XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING

494-Modeling and Simulation of a Landing Gear System with Euler-Lagrange Vector Bond-Graph Approach.

Yuri Silva Souza[1], Daniel Bueno Silveira Lima[2], Luiz Carlos S. Góes[1]

[1]*Instituto Tecnológico de Aeronáutica-DCTA*
Pc. Mal-Ar Eduardo Gomes, 50. 12228-900 S.J.Campos, SP-Brazil
yuri.meca@gmail.com; goes@ita.br
[2]*Empresa Brasileira de Aeonáutica*
EMBRAER, 12227-901, S.J.Campos, SP-Brazil
daniel.lima@embraer.com.br

Abstract. This work aims to develop and simulate a model of landing gear system during an aircraft touch down. During an aircraft development process, many experimental tests are made to ensure the functionality and safety of the landing gear systems. Instead of using iron bird prototypes to perform the tests, using digital twins as high-fidelity model simulations to predict their physical behavior is safer and less expensive. This work applies the vector bond-graph method to model the landing gear dynamics during touchdown. The aircraft landing gear dynamics are modeled as a multi-body system. The methodology uses modern modeling tools based on a combination of vector bond graph representation allied to the Euler-Lagrange variational method for system dynamics modeling. The system consists of a planar aircraft model composed of three main modules: a nose landing gear, the airframe, and the aircraft's main landing gear. The landing gear system was first modeled using a vector bond graph representation, and the resultant vector bond graph was analyzed from the Euler-Lagrange standpoint by determining the system's generalized coordinates and total Lagrangian function. Symbolic algebraic processing was applied to determine the coupled nonlinear differential equations. The energy interactions of all system elements were calculated to generate the equations of motion in the form of Euler-Lagrange equations. The analytical equations were implemented in a MATLAB/Simulink to simulate a typical aircraft landing condition. A vertical load was applied to the model, simulating a smooth aircraft touchdown. When the aircraft speed reached 40 m/s, and the vertical load reached the aircraft's total weight, braking force was applied to decelerate the aircraft until it stopped. The simulation results showed the effectiveness of the proposed methodology. The combined vectorized Lagrangian bond graph modeling can be applied to the Multiphysics domain typical of several aeronautical systems of interest.

Keywords: Landing gear, Vector Bond-Graph, Euler-Lagrange.

1 Introduction

The objective of this work is to develop and simulate a model of landing gear system during an aircraft touchdown. During an aircraft development process, many tests must be done to ensure the functionality and safety of the onboard systems. Instead of using prototypes to perform the required tests, it is safer and less expensive to use high-fidelity digital twins, to simulate and predict the system behavior. In this endeavor, it is presented the development, of an aircraft landing gear system representation by a tricycle landing gear type shown in Figure 1.

According to this generic planar representation, the airframe is represented by a concentrated mass. It will carry all the aerodynamic forces of a generic aircraft. Both, the main and nose landing gear consist on two main elements. A shock absorber (suspension), represented by a pair spring-damper and the wheel-tire assembly, represented by a concentrated mass and spring. There is still another pair of spring-damper, to represent the structural flexibility in longitudinal axis, as depicted in Figure 1. It is important to observe that for this case study,

roll and yaw moments were not considered. The possible degrees of freedom are pitching motion, longitudinal and vertical displacements.

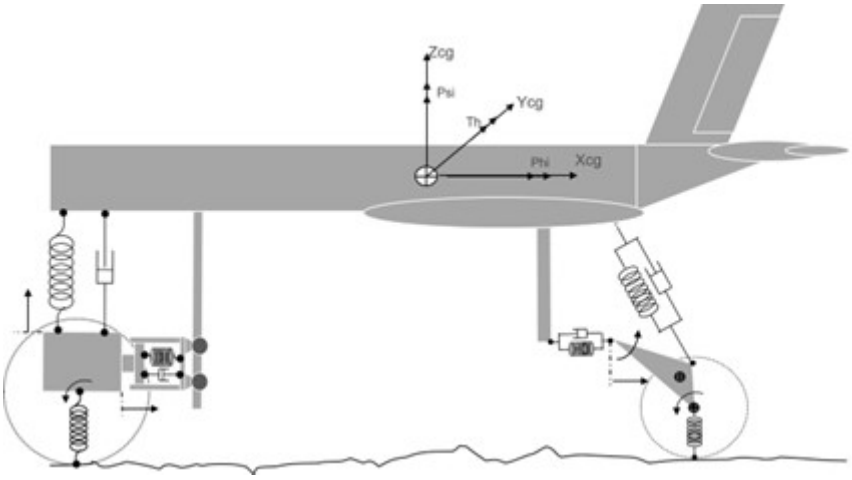


Fig. 1: Generic planar aircraft representation

One of the requirements for computer simulation is a representative model of the system to be simulated. For this work, it was decided to explore the modern tools of graphic formulations. The modeling methodology chosen is based on the Lagrangian method associated with the Bond-Graph (BG) graphical representation. This approach is based on past studies that combined both modeling methods Karnopp [1]. A bond-graph is a graphical representation of the system and its energy flow, energy storage and dissipation. Every component of the physical system can be translated into a bond-graph element. These elements represent how each component interacts with the power and energy that flows through the system. The bonds are bilateral connections between two elements through power ports. The energy exchange between the two elements is power conservative and it is represented by the transmission of flow and effort, characterizing the instantaneous power, where the power is the product of the flow and effort variables. This concept is domain-independent when describing the dynamic behavior of multiphysical systems, Borutzky [2]. The Euler-Lagrange equations describe the dynamics of the systems and model their behaviors. Differently from considering the relations of forces and torques on the system, like applying directly the Newton's Second Law, the Euler-Lagrange equations are built by analyzing the energy transformations across the system, Morin [3].

2 System modeling with Bond-Graph

The bond-graph is a graphic intuitive representation of the power flow through the system analyzed. Effort and flow will always be exchanged between two elements, where one of them will impose flow and the other one will respond with an effort (refer to Figure 2). The semi-arrow indicates the energy flow, and the vertical trace (causality) indicates which element is transmitting effort and which is reacting with flow (the element closer to the trace).

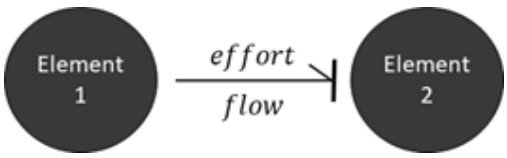


Figure 2: Elements and bond representation

The system and its elements are in linear mechanical domain, which means that effort is the same as force and flow is the same as linear speed. In addition to that, one can imagine that since it is a planar aircraft landing gear system, the elements will interact with each other in two different axes. To represent the complete system

dynamics, the vector bond-graph approach is very useful tool. When using vector bond-graph representation, the transmitted effort becomes a vector of efforts (the same for flow), as described by eq.(1).

The power bond for a B-G vector connection can be generalized to a n-dimensional vector bond as depicted in Figure 3.



Figure 3. Multidimensional BG connection represented by n-scalar bonds, or a single vector BG.

The BG vectors, ($\underline{e}$, $\underline{f}$) shown in Fig 3 are defined by the relations in eq.(1), bellow.

$$\underline{e} = (e_1, e_2, \ldots, e_n)^T \; ; \; \underline{f} = (f_1, f_2, \ldots, f_n)^T \qquad \text{Eq.(1)}$$

In order to generate the bond graph of the system represented in Figure 1, the system was divided into three main components, namely: the airframe, the nose landing gear and the main landing gear. For each one of these sub-systems, a systematic method to model the mdof mechanical systems by bond-graph approach was applied [2].

2.1 Nose landing gear BG modeling

The airframe interacts with the nose landing gear (NLG) by transmitting effort to the shock absorber. When separating the subsystems, it is necessary to represent the proportional aircraft mass that interacts with the NLG. The Figure 4 illustrates the NLG system schematic and its BG representation.

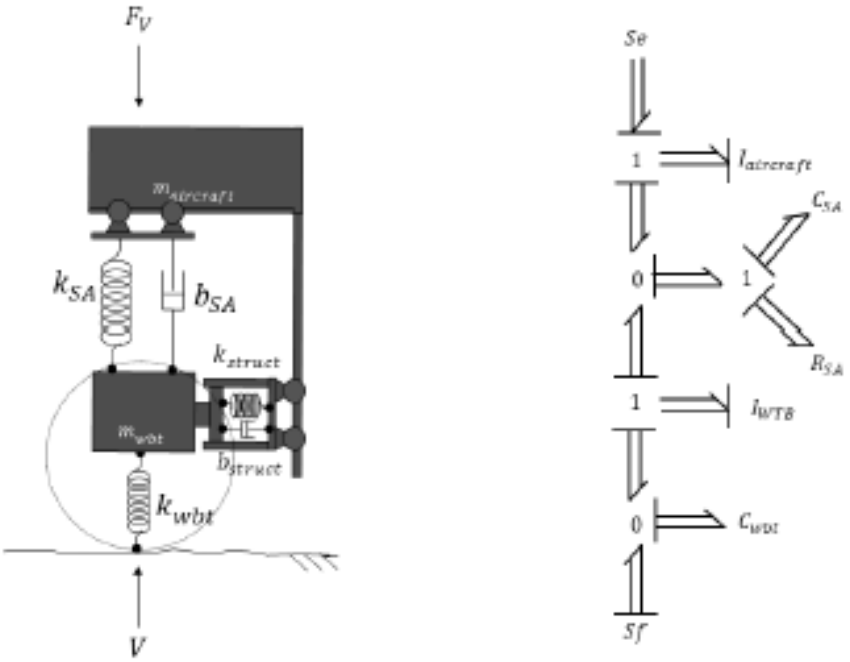


Figure 4. Nose landing gear schematic and its vector bond-graph representation

One may notice that, for vector BG nose landing gear model, the elements that represent the concentrated stiffness of the gear (C_{struct} and R_{struct}) were suppressed. In fact, those elements were introduced in the Shock Absorber elements matrices. It means that CSA and RSA represent the matrices below (Eq.2):

Eq(2)

$$C_{SA} = \begin{pmatrix} {k_{struct}}^{-1} & \infty \\ \infty & {k_{SA}}^{-1} \end{pmatrix}, R_{SA} = \begin{pmatrix} b_{struct} & 0 \\ 0 & b_{SA} \end{pmatrix}$$

Furthermore, the wheel-brake-tire (WBT) assembly elements, C_{wbt} and R_{wbt}, are mainly influenced by the tire dynamics. It was then considered that the tire's damping coefficient is negligible as compared to the shock absorber's damping coefficient; this implies there is no damper element between the WBT mass and the ground. Considering mainly the tire dynamics, it can be said that the spring effects in X-axis are also negligible.

2.2 Main landing gear BG modeling.

The airframe interacts with the main landing gear (MLG) by transmitting effort to the shock absorber. When separating the subsystems, it is necessary to represent the proportional aircraft mass that interacts with the MLG. The Figure 5 below illustrates the MLG system schematic and its BG representation.

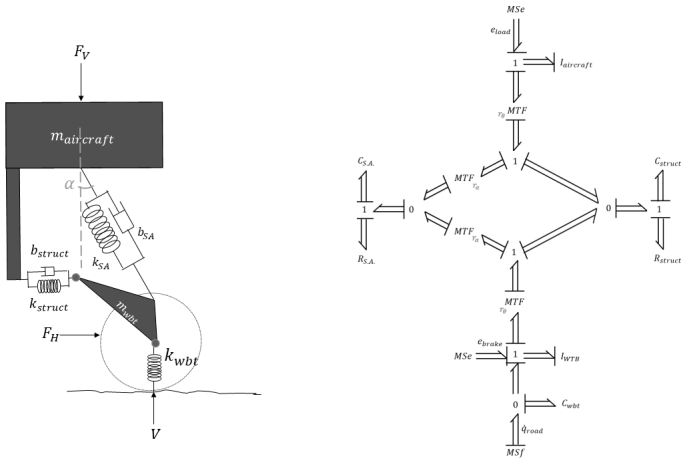


Figure 5: Main landing gear schematic and its bond-graph representation

The alpha angle, depicted in the above figure, is obtained by the trigonometric relation between the partial aircraft concentrated mass and the WBT mass elements. The structural spring-damper elements are not affected by the alpha angle but only by the aircraft body pitch angle, theta. The tire-spring element is not affected by any rotational dof. It will always react vertically. The effects alpha and theta are represented on the BG by the modulated transformers (MTF) elements, which are rotational matrices. With this representation, it one can see that the theta rotation is applied only to the structural stiffness and to the shock absorber elements, while the alpha rotation only applies to the shock absorber elements. Besides, one may notice that the Wheel-tire-brake assembly elements are not affected by any of these rotational dof. Otherwise, the representation of both structural and shock absorber reactions in the same matrix was not possible for the main landing gear model. It was necessary to separate those effects because, since in that case, there are a relative motion between the structural and the shock absorber elements.

2.3 Airframe BG modeling

As shown in Figure1, the airframe transmit the dynamic forces concentrated on the aircraft CG to the landing gear interfaces, causing a sum of speeds, combined with the nose and main gear motions. Those sub-systems react with applied forces to the airframe at both translational and rotational aircraft degrees with force. This relation was

represented by the airframe BG on Figure 6.

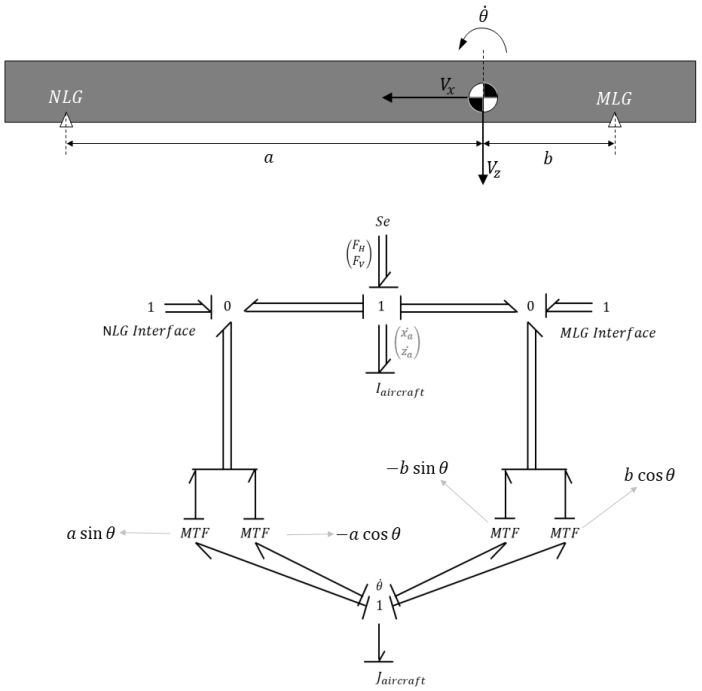


Figure 6. Airframe schematic and its bond-graph representation

In Figure 6, it is possible to identify the velocities acting on the aircraft CG, represented on the airframe, the nose landing gear attachment point velocity, the main gear attachment point velocity, and the angular velocity given by the aircraft pitch motion (theta). It is a reasonable assumption to say that the aircraft load and the linear and angular velocities are concentrated on its center of gravity. Those dynamic effects are transmitted as flow to the landingear legs through the airframe and it is dependent on the distance from the landing gear legs to the aircraft CG. In Figure 6, those distances are referenced by the parameters a and b, in the top figure.

3 Bond-Graph Euler-Lagrange Equations

The Euler-Lagrange Equations are derived from the principle of least action, and is written as [1],

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = E_i, i = 1, 2, ..., n \tag{3}$$

$$L = T^*(q_1, q_2, ..., q_n, \dot{q}_1, \dot{q}_2, ..., \dot{q}_n, t) - V(q_1, q_2, ..., q_n) \tag{4}$$

In the above equations, L is the Lagrangian and it is the difference between kinetic co-energy (T*) and potential energy (V), as defined in Table 1. Note that the Lagrangian is function of the generalized coordinates (q) and its time derivatives, the generalized velocities. The generalized coordinates are independent of each other, and give the total number of dof of the dynamic system. The right-hand side of this equation (Ei) represents the non-conservative forces (external and dissipative forces) acting on the system. To apply the Lagrangian method, the first step comprises the selection of generalized coordinates, and identify the processes of energy storage and dissipation, associated with these coordinates.

In the proposed methodology the Lagrangian is written as function of the BG storage and dissipation elements, expressed in terms of state-functions. For the classical E-L equations formulation, one must find the inetic and potential energies of the system. The generalized elements of the BG representation give directly the stored energy (kinetic and potential energy), as well as the external forces and dissipated energy. The junction structure give power transmission and energy transformation in the physical system. Considering these elements,

the potential energy is associated with generalized capacitors (C-elements) and the kinetic co-energy is associated with generalized inductors (I-elements), as shown on Table 1, below.

Table 1: Energy equations in function of generalized elements.

Element		Constitutive Relation	State Functions
C	Linear	$e = \frac{q}{C}$	$V(q) = q^2/2C$
	Nonlinear	$e = e(q)$	$V(q) = \int^q e dq$
I	Linear	$f = \frac{p}{I}$	$T^*(f) = If^2/2$
	Nonlinear	$f = f(p)$	$T^*(f) = fp - \int^p f dp$

3.1 Landing-gear vector BG symbolic manipulation and computational simulation

After finding the Lagrangian, the E-L equations were obtained with Mathematica software, by applying the equation (3). The Mathematica file can be seen in Figure 7, where the E-L formulation is presented as well as two of the final equations.

Euler-Lagrange formulation

```
LagrangianEquations[T_, V_, Q_: 0, genCoords_List] :=
  Module[{L = T - V},
   (D[D[L, D[#, t]], t] == Q + D[L, #]) & /@ genCoords
  ]
```

Equations

```
ELNLGX = Simplify[LagrangianEquations[T1, V1, E1[[1]], {x1[t]}]]
ELNLGZ = Simplify[LagrangianEquations[T1, V1, E1[[2]], {z1[t]}]]
ELMLGX = Simplify[LagrangianEquations[T2, V2, E2[[1]], {x2[t]}]]
ELMLGZ = Simplify[LagrangianEquations[T2, V2, E2[[2]], {z2[t]}]]
ELAirX = Simplify[LagrangianEquations[T3, V3, E33[[1]], {x3[t]}]]
ELAirZ = Simplify[LagrangianEquations[T3, V3, E33[[2]], {z3[t]}]]
ELTheta = Simplify[LagrangianEquations[T3, V3, E34, {θ[t]}]]
{{mt x1''[t]}} == {{-ktx x1[t] -
    kshockZ Sin[θ[t]] (Sin[θ[t]] (x1[t] - xNLG[t]) + Cos[θ[t]] (-z1[t] + zNLG[t])) +
    kshockX Cos[θ[t]] (Cos[θ[t]] (-x1[t] + xNLG[t]) + Sin[θ[t]] (-z1[t] + zNLG[t])) +
    bshockX Cos[θ[t]]^2 (-x1'[t] + xNLG'[t]) + bshockZ Sin[θ[t]]^2 (-x1'[t] + xNLG'[t]) +
    bshockZ Cos[θ[t]] Sin[θ[t]] (z1'[t] - zNLG'[t]) +
    bshockX Cos[θ[t]] Sin[θ[t]] (-z1'[t] + zNLG'[t])}}
{{mt z1''[t]}} == {{Weight +
    kshockZ Cos[θ[t]] (Sin[θ[t]] (x1[t] - xNLG[t]) + Cos[θ[t]] (-z1[t] + zNLG[t])) +
    kshockX Sin[θ[t]] (Cos[θ[t]] (-x1[t] + xNLG[t]) + Sin[θ[t]] (-z1[t] + zNLG[t])) -
    ktz (z1[t] + zroad[t]) + bshockZ Cos[θ[t]] Sin[θ[t]] (x1'[t] - xNLG'[t]) +
    bshockX Cos[θ[t]] Sin[θ[t]] (-x1'[t] + xNLG'[t]) +
    bshockZ Cos[θ[t]]^2 (-z1'[t] + zNLG'[t]) + bshockX Sin[θ[t]]^2 (-z1'[t] + zNLG'[t])}}
```

Figure 7. Extract from Mathematica program to generate Euler-Lagrange equations.

The above equations were implemented in Matlab/Simulink computational environment to simulate a typical

landing condition. The parameters and detailed description of the simulation process is described in Souza [4].

A typical result from this simulation can be seen in Figure 8. Firstly, the aircraft was accelerated and at the same time, vertical load was applied to the model, simulating a smooth aircraft touchdown. When the aircraft speed reached 40 m/s and the vertical load reached aircraft total weight (around 5s), braking force was applied to decelerate the aircraft until it stops. When braking was applied, a momentum was generated at the aircraft CG. It is possible to see that both the nose and the main gear reacted to this dynamic input, applied at 5 s.

4 Conclusions

In this work, the combination of Lagrangian and Bond-Graph modeling methods was applied to a landing gear system with planar aircraft model. Firstly, the system was analyzed under the bond-graph standpoint. It was possible to explore not only the concepts of this methodology but mainly its main advantage for this work: an intuitive power distribution representation, built following a systematic process. Once proved the effectiveness of this modeling methodology, the resultant bond-graph was analyzed under the Lagrangian standpoint. In this case, the energy interactions of all the system elements were calculated to generate equations of motion in the form of Euler-Lagrange equations. The process for finding E-L equations from a bond-graph model is also a systematic process that simplifies the formulation of such equations, compared to the classical Lagrangian modeling methodology. The simulations considered simplified linear elements, and yet reasonable results were obtained. The equations implemented, however, can be easily adapted to non-linear behavior. The process for generating Euler-Lagrange equations uses algebraic approach. The elements on these equations are variables which can represent both constants or entire functions. It means that for representing linear or non-linear behaviors, the process for generating the Euler-Lagrange is the same. The suitability to simulate non-linear elements is very useful and can have a variety of applications. It was an opportunity to apply a relatively new methodology into the aeronautical field. Despite the study case chosen, a mechanical system, the combined modeling methodology can be applied to multi-domain systems. It is an effective engineering tool that helps comprehending and developing complex systems.

Acknowledgements. The authors are grateful for the support of the Aeronautical Institute of Technology and Fundação Casimiro Montenegro Filho (FCMF) for support during the realization of this research.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors or has the permission of the owners to be included here.

References

[1] Karnopp D, Lagrange's equations for complex bond graph systems, Journal of Dynamic Systems, Measurement and Control, Volume 99, 300-306 (1977).
[2] Borutzky W, Bond graph modelling and simulation of multidisciplinary systems - An introduction, Simulation Modelling Practice and Theory, Volume 17 Issue 1, 3-21. Elsevier (2009).
[3] Morin D, Introduction to Classical Mechanics: With Problems and Solutions, 218-280, Cambridge University Press (2008)
[4] Yuri Silva Souza, Modeling and computational simulation of a landing gear system with rigid and flexible body movement, Master of Science in Aeronautical Angineering, ITA-DCAT, São José dos Campos, SP, 2019.