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Stability analysis of arches under contact constraints: a MASTAN2-based study

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Abstract. This work presents a nonlinear analysis of a slender circular ring in unilateral contact with a rigid support, a configuration found in applications such as tunnel lining rings, buried reservoirs, pipelines, and circular structures partially supported on concrete saddles. The numerical formulation considers large displacements as well as geometric and material nonlinearities. The study employs the finite element method, with both elastic and inelastic analyses carried out using MASTAN2 and CS-ASA software, enabling a comparison between the obtained responses. Three configurations of rigid support height are investigated: h=0, h=R/2, and h=R. The aim is to understand the effect of contact on stability and post-buckling behavior. Numerical results are compared with previous studies for validation and discussion.

Keywords: Structural Analysis, Finite Element Method, Nonlinear Analysis, Geometric Nonlinearity, Material Nonlinearity, Unilateral Contact, MASTAN2, CS-ASA.

1 Introduction

Circular structures in partial contact with rigid surfaces appear in various engineering applications, including tunnel lining rings, buried reservoirs, pipelines, and circular structures partially supported on concrete saddles. These systems are common in underground and hydraulic works, where contact occurs only under compression, characterizing the so-called unilateral contact. Under such conditions, the structure may detach from the support when subjected to tension, significantly altering its structural response.

Unilateral contact leads to complex nonlinear responses. These effects include changes in the distribution of internal forces, contact zones that activate and deactivate, and abrupt variations in overall stiffness. Classical studies, such as that of Simo and Vu-Quoc [1], analyzed the problem in the context of soil–structure interaction. More recent research, such as Silva [2], has advanced modeling approaches by proposing reference parameters for numerical validation.

In this work, the behavior of a slender circular ring in unilateral contact with a rigid support is investigated. Three saddle geometries are evaluated, varying the support height. The analyses are performed considering second-order effects, both in the elastic and inelastic regimes. The objective is to understand how saddle height influences stability and post-buckling behavior, as well as to compare the results obtained using different computational tools.

2 Methodology

The numerical formulation adopts the finite element method (FEM) due to its capability to represent large displacements, geometric nonlinearity, and material inelasticity. The ring is discretized into beam–column elements using a corotational formulation.

Second-order elastic and inelastic analyses are performed in two programs: MASTAN2 [3] and CS-ASA [4]. The combined use of these tools allows for comparison of results obtained on platforms with different implementations, enabling verification of consistency and identification of possible variations. In MASTAN2, nonlinear analysis features are fully explored, while in CS-ASA, equivalent modeling is applied to ensure comparability.

Unilateral contact is modeled using spring elements in the normal direction. These elements transmit only compressive forces, being deactivated when tension occurs. The technique follows the procedure described by Silveira and Gonçalves [5] and is applied to the predicted contact region between the ring and the saddle.

Loading is imposed as a controlled vertical displacement at the top point of the ring. The response is monitored by tracking the displacement at the same point. Three saddle geometries are considered:

- h=0: base tangent to the bottom point of the ring.
- h=R/2: support at half the radius height.
- h=R: support over a half circumference.

The numerical solution uses the incremental–iterative Newton-Raphson method. This procedure enables tracing of the equilibrium path up to collapse, identifying instabilities and failure modes.

3 Numerical Example – Ring in Contact with a Concrete Saddle

This example investigates the structural behavior of a slender circular ring in unilateral contact with a horizontal rigid base. The loading consists of a concentrated point load applied at the top of the ring. This is a classical problem in unilateral contact analysis with large deformations. Similar cases were studied by Simo and Vu-Quoc [1] and Silva [2], enabling a direct comparison of the results.

The objective is to analyze nonlinear effects related to the activation and deactivation of contact and the influence of curvature on the propagation of the contact zone.

Figure 1 presents the geometry and loading scheme. The ring has a radius of 100, an elastic modulus of 10^9, a moment of inertia of 10^{-6}, a cross-sectional area of 0.1, and a yield stress of 36518. The contact spring stiffness is 1200, representing a rigid flat base. These values are given in units compatible with reference studies to allow direct comparison of results [1, 2].

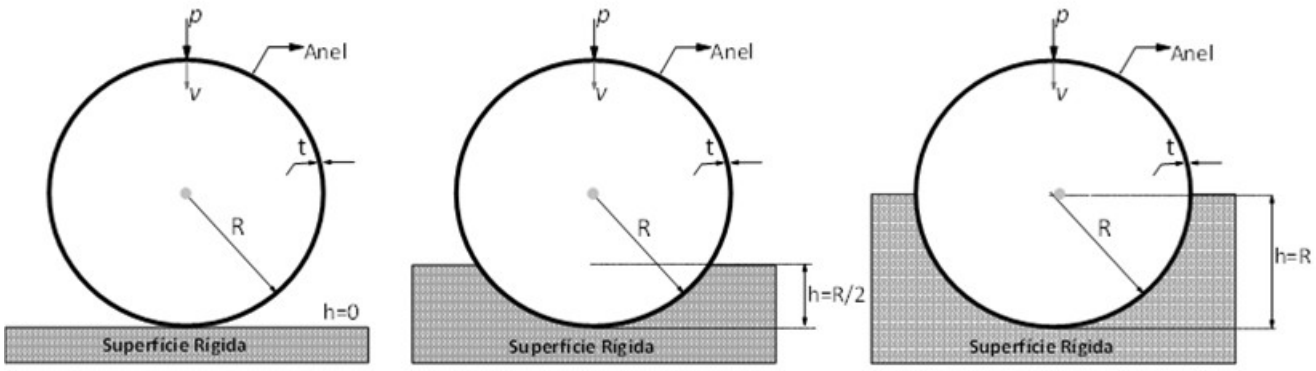


Figure 1. Geometry of the ring and configuration of the rigid saddle.

The discretization uses 50 beam–column finite elements for half of the ring, exploiting structural symmetry. The model employs a corotational formulation. The upper end is hinged, and a controlled vertical displacement is applied at the top point, where the structural response is also monitored throughout the loading process.

The rigid saddle is evaluated in the three support height configurations presented in Section 2, allowing investigation of its influence on stability and post-buckling behavior.

Unilateral contact is represented by active springs in the normal direction, transmitting only compressive forces, following Silveira and Gonçalves [5]. Figure 2 shows the finite element numerical model, highlighting the discretization of the ring and the position of the contact springs. This representation provides a clear visualization of the modeling approach used to simulate the interaction between the ring and the rigid saddle.

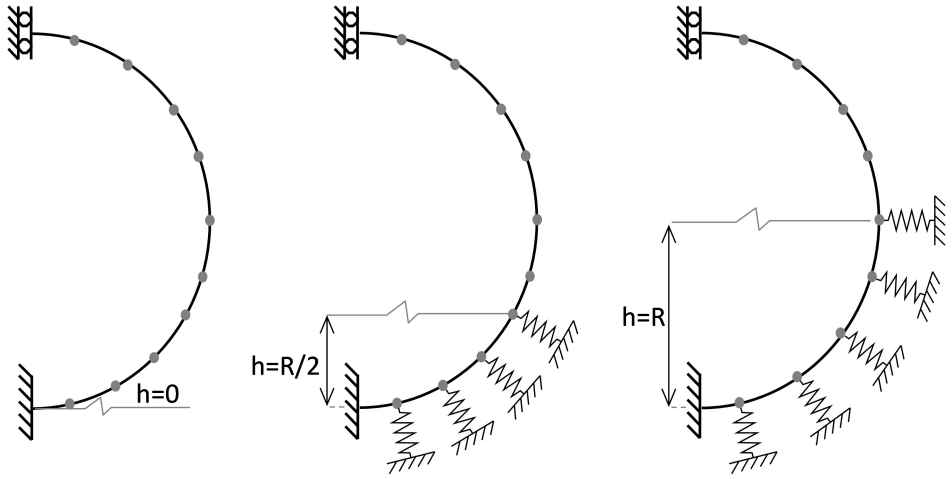


Figure 2. Finite element numerical model with ring discretization and contact springs.

4 Elastic and Inelastic Analyses (MASTAN2 and CS-ASA)

The analyses consider second-order geometric nonlinearity and the inelastic behavior of the material. Two computational platforms were employed: MASTAN2 and CS-ASA.

The tracing of the equilibrium path up to collapse is carried out using the incremental–iterative Newton-Raphson method, available in both programs.

For each saddle height, the results obtained with MASTAN2 and CS-ASA were compared with reference values from the literature [1, 2]. This approach allows verification of consistency between the platforms and identification of possible differences in predictions for elastic and inelastic behavior.

Figure 3 presents an example of the equilibrium path obtained for the ring on a rigid base, illustrating the influence of contact conditions on the curve shape and on the critical instability points.

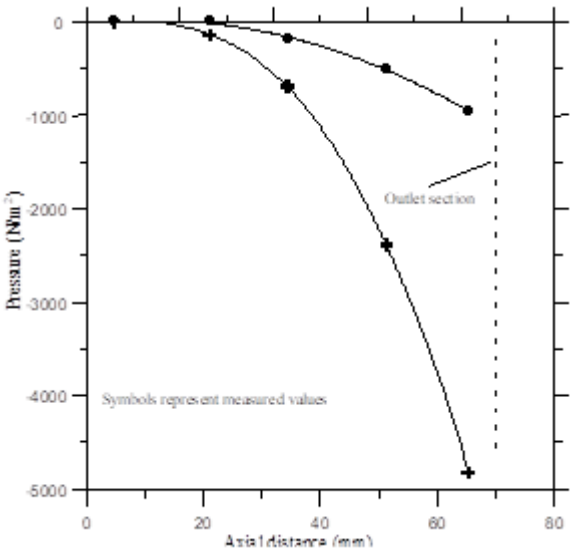


Figure 3. Equilibrium path for the ring on a rigid base (elastic and inelastic)

5 Conclusions

The study shows that the unilateral contact condition significantly influences the stability of the ring.
Variation in the saddle height modifies the instability mode and the critical load.
Differences between elastic and inelastic analyses are relevant for safety assessment.

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