.L.B. Ribeiro. "Artificial neural networks for predicting the ultimate strength of RC beams". *Ibracom Structures and Materials Journal*, vol. 12, n. 5, pp. 1088–1109, 2019.
[7] P.G. Asteris, K.G. Kolovos, M.G. Douvika, and K. Roinos. "Prediction of self-compacting concrete strength using artificial neural networks". *European Journal of Environmental and Civil Engineering*, vol. 20, n. 1, pp. 99–122, 2016.
[8] J.R. Koza. *Genetic Programming: On the Programming of Computers by Means of Natural Selection*. MIT Press, 1992.
[9] J.S. Rajeswari. Investigation on batter pile Foundation in liquefiable and Laterally spreading ground. Ph.D. thesis, Indian Institute of Technology, 2021.
[10] W.S. Alcívar, N.J. Párraga, and J.C. Chunga. "Comparación teórico-experimental del comportamiento a flexión de vigas de hormigón armado mediante gráficas momento-curvatura". *Revista Riemat*, vol. 6, n. 1, 2021.
[11] A.S. Nowak. *Reliability of Structures*. McGraw-Hill, Boston, US, 2000.
[12] CSA S408-11. Guidelines for the development of limit states design standards. Canadian Standard Association, Mississauga, Ontario, Canada, 2011.
[13] JCSS - Joint Committee on Structural Safety. Probabilistic Model Code. 2001.
[14] O. Casaverde. Comparison of Safety Levels of Reinforced Masonry and Concrete Walls Using Reliability Analysis. MSc. thesis, University of Alberta, Edmonton, Canada, 2023.
[15] ABNT NBR 6118. Projeto de estruturas de concreto - Procedimento, Rio de Janeiro, 2014.
[16] ABNT NBR 8681. Ações e segurança nas estruturas - Procedimento, Rio de Janeiro, 2003.
[17] H.O. Madsen, S. Krenk, and N.C. Lind. *Methods of structural safety*. Dover, New York, US, 1986.
[18] W. C. Santiago, H. M. Kroetz, and A. T. Beck. Reliability-based calibration of Brazilian structural design codes used in the design of concrete structures. *Ibracom Structures and Materials Journal*, vol. 12, n. 6, pp. 1288-1304, 2019.
[19] F. Bartlett, H. Hong, and W. Zhou. "Load factor calibration for the proposed 2005 edition of the National Building Standard of Canada: Statistics of loads and load effects." *Canadian Journal of Civil Engineering*, vol. 30, n.1, 429-439, 2003.
[20] T.V. Galambos, B. Ellingwood, B., J.G. MacGregor, C.A Cornell. "Probability based load criteria: Assessment of current design practice". *Journal of the Structural Division*, 108(5), 1982.