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Time-dependent reliability analysis of corroded pipelines using a hybrid successive Pareto simulation and importance sampling approach

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Abstract. The present study introduces a methodology for time-dependent structural reliability analysis of corroded pipelines by integrating the successive Pareto simulation (SPS) method with importance sampling (IS). This paper extends the application of SPS to assess the probability of failure evolution over time due to corrosion defects. First, the methodology is validated against results from the literature, where the DNV RP-F101 equation is applied. Then, the pipeline is modeled using a 2D axisymmetric finite element mesh automatically generated by using the PIPEFLAW software. Structural non-linear analysis is performed using the HYPLAS finite element code, considering failures due to small leak and burst. The reduction in the number of evaluations, compared to IS, is up to 34.52%. The hybrid SPS-IS approach improves the efficiency of reliability analysis for corroded pipelines by combining the lower number of function evaluations achieved by SPS with the variance reduction of IS.

Keywords: reliability analysis, successive Pareto simulation, importance sampling, finite element.

1 Introduction

Corrosion is the most common threat to the structural integrity of oil pipelines [1], potentially leading to failure with catastrophic consequences. The structural assessment of such systems should, ideally, account for the time-dependent evolution of defects and the randomness of the variables involved. Reliability analysis methods have increasingly been adopted to address these uncertainties, but they often involve high computational costs, especially when combined with finite element models (FEM) [2].

The objective of reliability analysis is to estimate the probability of failure (P_f) of the structure, which can be done through different methodologies. The two most traditional methods are the first-order reliability method (FORM) [3, 4] and Monte Carlo (MC) [5]. While the first has great efficiency for some problems, it also may present convergence and accuracy problems [6, 7]. The second method can be more accurate given a sufficient sample size, but may require a higher computational cost [2]. This work focuses on MC-based methods, which aim to preserve the accuracy of MC while reducing its cost [8].

A hybrid approach is proposed here for the time-dependent reliability analysis of corroded pipelines by combining successive Pareto simulation (SPS) method [9] with importance sampling (IS) [10]. The methodology is first validated using the semi-empirical burst pressure equation from the DNV RP-F101 standard [11]. Then, a reliability analysis is conducted based on nonlinear numerical simulations performed by the HYPLAS finite element code [12], where defect growth is modeled as a stochastic process over time. Geometry generation is fully automated using the PIPEFLAW software [13], considering an axisymmetric model for the pipe.

2 Corroded pipeline assessment

The assessment of pipelines with corrosion defects can be performed in three different levels, following the American Pipeline Institute (API) recommendations [14], where the third level is considered the most accurate and the first level is considered less accurate [15]. The first level uses a simplified version of a single defect, while the second level considers complex-shaped defects and the interaction between multiple defects. The assessment of corroded pipelines using both levels considers well known established equations from standards. The third level consists of a non-linear numerical analysis using the finite element method (FEM), providing more reliable results to the remaining strength of the pipe. Cosham and Hopkins [16] considers a level beyond the FEM analysis, where reliability analysis is applied to obtain a more complete assessment of the pipe, taking into account the probabilistic behavior of the variables involved.

In this work, the time-dependent reliability analysis is conducted using a semi-empirical criterion (level 1) and an axisymmetric FEM analysis (level 3). The semi-empirical criterion is considered to validate the proposed hybrid approach, comparing against results obtained in the literature for a pipe with a single corrosion defect. The FEM is conducted to provide a deeper insight in the behavior of the same pipe through time and measure the benefits of the proposed reliability method when considering numerical models.

2.1 Semi-empirical criterion

The DNV RP-F101 criterion [11] for the burst pressure calculation is considered in the first part of this work, in order to provide direct comparison against results found in the literature. Those results have the purpose of validating the proposed approach for reliability analysis and to observe the major difference with respect to the numerical-based results. The burst pressure estimated by the DNV RP-F101 criterion is

$$P_b = \frac{1.05\sigma_u 2t}{D-t}\left[\frac{1-(d/t)}{1-(d/t)M^{-1}}\right], \tag{1}$$

where D is the external diameter of the pipe, d is the defect depth, t is the pipe wall thickness, σ_u is the ultimate tensile strength and M is the Folias factor. The latter is obtained through Eq. (2), where l is the defect length.

$$M = \sqrt{1 + 0.31\left(\frac{l}{\sqrt{Dt}}\right)^2} \tag{2}$$

2.2 Finite element analysis

The FEM analysis is performed using the HYPLAS software [12] to analyze the mechanical behavior of the pipe through an incremental loading process, assuming finite deformations. Material behavior is described by an elastoplastic model incorporating von Mises yielding and associative flow. The Newton-Raphson algorithm, coupled with an arc-length procedure, is applied to solve the equilibrium equations for each load increment. The burst pressure is obtained through the peak load, when the von Mises stress reaches the ultimate strength of the material or the algorithm fails to converge.

A bi-dimensional model with an axisymmetric defect is adopted to perform the FEM analysis, in order to reduce the computational cost of the multiple simulations required by reliability analysis [17]. The mesh geometry in this work is the same used by Rodrigues *et al.* [18], as it provides good results compared to experimental tests and 3D FEM simulations [17, 18]. The mesh is automatically generated using the PIPEFLAW software [13].

Figure 1 shows the adopted mesh configuration, where DL is the defect length, t the pipe wall thickness, d the defect depth and r the frontal fillet radius of the defect region. Symmetric boundary conditions are applied by fixing Y-displacements at the upper edge nodes through the pipe wall. To emulate the effect of an infinitely long pipe, the lower boundary (far from the defect) is also constrained in the vertical direction. All simulations consider internal pressure as the only acting load. The applied loads and boundary conditions are also illustrated in Fig. 1.

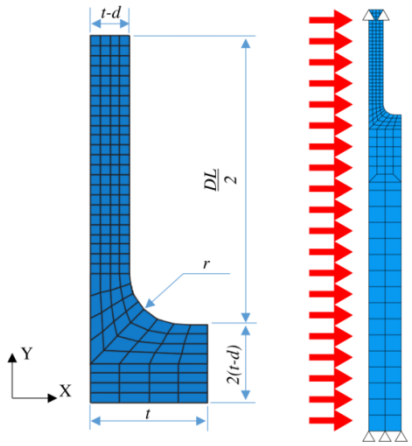


Figure 1. Adopted mesh configuration, loads and boundary conditions, taken from Oliveira *et al.* [17] and Rodrigues *et al.* [18].

3 Reliability analysis

The reliability analysis considers the randomness of each variable present in the problem. Each random variable (RV) is associated with a probability density function (PDF) according to its expected behavior in practice [19]. A limit state function $G(\boldsymbol{X})$ can then be built with these RVs, where the failure event is defined by $G(\boldsymbol{X}) \leq 0$ and $\boldsymbol{X}$ is a vector containing the RVs.

For the reliability analysis of corroded pipelines, two limit state functions are considered. The first limit state function (G_1) is related to the burst pressure, while the second one (G_2) is related to the small leak event [20]. The two functions are presented in Eqs. (3) and (4), where P_o is the operation pressure. In this work, these two failure modes are treated separately.

$$G_1(P_b, P_o) = P_b - P_o \tag{3}$$

$$G_2(d, t) = 0.8t - d \tag{4}$$

3.1 FORM

The method known as FORM [3, 4] simplifies the reliability analysis by converting the variables to a standard Gaussian space. Using an specific search algorithm, as the Hasofer-Lind-Rackwitz-Fiessler (HL-RF) [19], the most probable point (MPP) is identified, which is the point inside the failure region nearest to the origin in the standard Gaussian space. The probability of failure is estimated as $\hat{P}_f^{FORM} = \Phi(-\beta)$, where β is the norm of the vector that connects the origin to the MPP and Φ is the cumulative distribution function (CDF) of the standard Gaussian distribution.

3.2 Monte Carlo

The MC method [5] consists of counting the number of points in the sample that are in the failure domain and dividing this number by the total sample size N. The identification of the failed sample points can be done by introducing an indicator function $I[G(\mathbf{x}) \leq 0]$, which is equal to 1 if $G(\mathbf{x}) \leq 0$ and equal to zero otherwise. The probability of failure estimate is then

$$\hat{P}_f^{MC}(\boldsymbol{x}) = \frac{1}{N}\sum_{i=1}^{N} I[G(\boldsymbol{x_i}) \leq 0]. \tag{5}$$

3.3 Importance sampling

The IS method [10] uses a different PDF to generate the sample, named sampling function $h_{\boldsymbol{X}}(\boldsymbol{x})$, which is

usually the multidimensional standard Gaussian PDF centered at the MPP. The sampling function has the objective of generating more points in the failure domain, in order to reduce the variance of $\hat{P}_f$. Adapting the equation of the probability of failure estimate obtained by MC, the IS estimate can be written as

$$\hat{P}_f^{IS}(\boldsymbol{x}) = \frac{1}{N}\sum_{i=1}^{N}\frac{f_{\boldsymbol{X}}(\boldsymbol{x_i})}{h_{\boldsymbol{X}}(\boldsymbol{x_i})} I[G(\boldsymbol{x_i}) \leq 0]. \tag{6}$$

3.4 Successive Pareto simulation

The Pareto optimality concept [21] is applied in this method to search for the most extreme sample points in the failure direction, *i.e.* the most distant points from the mean point towards the failure domain. In this method, a point $\boldsymbol{x_i}$ is said to be dominated by another point $\boldsymbol{x_p}$ if $x_{i,j} \leq x_{p,j}$ for any $j = 1, 2, \ldots, n$, and $x_{i,j} < x_{p,j}$ for at least one value of j, where n is the number of RVs. The RVs are classifed as capacity or demand, where the capacity variables are multiplied by -1 during the search step to unify the failure direction for all RVs.

The SPS method [9] is an iterative process where, at each iteration, the Pareto front (extreme points) is identified and evaluated. Every point dominated by a point from the Pareto front that is in the safe domain is considered safe and, as a consequence, is removed from the sample for the next iteration. This process allows the estimation of P_f without evaluating the whole sample. This methodology is applied in the same sample used in MC, so the initial sample size of SPS is the same of MC. The probability of failure estimated through SPS is given by Eq. (7), where $\boldsymbol{\mathcal{P}}$ is a set formed by the Pareto fronts of all iterations. In total, much less function evaluations than traditional MC are carried out.

$$\hat{P}_f^{SPS}(\boldsymbol{x}) = \frac{1}{N}\sum_{\boldsymbol{x_i}\in\boldsymbol{\mathcal{P}}} I[G(\boldsymbol{x_i}) \leq 0] \tag{7}$$

3.5 Hybrid SPS-IS

The proposed hybrid approach claims for the variance reduction of IS with the reduction in the number of function evaluations promoted by SPS. The search mechanism from SPS, to identify the sample points in the failure domain, is coupled within IS after the sample generation around the MPP. The main idea of this approach is, then, evaluate the limit state function only at the required sample points to next obtain the probability of failure by IS. The proposed algorithm can be summarized in the following steps:

1. Find the MPP using FORM;
2. Generate the sample around the MPP using the IS sampling function;
3. Perform the SPS method on the sample generated in step 2, identifying and evaluating the failed points of the sample;
4. Estimate the probability of failure applying Eq. (8) to the points evaluated in step 3.

$$\hat{P}_f^{SPS,IS}(\boldsymbol{x}) = \frac{1}{N}\sum_{\boldsymbol{x_i}\in\boldsymbol{\mathcal{P}}}\frac{f_{\boldsymbol{X}}(\boldsymbol{x_i})}{h_{\boldsymbol{X}}(\boldsymbol{x_i})} I[G(\boldsymbol{x_i}) \leq 0] \tag{8}$$

4 Time-dependent reliability

The size of the corrosion defect change with time. Different models can be found in literature to predict the behavior of the defect growth rate, for instance, a linear function or a power law [22]. In this work, the defect depth growth d_g is treated as a stochastic process with gamma distribution, as it adequately represents the deterioration process with time [22]. For each elapsed time τ since the initial corrosion time t_0, d_g is a random variable following the PDF

$$f(d_g(\tau)) = \frac{\lambda^{\gamma\tau}}{\Gamma(\gamma\tau)} d_g(\tau)^{\gamma\tau-\tau} e^{-\lambda d_g(\tau)} I_{(0,\infty)}(d_g(\tau)), \tag{9}$$

where λ and γ are the scale and shape parameters of the gamma PDF, respectively, Γ is the gamma function and $I_{(0,\infty)}$ is the indicator function. The function $I_{(0,\infty)}$ is equal to 1 if $x \geq 0$ and equal to 0 otherwise.

The defect depth is then estimated as the sum of the initial defect depth d_0 and the defect depth growth, as described by Xiao *et al.* [22]. Since the statistics of d_g varies with time, the defect depth is also treated as a stochastic process in the form shown in Eq. (10). The defect length is a linear function as shown in Eq. (11),

where $\dot{l}$ is the growth rate and l_0 is the initial defect length, both random variables.

$$d(\tau) = d_0 + d_g(\tau) \tag{10}$$

$$l(\tau) = l_0 + \dot{l}\tau \tag{11}$$

5 Results and discussion

The problem presented by Xiao *et al.* [22], with a pipe made of API 5L X60 steel, is considered in this section. The random variables and stochastic processes adopted here are presented in Tab. 1, where σ_y is the yield stress, $\gamma = 4$ year^{-1} and $\lambda = 20$ mm^{-1}, following representative values of typical corrosion growth rates [22]. The initial time for corrosion is set at 2.88 years.

Table 1. Variables of the problem presented in Xiao *et al.* [22].

Variable	Mean	Standard deviation	Distribution
D (mm)	508	5.08	Normal
t (mm)	8.74	0.263	Normal
σ_y (MPa)	413.40	20.67	Normal
σ_u (MPa)	516.75	15.503	Lognormal
P_o (MPa)	8.19	0.819	Gumbel
d_0 (mm)	0.874	0.087	Weibull
d_g (mm/year)	$\gamma\tau/\lambda$	$\sqrt{\gamma\tau}/\lambda$	Gamma
l_0 (mm)	50	5	Lognormal
$\dot{l}$ (mm/year)	3	0.6	Lognormal

The time-dependent reliability analysis is performed in two parts, measuring the impact of the proposed approach on $\hat{P}_f$ and the number of function evaluations. The $\hat{P}_f$ is evaluated at each 5 years, where the elapsed time τ is the difference between the current year and t_0. In this part of the reliability analysis, the sample size for MC and SPS is 10^7, while the sample size for IS and the hybrid SPS-IS is 5000. Results are presented in Fig. 2. It is important to emphasize that the hybrid approach and IS yield identical $\hat{P}_f$ values when using the same sample, *i.e.* the proposed approach introduces no additional error to the estimate for this type of problem.

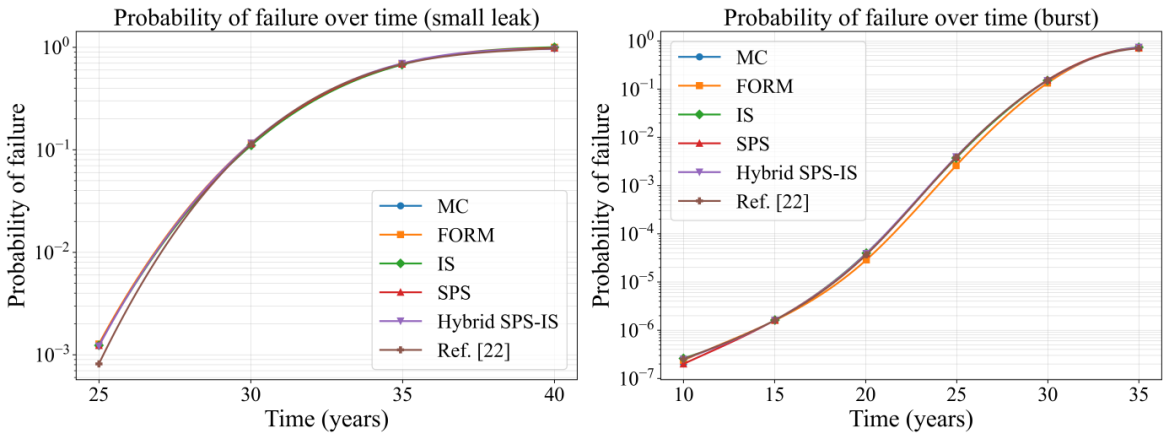


Figure 2. Probability of failure versus time, considering small leak and burst failure events.

For the second reliability analysis, the FEM software HYPLAS is adopted to estimate the burst pressure. The sample size for each time is chosen to provide a coefficient of variation $\delta(\hat{P}_f) = 5\%$, which allows the comparison of the computational cost required for each method to reach similar accuracy. D and t are treated as deterministic here, following the sensitivity analysis presented by Teixeira *et al.* [23], in order to reduce the computational cost. Table 2 shows the required number of limit state function evaluations to obtain $\delta(\hat{P}_f) = 5\%$

for both IS and the hybrid SPS-IS. Figure 3 presents the values of $\hat{P}_f$ for both IS and the hybrid SPS-IS approach, where the semi-empirical reference values from Xiao *et al.* [22] are also presented.

Table 2. Number of function evaluations for each method to reach $\delta(\hat{P}_f) = 5\%$ and reduction in the number of function evaluations promoted by the Hybrid SPS-IS when compared to IS. Enclosed values are the number of iterations.

Time (years)	Number of function evaluations			Reduction
	FORM	IS	Hybrid SPS-IS	
10	6142 (8)	5930	3883	34.52%
15	5378 (5)	5250	3789	27.83%
20	7716 (11)	7420	5307	28.48%
25	9988 (100)	3840	3037	20.91%
30	4288 (100)	1500	1162	22.53%
35	3278 (100)	490	428	12.65%

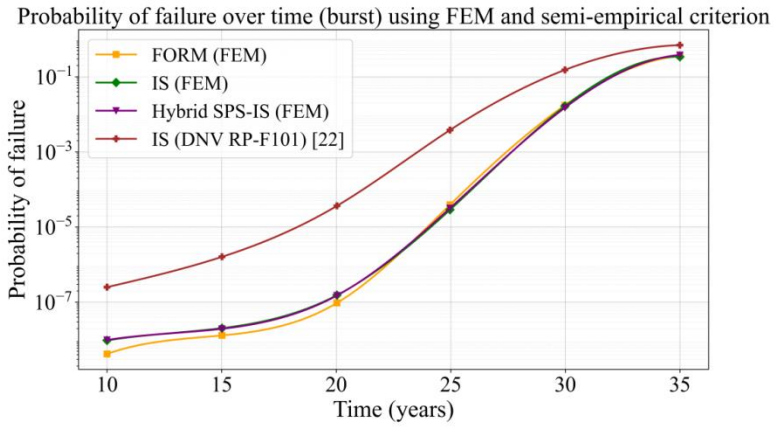


Figure 3. Probability of failure over time, considering the results for the FEM analysis and the semi-empirical criterion (DNV RP-F101).

When using the nonlinear FEM analysis with the axisymmetric model to estimate the burst pressure, as expected, less conservative results are obtained for $\hat{P}_f$ during the observed times. It can also be observed that the hybrid approach promoted a reduction in the number of function evaluations of traditional IS between 12.65% and 34.52%. As expected, the reduction promoted by SPS becomes more important for smaller values of $\hat{P}_f$. In general, the results indicate the benefit of combining the SPS method with MC-based variance reduction techniques. It is also important to observe that coupling SPS with IS did not change the accuracy of IS for this kind of problem.

6 Conclusions

This work proposed a hybrid approach to efficiently estimate the probability of failure of corroded pipelines, combining importance sampling and successive Pareto simulation. Both a semi-empirical limit state function using the DNV RP-F101 equation and a nonlinear FEM analysis were considered. As expected, the DNV RP-F101 equation produced higher values of $\hat{P}_f$ when compared to the FEM results. The variation rate of $\hat{P}_f$ with time, however, was similar for both limit state functions.

A significant reduction is obtained in the number of function evaluations required by IS when using the proposed methodology. The hybrid SPS-IS approach successfully combined the variance reduction of IS and the reduced number of function evaluations of SPS, while maintaining the accuracy of IS. The results indicate that the SPS-IS method can be a valuable alternative for reliability analysis in the assessment of corroded pipelines where defect evolution must be considered over time. This also shows that the selective process within the SPS method can be combined with variance reduction methods to enhance its potential.

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