XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGENEERING

Many-objective truss structural optimization considering stability and vibrational aspects

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Abstract. Many-objective structural optimization problems are well-known and have been discussed in the literature. Concerning framed structures, most formulations consider two objective functions: minimizing weight and minimizing the maximum nodal displacement. This paper aims to expand on and discuss many-objective truss structural optimization problems (MaOSOPs) with seven objectives. The objective functions are the weight, the number of different cross-section types, the maximum nodal displacement, the first natural frequency of vibration, the first load factor concerning the structure's global stability, the sum of the absolute values of the first vibration mode, and the sum of the absolute values of the first stability mode. The sizing design variables are the cross-sectional areas of the bars, and the constraints are the nodal displacements and axial stress at the bars. The experiments are traditional benchmark problems in the literature. The algorithms used are the multi-objective meta-heuristic with iterative parameter distribution estimation (MM-IPDE), the success history–based adaptive multi-objective differential evolution (SHAMODE), the success history–based adaptive multi-objective differential evolution with whale optimization (SHAMODE-WO), and the third non-dominated sorting genetic algorithm (NSGA-III). Pareto fronts are obtained by providing the decision-maker with a set of non-dominated solutions that can be chosen according to their preferences.

Keywords: Many-objective structural optimization, automatic member grouping, differential evolution algorithms, global stability aspects, vibrational structural aspects.

1 Introduction

In this paper, a new many-objective truss structural optimization problem (MaOSOP) is proposed, formulated, and solved using four multi-objective evolutionary algorithms (MOEAs). To the best of the authors' knowledge, this MaOSOP has not been previously reported in the literature. Seven objective functions are considered in the formulation such as the structure's weight, the number of different cross-sectional areas of the bars, the maximum nodal displacement, the first natural frequency of vibration, the first load factor concerning the structure's global stability, the sum of the absolute values of the first vibration mode, and the sum of the absolute values of the first stability mode. The minimization of the weight of the structure is a conflicting objective function with the others considered in the formulation of the structural optimization problem proposed in this paper. That is, to minimize the number of different transverse sections, maximize the first natural vibration frequency, and also, the critical load factor implies a mass increase in the structure. On the other hand, minimizing the amplitudes of the first vibration mode and the first mode of global instability are conflicting objectives with the minimization of structural weight. The proposal of this many-objective structural optimization problem is justified considering the following aspects: (i) the minimization of the weight of the structure is undoubtedly the objective function of greater interest, besides being widely used, in structural optimization problems; (ii) the minimization of the

number of transverse sections of the bars will provide bars grouping that can generate savings in manufacturing, transportation, execution, welding, as well as architectural and aesthetic aspects; (iii) maximization of the first natural frequency of vibration and the first critical load factor regarding the global stability of the structure can avoid problems with structure resonance and instability, respectively; and, finally, (iv) the minimization of the amplitudes of the first vibration and instability modes can provide undesirable structural configurations.

To solve the proposed MaOSOP, four MOEAs are adopted, such as the multi-objective meta-heuristic with iterative parameter distribution estimation (MM-IPDE) [1], the success history–based adaptive multi-objective differential evolution (SHAMODE), the success history–based adaptive multi-objective differential evolution with whale optimization (SHAMODE-WO) [2], and the third non-dominated sorting genetic algorithm (NSGA-III) [3]. The results are presented in parallel coordinates for each algorithm and by the extreme points of the Pareto front (PF) for each objective function. In addition to the PFs considering the seven objective functions, a PF is also presented with only the objective function concerning the weight of the structure and the number of distinct cross-sectional areas of the bars. In the final unified PFs, the contributions of non-dominated solutions obtained from each MOEA are provided. Afterwards, a multi-criteria decision-making (MCDM) [4] is applied to extract the desired solutions from the Pareto front (PF), according to an artificial decision-maker DM's preferences. In this MCDM, two scenarios are established, and two non-dominated solutions are extracted, presenting the complete data for each one.

This paper is organized as follows: The formulation of the MaOSOP is defined in Section 2. The MOEAs and the MCDM adopted in this paper are briefly described in Section 3. Section 4 describes the computational experiment. Section 5 discusses the results, and, finally Section 6 reports the main inferred findings in this paper and future works.

2 Many objective optimization problem

The formulation of the proposed many-objective optimization problem is written as:

$$\begin{aligned} \min\ W(\mathbf{x}),\quad \min\ \text{ncs},\quad \min\ u_{\max}(\mathbf{x}),\quad \min\ -f_1(\mathbf{x}),\\ \min\ -\lambda_1(\mathbf{x}),\quad \min\ |\sum_{i=1}^{z}\phi_i(\mathbf{x})|,\quad \min|\sum_{i=1}^{z}\Delta_i(\mathbf{x})|\\ \textbf{s.t.}\quad \frac{\sigma_i(\mathbf{x})}{\bar{\sigma}}-1\leq 0,\quad 1\leq i\leq m_\sigma\\ \mathbf{x}^L\leq\mathbf{x}\leq\mathbf{x}^U, \end{aligned} \tag{1}$$

where W(**x**) is the total weight of the structure, written as:

$$W(\mathbf{x})=\sum_{i=1}^{N}\rho A_i L_i \tag{2}$$

where ncs is the number of different cross-sectional areas, $u_{\max}(\mathbf{x})$ the maximum nodal displacement, $f_1(\mathbf{x})$ the first natural frequency of vibration, $\lambda_1(\mathbf{x})$ the first load factor concerning the structure's global stability, $|\sum_{i=1}^{z}\phi_i(\mathbf{x})|$ the sum of absolute values of the first vibration mode, and $|\sum_{i=1}^{z}\Delta_i(\mathbf{x})|$ the sum of absolute values of the first stability mode, $\bar{\sigma}$ is the allowable axial stress at the bars. The sizing design variables are discrete defined by the lower $\mathbf{x}^L$ and upper $\mathbf{x}^U$ bounds. The evaluation of the eigenvalues of the matrix obtains the natural frequencies of vibration:

$$\left([K]-f_{m_f}^2\,[M]\right)\phi_{m_f}=0, \tag{3}$$

where $[K]$ is the elastic stiffness matrix, $[M]$ is the mass matrix, and ϕ_{m_f} is the m_f-th eigenvector corresponding to the m_f-th eigenvalue [5]. The load factors λ regarding the global stability are obtained by the evaluation of the eigenvalues of the matrix:

$$\left([K]+\lambda_{m_\lambda}\,[K_G]\right)\Delta_{m_\lambda}=0, \tag{4}$$

where $[K_G]$ is the geometric matrix of the structure, and λ_{m_λ} are the equivalent eigenvalues with respect to the m_λ load factors of the structure. The lowest value λ_{cr} of λ_{m_λ} gives the buckling load factor or critical load factor of the structure. The eigenvector Δ_{m_λ} represents the corresponding shape modes of the structure [6].

3 The multi-objective evolutionary algorithms and MCDM

Differential Evolution (DE) is one of the most widely used meta-heuristics for solving optimization problems. Originally introduced by Storn and Price [7], DE operates by generating and evolving a population of candidate solutions defined by continuous variables. Building upon prior research in the literature [8, 9], this study applies three DE-based multi-objective structural optimization algorithms (MOEAs) to address the MaOSOP formulated herein: the Success-History based Adaptive Multi-Objective Differential Evolution (SHAMODE) and its variant incorporating Whale Optimization (SHAMODE-WO), both proposed by Panagant *et al.* [2], as well as the Multi-Objective Meta-Heuristic with Iterative Parameter Distribution Estimation (MM-IPDE), introduced by Wansasueb *et al.* [1]. Except for NSGA-III, the three other MOEAs were adopted in this paper due to their superior performance in MOSOPs when compared with 13 other MOEAs recently analyzed by [10]. Some main aspects of these MOEAs are:

- SHAMODE [2] is an extension of the SHADE [11] algorithm for multi-objective structural optimization, featuring adaptive parameter control based on past successes and an external archive to store non-dominated solutions. SHAMODE-WO enhances SHAMODE by incorporating the Whale Optimization Algorithm to diversify the search process.
- MM-IPDE [1] combines a modified DE algorithm (MODE) with a population-based learning method (PBILM), running them in parallel — MODE explores solutions, while PBILM adapts DE parameters. It also uses multiple DE strategies, non-dominated sorting, and an external archive to manage non-dominated solutions.
- NSGA-III builds [3] upon NSGA-II by using fast non-dominated sorting to rank solutions into Pareto fronts. To maintain diversity, it introduces reference point association and niche preservation, especially when selecting solutions from the last front. It is well-suited for high-dimensional objective spaces, offers a well-distributed Pareto front, and does not require prior preference information from the decision-maker.

Constraint handling is performed using the constraint-based non-dominated sorting method [12] or the constraint dominance principle. Under this approach, for any two candidate solutions, x and y: (i) if x is feasible and y is not, x is preferred; (ii) if both are infeasible, the one with the lower total constraint violation is favored; and (iii) if both are feasible, the one that dominates the other according to objective function values is ranked higher. All algorithm parameters were set according to their respective original publications. No parameter sensitivity analysis or benchmarking of the adopted MOEAs was conducted.

The non-dominated solutions generated by these MOEAs are plotted using Pareto fronts and parallel coordinate plots. To identify preferred solutions from the Pareto front, the Multi-Criteria Tournament Decision (MTD) method, proposed by Parreiras and Vasconcelos [4], is employed. Originating from the Multi-Criteria Decision-Making (MCDM) framework, this approach ranks the best and worst solutions based on the objective function values and the weights (w_i), which are defined by the decision-maker (DM) according to the relative importance of each objective. The weights must satisfy the condition $\sum_{i=1}^{Nof} w_i = 1$, in which N_{of} denotes the number of objective functions. Further details about this method can be found in [4, 8, 13].

4 Computational experiment

The computational experiment is the traditional benchmark 10-bar truss, illustrated in Figure 1 and first proposed by [14]. The sizing design variables are the cross-sectional areas of the bars, which can be chosen from a discrete set of 42 options (in^2): 1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50. The material has specific mass ρ = 0.1 lb/in^3 and Young's modulus E = 10^4 ksi. There are vertical downward loads of 100 kips applied at nodes 2 and 4. The stress bounds for each bar are $\pm$ 200 ksi. For dynamic analyses, a non-structural mass of 1000 lb is attached to the free nodes. The experiment was evaluated in 20 independent runs with a population size of 100 individuals and 2000 generations.

5 Results

In this section, the results obtained are presented and discussed. The parallel coordinates for the dominated and non-dominated solutions are illustrated in Figure 2 for each algorithm, while Figure 3 provides a 3D visualization of the solutions provided by all MOEAs. It is possible to observe how the lines are widely spaced on the y-axis and how the objectives are conflicting. Table 1 shows the extreme values of each objective function, which corresponds to the bounds of the parallel coordinates in the vertical axis. MM-IPDE presents the values with more

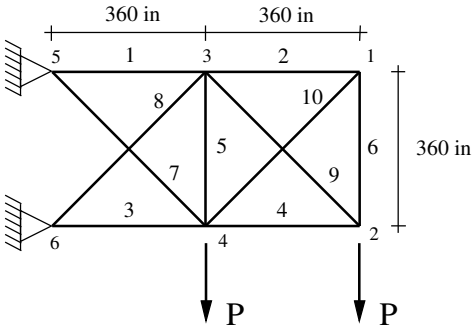
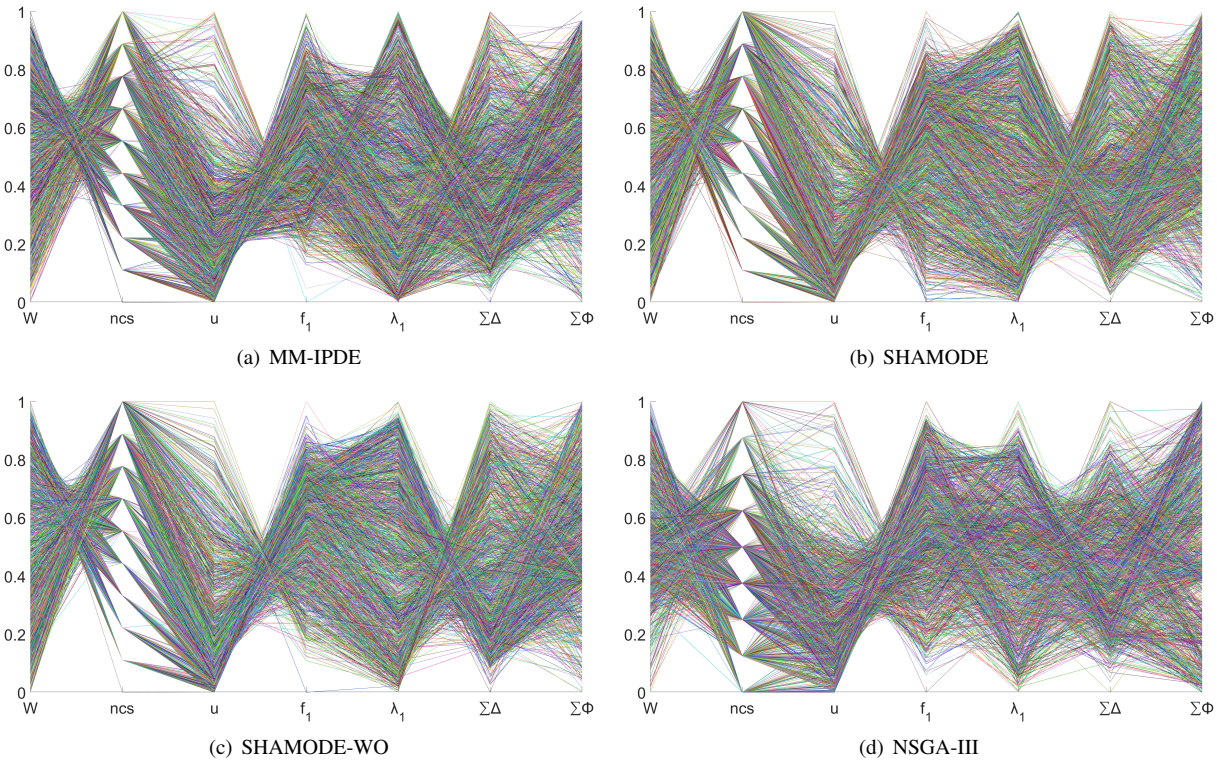


Figure 1. The 10-bar truss.

amplitude. Table 4 indicates the contribution of each MOEA to the unified PF from the complete set of feasible solutions. The percentages are similar, which means competitive performance of the algorithms.

After the unified PFs, an artificial decision-maker must choose the non-dominated solutions according to their preferences. This procedure is based on multi-criteria decision-making (MCDM), as proposed by Parreiras and Vasconcelos [4]. Two scenarios are set to extract the desired non-dominated solutions, simulating the preferences of an artificial DM. They are applied respectively to W(**x**), ncs, $u_{\max}(\mathbf{x})$, $f_1(\mathbf{x})$, $\lambda_1(\mathbf{x})$, $|\sum_{i=1}^{z} \phi_i|$, $|\sum_{i=1}^{z} \Delta_i|$ as follows: (i) Scenario 1: 70%, 5%, 5%, 5%, 5%, 5%, 5% and (ii) Scenario 2: 40%, 10%, 10%, 10%, 10%, 10%, 10%.



(a) MM-IPDE (b) SHAMODE

(c) SHAMODE-WO (d) NSGA-III

Figure 2. Parallel coordinates of solutions for the 10-bar truss.

Figure 4 provides a PF considering only the weight ($W(\mathbf{x})$) and ncs objective functions. This FP was extracted from the unified FP to illustrate how the decrease in the structure's weight is proportional to the increase in ncs, that is, the lower ncs, the greater the structure's weight.

Table 2 provides the design variables (dv) and Table 3 the objective functions (obj) values of the extracted non-dominated solutions. Both solutions extracted are from NSGA-III, which indicates the good performance of the algorithm for many-objective optimization problems. These extracted solutions are illustrated by the members of the bars in Figure 5. The difference in the weights of the trusses is 911.013 lbs, representing 25.3% less material needed. This also represents one more cross-sectional view and three different bars in the model, which affect processes such as fabrication, transportation, and execution in the field. Therefore, it is possible to notice the

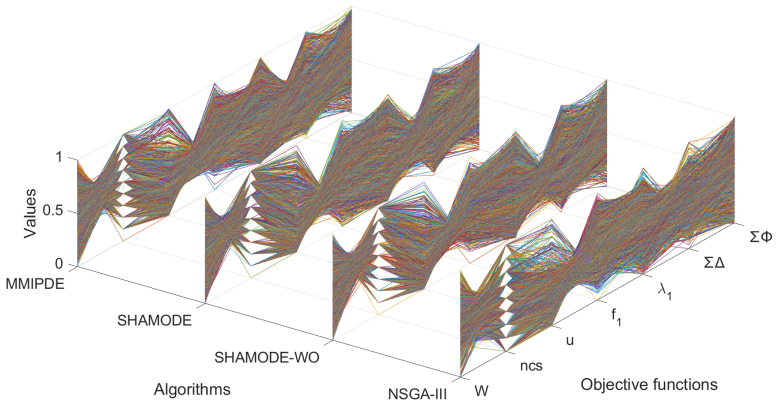


Figure 3. PFs for dominated and non-dominated solutions.

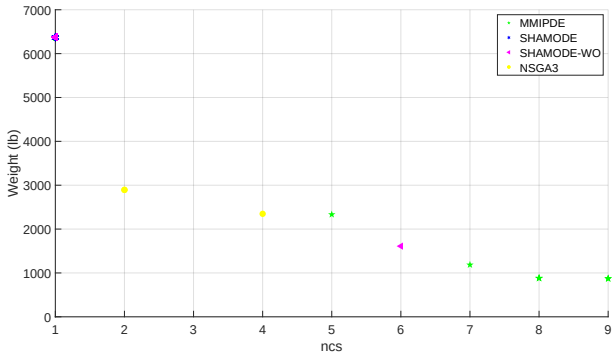


Figure 4. Pareto front of the non-dominated solutions considering $W\mathbf{x}$ and ncs.

conflict faced by the decision-maker when considering a MOSOP, as the objective functions are conflicting and the results provide benefits in some aspects but disadvantages in others.

Table 1. Extreme non-dominated solutions of the PFs.

		MM-IPDE	SHAMODE	SHAMODE-WO	NSGA-III
$W(\mathbf{x})$	min	872.492	969.573	927.598	1098.135
	max	6374.395	6374.395	6374.395	5541.086
ncs	min	1	1	1	3
	max	10	10	10	10
$u_{\max}(\mathbf{x})$	min	0.030	0.030	0.030	0.032
	max	0.172	0.167	0.169	0.155
$f_1(\mathbf{x})$	min	7.003	7.834	7.632	7.879
	max	14.412	14.041	14.041	14.341
$\lambda_1(\mathbf{x})$	min	163.944	174.548	180.016	216.719
	max	2804.937	2374.371	2364.195	2059.012
$\lvert\sum\phi_i\rvert(\mathbf{x})$	min	0.059	0.060	0.060	0.062
	max	0.098	0.097	0.097	0.095
$\lvert\sum\Delta_i\rvert(\mathbf{x})$	min	1.133	1.669	1.614	1.461
	max	6.144	5.014	5.180	5.752

dv (in)^2	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Scenario 1	13.90	33.50	33.50	7.97	2.63	2.63	13.90	13.90	7.97	13.90
Scenario 2	7.97	33.50	33.50	7.97	33.50	33.50	13.90	13.90	7.97	13.90

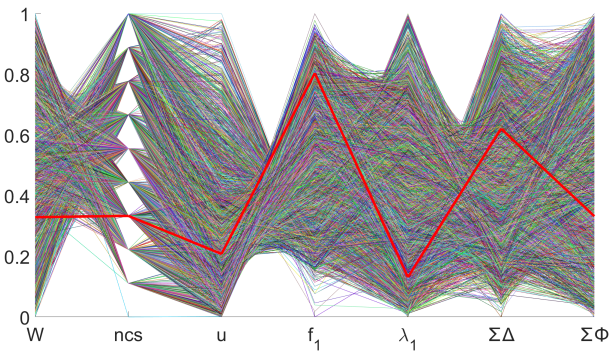
Table 2. Design variables of the extracted non-dominated solutions.

obj	$W(\mathbf{x})$ (lbs)	ncs	$u(\mathbf{x})$ (in)	$\lambda_1(\mathbf{x})$	$f_1(\mathbf{x})$ (Hz)	$\lvert\sum\phi_i\rvert(\mathbf{x})$ (in)	$\lvert\sum\Delta_i\rvert(\mathbf{x})$ (in)
Scenario 1	2683.156	4	0.060	13.180	510.576	0.083	2.761
Scenario 2	3594.169	3	0.054	13.331	1305.177	0.073	2.119

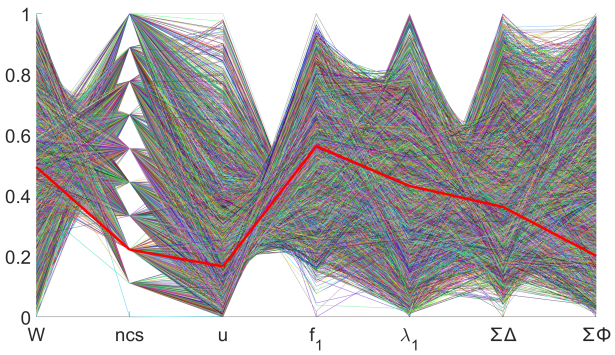
Table 3. Objective of the extracted non-dominated solutions.

Table 4. Contributon of each MOEA to the unified PF.

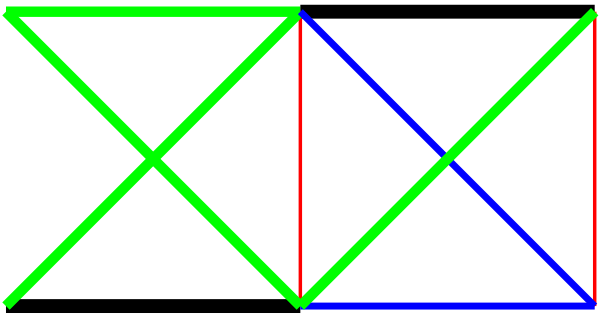
MM-IPDE	1519 (22.87%)
SHAMODE	1734 (26.11%)
SHAMODE-WO	1748 (26.32%)
NSGA-III	1640 (24.70%)
Set of non-dominated solutions	6641 (100.00%)
Complete set of all solutions	7820



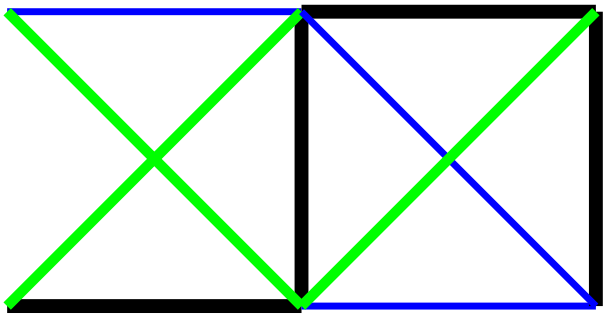
(a) Scenario 1



(b) Scenario 2



(c) Scenario 1



(d) Scenario 2

Figure 5. PFs using parallel coordinates and visual representation of extracted solutions for the scenarios.

6 Conclusions

This paper proposed and solved a new MaOSOP considering seven objectives: the total weight of the structure, the number of different cross-section types, the maximum nodal displacement, the first natural frequency of vibration, the first load factor concerning the structure's global stability, the sum of absolute values of the first vibration mode, and the sum of absolute values of the first stability mode. To the best of the authors' knowledge, this MaOSOP has not yet been found in the literature. The computational experiment was evaluated by four MOEAs: MM-IPDE, SHAMODE, SHAMODE-WO, and NSGA-III. The results were non-dominated and dominated solutions obtained for each algorithm, represented in parallel coordinates and presented from both ends of the PF. A PF of the non-dominated solutions concerning the total weight of the structure and ncs is also illustrated. The dominance of non-dominated solutions was analyzed by contributions for each algorithm from the complete set of solutions obtained. Two solutions were extracted according to the decision maker's preferences. Future work can be conducted to explore other structures and evolutionary algorithms in the context of the MaOSOP formulation presented in this paper.

Acknowledgements. The authors wish to thank the Federal University of Juiz de Fora and CNPq (Grants 308105/2021-4, 303221/2022-4, and 4128462/2024-8) for their support.

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