XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGENEERING

Non-Intrusive Parallel Implementation of GFEM with Global-Local Enrichment for Multi-Domain Simulations

Túlio Roberto Eládio Marques[1], Ana Clara Pedras Bueno[1], Ramon Pereira da Silva[2], Felício Bruzzi Barros[1]

[1]*Graduate Program in Structural Engineering - PROPEEs, Federal University of Minas Gerais*
Av. Antônio Carlos, 6627– Pampulha, 31270-901, Minas Gerais, Brazil
tulioeladio10@ufmg.br, anaclarabueno@ufmg.br, felicio@dees.ufmg.br
[2]*Dept. of Structural Engineering, Federal University of Minas Gerais*
Av. Antônio Carlos, 6627– Pampulha, 31270-901, Minas Gerais, Brazil
ramon@ufmg.br

Abstract. This paper expands the non-intrusive implementation of the Generalized Finite Element Method with Global-Local enrichment (IGL-GFEM^{gl}) for multi-domain analyses simulated under a parallel algorithm approach. In IGL-GFEM^{gl}, the global problem is initially discretized using a coarse mesh without considering localized phenomena. This problem is solved by the Abaqus commercial software using the Finite Element Method. The localized phenomena of interest, such as cracks, are effectively represented on a local scale, defining problems associated with one or more local domains. Mesoscale problems are created as intermediate scales between the global and local domains. The association between each mesoscale and its respective local problem is carried out through the global-local enrichment of GFEM^{gl}. The coupling of the mesoscales to the global problem is established through the transfer of displacements and generalized forces, characterizing the non-intrusive strategy known as Iterative Global-Local (IGL). Numerical simulations using GFEM^{gl} are performed in the computational system INSANE (Interactive Structural ANalysis Environment – *www.insane.dees.ufmg.br*), a free software project developed at the Department of Structural Engineering at the Federal University of Minas Gerais. Numerical examples are presented to demonstrate the simulation's performance and to investigate the influence of the main parameters related to the proposed strategy. The quality of the results is evaluated based on the state of stresses and strains at critical points of the models and Linear Elastic Fracture Mechanics parameters. The investigations indicate that the convergence process is not affected by the consideration of multiple local domains, as long as these do not result in a significant increase in the stiffness difference between the coupled scales. Additionally, the computational performance of the algorithm was evaluated, comparing simulations in which the mesoscale processing was executed sequentially and in parallel.

Keywords: Generalized Finite Element Method, Global-Local Analysis, Non-intrusive Coupling, Linear Elastic Fracture Mechanics, Parallel Computing

1 Introduction

In Fracture Mechanics problems, the phenomena of nucleation and coalescence of microvoids can lead to the failure of structural components through the formation of cracks. Such cases involve discontinuities and stress concentrations, requiring excessively refined meshes in confined regions of the domain, which makes the discretization process cumbersome in the standard Finite Element Method (FEM) approach. To overcome these limitations, strategies such as the Iterative Global-Local (IGL) technique have been developed, enabling the separation of the global behavior of the structure, represented by a coarse discretization, from the crack simulation, described by an independent fine mesh in a restricted region of the domain.

The IGL approach allowed the development of non-intrusive coupling between academic codes and commercial software, enabling the application in industry of methods such as the Generalized Finite Element Method (GFEM) [1], with Global-Local enrichment (GFEM^{gl}) [2]. The enrichment strategy of GFEM^{gl} allows distinct discretizations between the global and local models, without using distorted elements or remeshing techniques, minimizing deleterious effects on the approximation.

The non-intrusive implementation of GFEM^{gl} in multiscale problems was proposed by Li et al. [3] [4], with GFEM^{gl}-IGL. This formulation eliminates the need for modifications in solvers of closed-source programs, being

particularly efficient in problems with localized phenomena where conventional remeshing is recurrent [5].

In this research, GFEM^{gl}-IGL is coupled to the commercial software Abaqus, responsible for the global model, while INSANE (INteractive Structural ANalysis Environment, available at *www.insane.dees.ufmg.br*) models the local phenomena with GFEM^{gl}. The present work expands the implementation of GFEM^{gl}-IGL in INSANE [6], allowing multiple local domains and, additionally, develops a parallelism manager for distributing the local analyses among multiple processors. This approach reduces computational cost and is enabled by the independence between local domains, whose influences are subsequently incorporated into the enriched global model.

2 Formulation Aspects: GFEM^{gl}-IGL applied to problems with multiple mesoscales

Li et al. [3] proposed the GFEM^{gl}-IGL strategy, which combines the iterative global-local algorithm of FEM-IGL with GFEM^{gl}, requiring only the transfer of forces and displacements between the problem's scales. This feature eliminates the limitations associated with substructuring-based strategies. This methodology enables the integration between commercial software, responsible for the global model analysis, and academic codes, applied to local modeling. In GFEM^{gl}-IGL, the problem is divided into three scales. The **Global Scale** corresponds to the complete domain, excluding localized phenomena, and is discretized with a coarse mesh. The **Mesoscale** is a subdomain contained within the global scale that acts as a non-intrusive coupling interface between the global and local scales. It generally uses the same mesh as the global scale, which facilitates information transfer and eliminates limitations of FEM-IGL [7]. Finally, the **Local Scale** represents the regions with the local phenomena of interest (such as cracks), with a fine discretization.

The global model is coupled to the mesoscale via IGL, which is enriched with the local scale solution through GFEM^{gl}. The steps of the solution process are described below, where k refers to each IGL iteration.

1. **Global model analysis**: solve the global model and obtain the solution $\boldsymbol{u}_G^k$;
2. **Mesoscale analysis**: define and solve the mesoscale models by imposing the global model solution ($\boldsymbol{u}_G^k$) as boundary condition. The solution $\boldsymbol{u}_M^k$ is obtained;
3. **Local model analysis**: define and solve the local models using the mesoscale solution ($\boldsymbol{u}_M^k$) as boundary condition (Neumann, Dirichlet, or Cauchy). The solution $\boldsymbol{u}_L$ is obtained;
4. **Enriched mesoscale analysis**: apply the enrichment constructed from the local problem solution to the mesoscale and solve the corresponding domain, obtaining $\boldsymbol{u}_{ME}^k$. In its original formulation, the global-local cycles of GFEM^{gl} described in this step and in step 3 occurred only once. However, the number of iterations can be defined to improve the quality of boundary conditions and reach the desired convergence;
5. **Residual force vector computation**: compute the residual vector, $\boldsymbol{f}_R^k$, as defined in Equation (1), using the values of $\boldsymbol{u}_G^k$ and $\boldsymbol{u}_{ME}^k$ at the interface between the mesoscale and the global model;

$$\boldsymbol{f}_R^k = -(\boldsymbol{f}_{ME}^{\Gamma_I} + \boldsymbol{f}_G^{\Gamma_I}) = -[(\mathbf{K}_{ME} \cdot \boldsymbol{u}_{ME}^k - \boldsymbol{f}_{ME}) \mid_{\Gamma_I} + (\mathbf{K}_G \cdot \boldsymbol{u}_G^k - \boldsymbol{f}_G) \mid_{\Gamma_I}] \tag{1}$$

   where $\boldsymbol{f}_{ME}$ and $\boldsymbol{f}_G$ are the force vectors corresponding to the interface boundary Γ_I, related to the mesoscale and global models, respectively. The term $\mathbf{K}$ denotes the stiffness matrices of the domains.

6. **Convergence**: Verify whether the convergence criterion has been satisfied, and in this case, finish the process. The final solution is given by $\boldsymbol{u}_{ME}^k$ in the mesoscale region and $\boldsymbol{u}_G^k$ in the remainder of the domain;
7. **Iteration**: if convergence has not been achieved, update the global model's force vector by adding the residual forces $\boldsymbol{f}_R^k$ with opposite sign. Set $k = k + 1$ and return to step 1 iteratively until convergence;

Steps 2 through 5 represent the parallelizable portion of the monolithic GFEM^{gl}-IGL approach, responsible for solving the mesoscale domains. This process covers the construction of local models and the simulation of the phenomena of interest, in this case cracks, using the global-local enrichment strategy characteristic of GFEM^{gl}. The independence among the mesoscales allows the distribution of these tasks across multiple processors, without requiring communication between them during the local analysis phase.

The parallelization of this methodology portion enables significant performance gains, especially in problems involving multiple localized phenomena, as the total simulation time tends to be dominated by the local analysis. The proposed strategy leverages this intrinsic feature of the global-local formulation, maximizing the use of modern multicore computational architectures, and enabling scalability in high-performance computing environments.

Li et al. [4] proposed a variant of the GFEM^{gl}-IGL method through an algorithm known as *staggered*. The key distinction of this new strategy lies in performing IGL iterations [7] between the global model and the mesoscale before coupling the GFEM^{gl} analysis. Thus, steps 3 and 4 are performed only after convergence of the IGL cycles. Only when the equilibrium between the mesoscale and global problems is reached the steps corresponding to

GFEM^{gl} are carried out. After this stage, new IGL iterations are performed until final convergence is achieved. In this way, the number of GFEM^{gl} simulations is reduced, minimizing possible stiffness changes in the mesoscale domain caused by global-local enrichment.

3 Numerical Experiments

The numerical problem consists of a two-dimensional frame illustrated in Figure 1, in which the results among the mesoscales are influenced by the applied loading, here represented by the wind force acting on the structure. The adopted constitutive model is linear elastic, considering a homogeneous and isotropic material, with Young's modulus $E = 2.0 \times 10^7$ and Poisson's ratio $\nu = 0.30$. The frame has a thickness of $t = 15.0$ and is subjected to a Plane Stress condition.

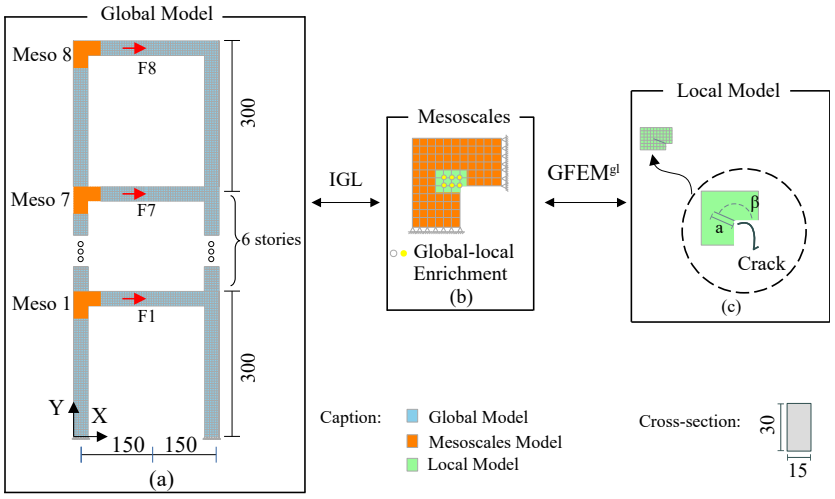


Figure 1. Frame with eight cracks under mixed-mode opening.

The meshes of the global scale and each mesoscale are composed of 8064 and 96 elements, respectively, with dimensions of 5.0×5.0. The elements in the local mesh have dimensions of 1.667×1.667. Eight mesoscales were evaluated, positioned based on the presence of cracks at the connections between beams and columns on the left side of the frame. The cracks considered in the local domains are identical and defined by the parameters $a = 7.9057$ and $\beta = 2.8198$ rad, and are kinematically represented using an enrichment system composed of singularity functions [8] and the Heaviside function [9]. All quantities presented are expressed in consistent units, which are omitted throughout the text for simplicity.

Four-node quadrilateral elements (Q4) were used, with bilinear shape functions and two degrees of freedom per node. Regarding numerical integration, Gauss quadrature was adopted: 2×2 for the global scale, 3×3 for the mesoscale, and 8×8 for the local scale in elements intersected by cracks, and 4×4 for the others. Cauchy boundary conditions were applied, with stress and displacement transfer enhanced by the ZZ-BD field [10]. Simulations were performed on an Intel® Core(TM) i5-10300H processor with 4 cores and 8 threads.

4 Conclusions

The computational performance of MEFG^{gl}-GLI was evaluated as a function of the number of mesoscales and processors used in the analysis. A model with two subdomains, mesoscales 1 and 2 - Figure (1), was initially considered, with the remaining local domains progressively incorporated into the analysis from bottom to top. Figure (2) illustrates the total time spent in each analysis relative to the sequential execution time of the monolithic approach with two mesoscales. The processing time of the analyses progressively decreases as more processors are added, stabilizing when their number matches the number of mesoscales. This behavior is illustrated in Figure (2) for the models with two, three, and four mesoscales. Regarding performance parameters, $SpeedUp > 1.0$ was observed in all analyses, with maximum values of 2.360 and 2.010 for the monolithic and staggered approaches, respectively. Concerning efficiency (E), it increased until the number of processors matched the number of mesoscales, minimizing idleness and approaching the maximum parallelization potential.

Therefore, the methodology demonstrated accurate crack simulation, preserving solution quality and model

consistency. While global-local enrichment handles singularities without compromising numerical stability, even with multiple mesoscales, the parallelism optimizes computation through load balancing and reusable threads.

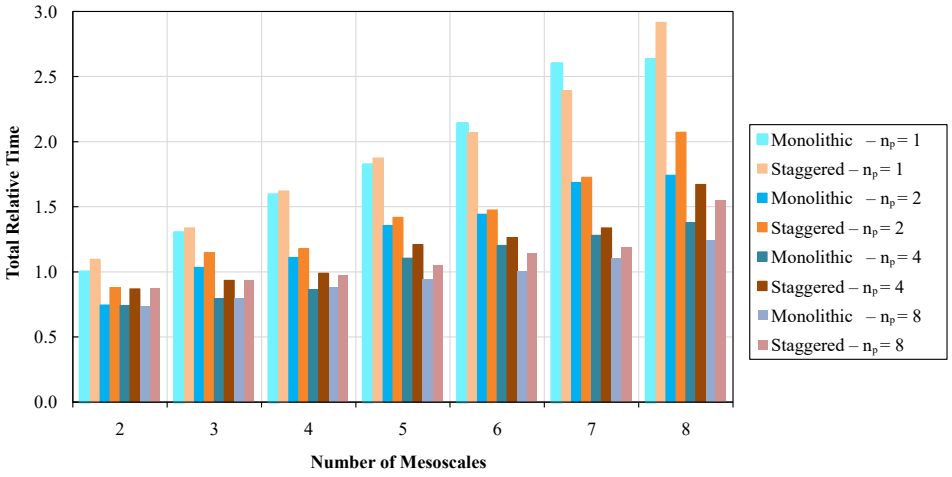


Figure 2. Total computation time as a function of the number of mesoscales and processors.

Acknowledgements. The authors gratefully acknowledge the important support of the Brazilian research agencies FAPEMIG (in Portuguese "Fundação de Amparo à Pesquisa de Minas Gerais"), CNPq (in Portuguese "Conselho Nacional de Desenvolvimento Científico e Tecnológico"- Grant 308444/2022-1) and CAPES (Coordination for the Improvement of Higher Education Personnel).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] T. Strouboulis, I. Babuška, and K. Copps. The design and analysis of the Generalized Finite Element Method. *Computer Methods in Applied Mechanics and Engineering*, vol. 181, n. 1-3, pp. 43–69, 2000.

[2] C. A. Duarte and D. J. Kim. Analysis and applications of a generalized finite element method with global-local enrichment functions. *Computer Methods in Applied Mechanics and Engineering*, vol. 197, n. 6–8, pp. 487–504, 2008.

[3] H. Li, P. O'hara, and C. A. Duarte. Non intrusive coupling of a 3 d generalized finite element method and abaqus for the multiscale analysis of localized defects and structural features. *Finite Elements in Analysis and Design*, vol. 193, pp. 103554, 2021.

[4] H. Li, J. Avecillas-Leon, N. Shauer, and C. A. Duarte. A non-intrusive iterative generalized finite element method for multiscale coupling of 3-d solid and shell models. *Computer Methods in Applied Mechanics and Engineering*, vol. 402, pp. 115408, 2022.

[5] M. Duval, J. C. Passieux, M. Salaun, and S. Guinard. Non-intrusive coupling: Recent advances and scalable nonlinear domain decomposition. *Archives of Computational Methods in Engineering*, vol. 23, n. 1, pp. 17–38, 2014.

[6] A. Bueno, N. Silveira Filho, and F. Barros. A non-intrusive multiscale framework for 2d analysis of local features by gfem — a thorough parameter investigation. *Finite Elements in Analysis and Design*, vol. 242, pp. 104258, 2024.

[7] J. D. Whitcomb. Iterative global local finite element analysis. *Computers & Structures*, vol. 40, n. 4, pp. 1027–1031, 1991.

[8] C. Duarte, I. Babuška, and J. Oden. Generalized finite element methods for three-dimensional structural mechanics problems. *Computers & Structures*, vol. 77, n. 2, pp. 215–232, 2000.

[9] N. Moës, J. Dolbow, and T. Belytschko. A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering*, vol. 46, pp. 131–150, 1999.

[10] T. R. Marques, G. M. Fonseca, R. M. Lins, and F. B. Barros. On the improvement of the local boundary conditions in gfemgl. *Engineering Analysis with Boundary Elements*, vol. 176, pp. 106219, 2025.