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Hinge joints in curved and straight elements of PAE-Grelhas 2.0

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Abstract. This paper presents the development of a Python program for the linear-elastic analysis of grillage structures composed of circumferential arc-shaped and straight elements, subjected to concentrated and distributed loads, with the addition of hinge joint connections. This work represents an upgrade to the Finite Element-based program PAE-Grelhas 2.0, which previously relied exclusively on fixed-end elements. The implementation is based on the Force Method, allowing for the computation of the flexibility matrix; its inversion yields the stiffness matrix, with the code adapted to account for the specific conditions introduced by hinge joints in each bar. For curved elements, accuracy is ensured through the use of Gauss-Legendre Quadrature, applying an appropriate number of points for effective numerical approximation. The results are validated through comparisons with cases developed by the authors, calculated using the Virtual Work Method.

Keywords: grillage structures; Finite Element Method; hinge joint; Python, PAE-Grelhas.

1 Introduction

According to Soriano [1], grillage structures are composed of bars arranged in a single plane and subjected to torsional moments, in-plane bending moments, and shear forces perpendicular to that plane. De Biagi [2] highlights their extensive adoption in civil, mechanical, and marine engineering, where the geometric configuration and moment-resistant connections enable flexural and torsional mechanisms with high load redistribution capacity. Moses & Onoda [3] complement that the grillage system is widely used in foundations, shipbuilding, and bridges, efficiently distributing concentrated loads. In foundations, they allow uniform load transfer to the soil; in shipbuilding, they form the basic hull structure, providing rigidity against dynamic wave forces; and in bridges, they optimize design by allowing multiple beams to work together. However, hinged joints, connections that provide rotational freedom at the nodes, make the structural behavior considerably more complex, requiring specialized analysis. Despite their relevance, there is a notable lack of educational and technical resources on grillages with hinged joints, as well as software capable of analyzing them natively, most programs assume rigid nodes or require manual adjustments that are time-consuming and error prone.

Rego et al. [4] presented a program for the analysis of grillages composed of circumferential arc-shaped elements and regular straight bars under various loading conditions, using Gauss-Legendre quadrature as the numerical integration method for deriving stiffness coefficients and load vectors. The program, titled “PAE-Grelhas 2.0,” was developed based on the Finite Element Method and implemented in Python, delivering highly accurate results. It was an updated version of the one developed by Lima and Soares [5]. However, its original formulation was based on fully fixed-end elements, offering no possibility of modeling hinged joints at the nodes. This paper presents the development of a computational program aimed at solving structural problems involving hinge joints applied to both circumferential arc-shaped and straight elements, called PAE-Grelhas 2.1. It represents a continuation of the PAE-Grelhas 2.0 platform, previously addressed, with its scope expanded to incorporate advanced features capable of handling complex geometries with greater analytical rigor and computational efficiency.

2 Methodology

The update to PAE-Grelhas 2.1, which include the analysis of elements with hinge joints was motivated by the lack of accessible software capable of handling such elements with sufficient accuracy. Although programs such as LESM [6] and VGPlan [7] offer solutions for structures composed of straight elements, they neither support curved elements nor allow for the rotation release individually (such as only releasing bending or torsion). As a result, approximations are often required, which can compromise the accuracy of the computed internal forces and displacements.

As in Rego et al. [4], the stiffness matrix coefficients and load vectors for the hinged elements were obtained from expressions derived using the Force Method, by computing the inverse of the flexibility matrix. The application of Gauss-Legendre quadrature ensured precise integration of the expressions associated with curved elements, allowing for the complete formulation of stiffness matrices and load vectors based on equilibrium conditions. After several validations using examples developed and solved by the authors themselves through the Virtual Work Method, it was concluded that using *10* Gauss-Legendre integration points was sufficient to achieve the desired level of accuracy.

This version is also based on the Finite Element Method and supports two types of grillage elements: the traditional straight bar with constant cross-section, two nodes, and three degrees of freedom per node; and the curved arc-shaped element, as illustrated in Figure 1. In the latter, angles α_1 and α_2 represent the initial and final angles, respectively, measured clockwise from the y-axis. Along with the radius of the arc, these parameters constitute the necessary input data for defining each curved element, which is evaluated in a polar coordinate system.

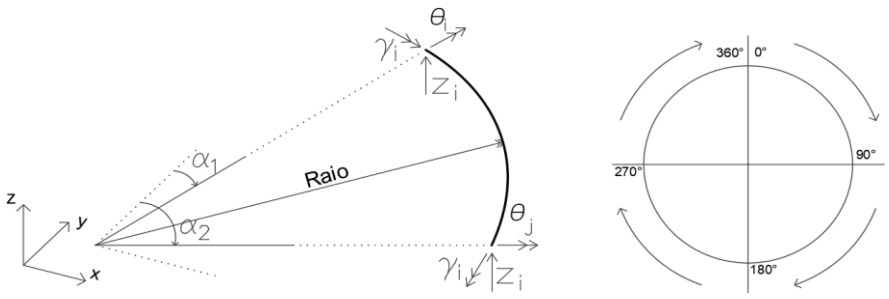


Figure 1. Generic circumferential arc-shaped element in isometric view and clockwise angular convention.

The main innovation in the data input process is that the program now accepts a .txt file with an additional column dedicated to specifying the type of hinge joint and the corresponding released degree of freedom (leading to a different stiffness matrix). This modification enables the program to correctly identify and process elements that contain hinge joints. Table 1 presents the adopted convention for encoding these joints in the input file.

Table 1. Hinge joint encoding.

Element Type	Code	Joint Type	Rotation Released
Straight	0	Fixed - Fixed	-
Straight	1	Fixed - Hinged	Torsion
Straight	2	Fixed - Hinged	Bending
Straight	3	Hinged - Fixed	Torsion
Straight	4	Hinged - Fixed	Bending
Curved	5	Fixed - Fixed	-
Curved	6	Fixed - Hinged	Torsion
Curved	7	Fixed - Hinged	Bending
Curved	8	Hinged - Fixed	Torsion
Curved	9	Hinged - Fixed	Bending

After processing, as described by Rego et al. [4], the new version generates two .txt output files. The first provides a detailed report with all input data, stiffness matrices, and load vectors, useful for academic or verification purposes. The second contains only the final structural results, including support reactions, internal forces (shear, bending, torsion), and nodal displacements and rotations. The program logic remains largely the same as the previous version, with the key addition of a module to interpret hinge joints.

The code was validated through several examples created by the authors. One of these is presented in the next section.

3 Results

An example of a grillage composed of straight and curved bars was developed and solved using the proposed code, and compared with an analytical solution. Since this is a statically indeterminate structure, the force method was initially applied to determine the internal forces and support reactions, considering the flexural and torsional stiffness properties presented in Table 2. Subsequently, unit loads were applied at specific points of interest and, through the Principle of Virtual Work, the particular displacements and/or rotations at these points were determined. These values served as reference for subsequent comparison with the results obtained from the matrix analysis in the program. Figure 2 shows the structure in isometric and plan views, while Table 2 presents the cross-sectional properties, including bending and torsional stiffness. Table 3 provides the data for the arc-shaped elements, and Table 4 presents the properties of the hinged joint elements.

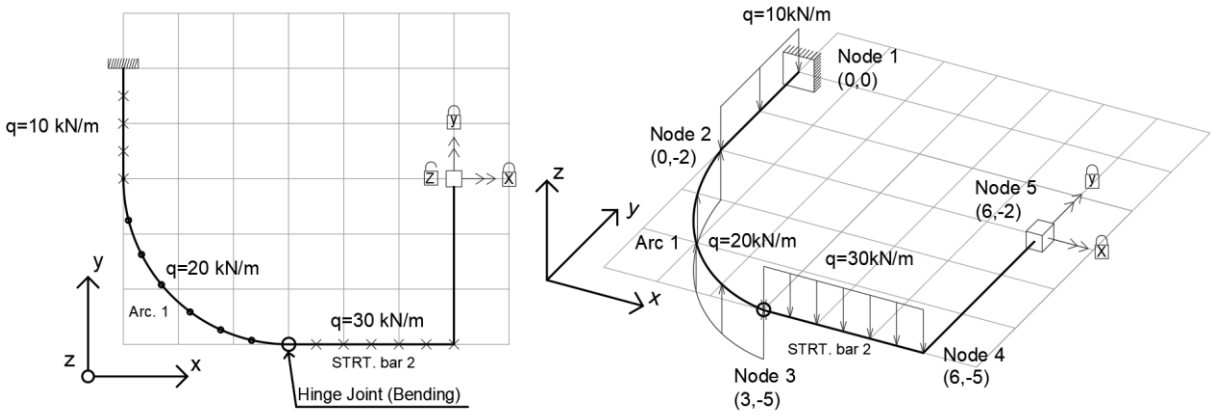


Figure 2. Grillage structure used as an example.

Table 2. Cross-sectional geometrical and material properties.

Parameters	Notation	Value
Young's modulus (Bending)	E	$2{,}5 \cdot 10^7$ kN/m^2
Shear modulus (Torsion)	G	$1{,}0 \cdot 10^7$ kN/m^2
Moment of Inertia (Bending)	I_b	$7{,}2 \cdot 10^{-3}$ m^4
Polar Moment of Inertia (Torsion)	I_t	$7{,}526 \cdot 10^{-3}$ m^4

Table 3. Input data for the circumferential arc-shaped elements.

Name	Notation	Value
Arc 1	Radius	3 m
	α_1	270°
	α_2	180°

Table 4. Input data for the hinged joint elements.

Name	Code	Joint Type	Rotation Released
Arc 1	9	Hinged - Fixed	Bending
Straight. Bar 1	4	Hinged - Fixed	Bending

Table 5 presents the input data for the example in Figure 2, including the number of nodes and elements, their coordinates, and nodal loads. The boundary conditions – vertical displacement (Z) and rotations about the X and Y axes – are defined using 0 for free and 1 for restrained. Load sign conventions follow the coordinate system. While no specific units are required, consistent magnitudes must be used; in this case, *kN*, *m*, and degrees were adopted. As previously discussed, a new column was added to the input file to define the type of hinge joint and the released degree of freedom, enabling the analysis of both rigid and articulated elements.

Table 6 presents a comparison of the support reactions and vertical displacements at nodes 2 and 4, obtained using the proposed improved PAE-Grelhas 2.1 code and an analytical solution derived from the Virtual Work Method. The results show excellent agreement, demonstrating the accuracy of the numerical model. It is worth emphasizing that the inclusion of hinged elements, integrated using Gauss-Legendre quadrature, preserves the precision observed in the previous version of the software, confirming the robustness of the proposed formulation for analyzing structures with curved elements and hinge joints.

Table 5. Input data in “PAE-Grelhas 2.1”.

|N. Nodes| & |N. Elements|										
5	4									
|Node|	|X Coord.|	|Y Coord.|	F_z	M_x	M_y	Displac. Z	Rot. X	Rot. Y		
1	0.0	0.0	0.0	0.0	0.0	1	1	1		
2	0.0	-2.0	0.0	0.0	0.0	0	0	0		
3	3.0	-5.0	0.0	0.0	0.0	0	0	0		
4	6.0	-5.0	0.0	0.0	0.0	0	0	0		
5	6.0	-2.0	0.0	0.0	0.0	0	1	1		
|Element|	| i |	| j |	|Joint|	|Radius|	$\lvert\alpha_1\rvert$	$\lvert\alpha_2\rvert$	|E|	$\lvert I_b\rvert$	|G|	$\lvert I_t\rvert$	$\lvert Q_z\rvert$
1	1	2	0	0.0	0.0	0.0	$2.5{\cdot}10^{7}$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^{7}$	$7.526{\cdot}10^{-3}$	-10.0
2	3	2	9	3.0	180.0	270.0	$2.5{\cdot}10^{7}$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^{7}$	$7.526{\cdot}10^{-3}$	20.0
3	4	3	4	0.0	0.0	0.0	$2.5{\cdot}10^{7}$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^{7}$	$7.526{\cdot}10^{-3}$	-30.0
4	4	5	0	0.0	0.0	0.0	$2.5{\cdot}10^{7}$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^{7}$	$7.526{\cdot}10^{-3}$	0.0

Table 6. Comparison of results.

-	Analytical	**PAE-Grelhas 2.1**	Error (%)
Support Reaction Node 1 Displac. z (kN)	15.75222	**15.75222**	0,0
Support Reaction Node 1 Rotation x (kN·m)	-90.26308	**-90.26308**	0,0
Support Reaction Node 1 Rotation y (kN·m)	-167.25666	**-167.25666**	0,0
Vertical Displac. Node 4 (m)	-0.0381387	**-0.0381387**	0,0

4 Conclusions

PAE-Grelhas 2.1 consolidates and enhances the functionality already presented in the previous version, maintaining the modeling of curved elements using circumferential arcs defined by only two angles and a radius. The application of Gauss-Legendre quadrature ensures the accuracy of the expressions used to form the stiffness matrices and load vectors.

The main innovation of this version is the implementation of articulated joints, with the possibility of freely releasing specific rotations (in the bending or torsional direction) in both straight and curved elements. This functionality, still uncommon in open-source software, allows for more accurate modeling of semi-rigid connections in grid systems, directly impacting the quality of the results obtained.

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