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Study of the influence of adaptive refinement strategy parameters to phase-field modeling of fracture

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Abstract. The phase-field model has consolidated itself as a promising tool to handle complex cracks. In this model, the crack is represented in a diffuse form where the phase-field variable indicates the degradation of the material. To better approximate this variable, extremely refined meshes are required, which demands expensive computational efforts. In this work, an adaptive refinement strategy is adopted to phase-field modeling of fracture. In this strategy, the domain is initially discretized with a coarse finite elements mesh and during the analysis the crack regions are automatically replaced by a Smoothed Point Interpolation Methods (SPIMs) and refined. The SPIM is a meshfree method that possess the Kronecker delta property. This property facilitates the application of boundary conditions and guarantees direct coupling with finite elements. Different criteria can be used to identify the crack regions. Furthermore, parameters such as refinement level and size of the substitution region are adopted in the strategy. For the SPIM, different smoothing domains and support nodes selection are applied. The aim of this work is to evaluate the influence of these adaptive strategy parameters on the crack propagation results. Numerical simulations are performed and the results are compared with results available in the literature.

Keywords: Adaptivity, Smoothed Point Interpolation Methods, Phase-Field Modeling.

1 Introduction

The phase-field modeling of fracture was initially proposed by Francfort and Marigo [1] to simulate complex cracks. In Bourdin et al. [2], the crack is represented as a diffuse entity where the phase-field variable indicates the material degradation and a scale length parameter indicates the width of the diffuse region. When this parameter is close to zero, the crack approaches a discrete crack. Different constitutive models have been proposed in the literature to model brittle fractures [3] and quasi-brittle fractures [4]. Each of these models indicates relationships between the mesh size in the crack region and the scale length parameter. The smaller the size of the diffuse region of the crack, the more refined meshes are required.

There are different methods in the literature adopted to discretize the phase-field models, among them finite element methods (FEM) and meshfree methods. In Novelli et al. [5], the meshfree method Smoothed Point interpolation Methods (SPIMs) was used for the discretization of phase-field modeling for brittle fracture. Despite the good results presented for the phase-field distribution, a high computational cost was observed because the mesh was refined throughout the crack growth region.

Several studies propose adaptive refinement strategies, where the mesh is automatically refined only in the regions of interest, reducing computational costs. In Novelli et al. [6], an adaptive FEM-SPIM coupling strategy was proposed for phase-field modeling of cracks. In this strategy, the analysis begins with a coarse FEM mesh, and the crack regions are automatically replaced by the SPIM meshfree method and refined. The present work aims to analyze the influence of different parameters of the adaptive strategy proposed in relation to the phase-field distribution and equilibrium trajectories. For the analyses, simulations are performed using INSANE (Interactive Structural ANalisys Environment) [1].

[1]More information on the project can be found at https://www.insane.dees.ufmg.br/; the development code is freely available at the Git

2 Phase-field modeling

In the phase-field modeling, the total energy functional (E_t) of the body is expressed by:

$$E_t = \int_\Omega \psi(\boldsymbol{\varepsilon}(\boldsymbol{u}), \phi) dV + \int_\Omega G_c \gamma(\phi, \nabla\phi) dV - \int_\Omega \boldsymbol{b} \cdot \boldsymbol{u} dV - \int_{\partial\Omega} \boldsymbol{t} \cdot \boldsymbol{u} dA \tag{1}$$

where ψ is the strain energy density, $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{u}$ is the displacement vector, G_c is the Griffith's critical energy release rate, γ is the crack surface density function, $\boldsymbol{b}$ are body forces and $\boldsymbol{t}$ are surface forces.

The strain energy is given by:

$$\psi(\boldsymbol{\varepsilon}(\boldsymbol{u}), \phi) = g(\phi)\psi_0^+(\varepsilon) + \psi_0^-(\varepsilon) \tag{2}$$

where the $g(\phi)$ is the energy degradation function responsible for degrading the tensile parts of the domain.

For quasi-brittle fractures, Wu [4] porposed the $g(\phi)$ function:

$$g(\phi) = \frac{(1-\phi)^p}{(1-\phi)^p + Q(\phi)}, \qquad Q(\phi) = a_1\phi + a_1a_2\phi^2 + a_1a_2a_3\phi^3 \tag{3}$$

where the exponent coefficient $p > 0$ and $a_i (i = 1, 2, 3)$ are calibrated from standard material properties. For the Cornelissen softening law, $\xi = 2$ and $p = 2$:

$$a_1 = \frac{4E_0G_f}{c_0 l_0 f_t^2} \qquad a_2 = 1.3868 \qquad a_3 = 0.6567 \tag{4}$$

where E_0 is the Young's modulus and f_t is the failure tension strength.

In Wu [4], the crack surface density function is proposed as:

$$\gamma(\phi, \nabla\phi) = \frac{1}{c_0}[\frac{1}{l_0}\alpha(\phi) + l_0|\nabla\phi|^2] \tag{5}$$

where $c_0 := 4\int_0^1 \alpha^{\frac{1}{2}}(\hat{\phi})d\hat{\phi}$, l_0 is the length scale parameter that control the crack width and α is called geometric crack function.

The geometric crack function is given by:

$$\alpha(\phi) = \xi\phi + (1-\xi)\phi^2 \in [0, 1] \qquad \forall\phi \in [0, 1] \tag{6}$$

where the parameter $\xi \in [0, 2]$. For quasi-brittle fracture, Wu [4] adopted the geometric crack function $\alpha = 2\phi - \phi^2$ with $\xi = 2$.

The value of displacements $\boldsymbol{u}$ and phase-field ϕ are found minimizing the total energy functional (Eq. 1). This minimization result in a system with two equations, an related to displacements and other related do phase-field.

3 Adaptive FEM-SPIM Coupling

The adaptive FEM-SPIM coupling strategy adopted in this work was proposed in Novelli et al. [6] and it is illustrated in Fig. 1. In this strategy, the analysis starts with a coarse FEM mesh, Fig. 1(a), and during the propagation of the crack, regions of the domain are substituted by the SPIM method and refined. The first stage of the analysis consists into identifying the finite elements that will be replaced by the SPIM method, Fig. 1(b). Different criteria can be used to identify the region of the domain and a scale parameter can be expand the substitution region. In this stage, the integration point of the triangular finite element can be converted into a new node and a new triangulation is generated, Fig. 1(c). The second stage aims to refine the meshfree region. There are criteria that identify cells that need a refinement. For the refinement, a new node is inserted at the centroid of each cell of the meshfree region, Fig. 1(d). A new triangulation is generated and new smoothing domains are created, Fig. 1(e). More levels of refinement can be added to the region depending on the needs of the analyzed model.

The adaptive process is activated at the end of each converged load step and it is divide in two stages: substitution of the FEM to SPIM and refinement of the SPIM region. These modifications of the mesh do not occur simultaneously, the mesh the model is either replaced by SPIM or refined. Correction iterations of the incremental-iterative process are performed after each modification to ensure the equilibrium of the model.

All the process is automatic and the user must provide some parameters for the analysis. The main parameters will be presented below. Information regarding the transfer of constitutive variables and state variables will not be approached here and can be found in Novelli et al. [6].

repository http://git.insane.dees.ufmg.br/insane/insane.git.

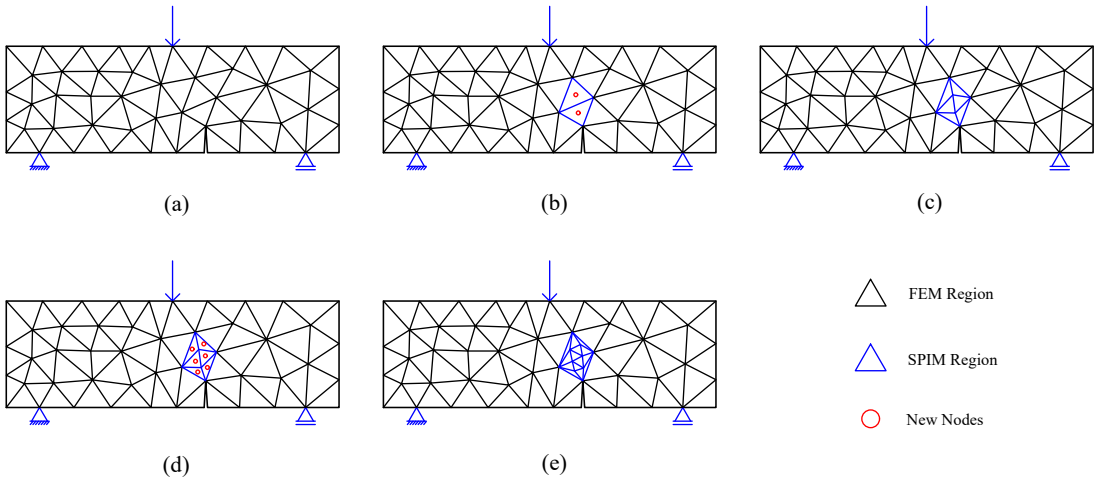


Figure 1. Adaptive strategy.

- **Substitution criterion:** This parameter refers to the criterion used to detect the finite elements of the domain that will be replaced by the meshfree region. Four different criteria can be used. The Shao criterion was proposed by Shao, Duan, and Qiu [7] and it considers the combination between the phase-field parameter and the strain energy. The phase-field criterion considers a critical value of the phase-field. In the strain energy criterion, the elements are replaced when the strain energy of the element is superior factor multiplied by the average strain energy of the domain. The equivalent strain criterion considers a limit of equivalent strain for substitution.
- **Factor for substitution:** The second parameter refers to the critical factor for the substitution.
- **Scale of substitution:** This parameter is responsible for the size of the substitution region. After an element has been detected for replacing, that element and the neighbouring elements within the size of the substitution region are replaced by an SPIM discretization.
- **Level of refinement:** This parameter indicates the final level of refinement of the meshfree region.
- **Refinement criterion:** After the substitution of the finite elements by the SPIM, the region can be refined. There are refinement criteria that are responsible for detecting the region of the meshfree domain that will be refined. The criteria are based on phase-field value, strain energy and equivalent strain. In all the criteria, the cells of the domain that present the value of the parameter greater than a factor multiplied by the average value of the domain are refined.
- **Critical factor for refinement:** This parameter is the factor that is multiplied by the average value of the meshfree domain and is used to indicate if the cell will be refined.
- **Smoothing domain and T-schemes:** The last parameter refers to the discretization of the meshfree domain. For this discretization can be used the smoothing domain of the type cell-based or edge-based and to selection of the support nodes used the T-schemes. T4 schemes is used for cell-based and T3 or T6/3 is used for edge-based.

4 Numerical simulations

The aim of this section is to illustrate the effects of the different parameters of the adaptive strategy. The test considered for this study is the 3 point bending test with a decentralized notch proposed by García-Álvarez et al. [8] illustrated in Fig. 2. The thickness is 50 mm. The constitutive model of Wu [4] is adopted and plane-stress is assumed. The Cornelissen softening law is employed with $p = 2$, $a_2 = 1.3868$ and $a_3 = 0.6567$. The crack shape function with $\xi = 2$ is used. The material parameters are: Young's modulus E = 33800 N/mm^2, Poisson's ratio $\nu = 0.2$, critical energy release rate G_c = 0.08 N/mm and length scale parameter l_0 = 4 mm. The RPIMp was used with the exponential function and $c = 1$ for the construction of the shape functions of the meshfree region.

The initial FEM mesh considers a element size of 20 mm in the whole domain as illustrated in Fig. 2. In the non linear analyses, the displacement control method was adopted incrementing vertical displacement of -0.00005 mm in the load point. The load applied was -6200 N. The equilibrium paths presented the factor load versus the *crack mouth opening displacement* (CMOD).

The parametric study was performed considering a fixed set of default parameters, and changing them one at the time for each case of the study. The default parameters are:

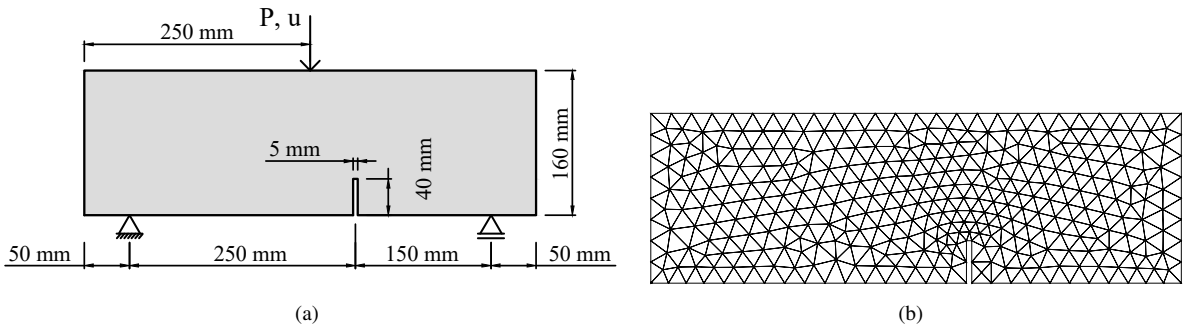


Figure 2. García bending: (a) Geometry, loading and boundary conditions, (b) Initial mesh.

- Substitution criterion - Shao criterion
- Critical factor for substitution - $factor = 0.2$ and $PF_{critical} = 0.01$
- Scale of substitution - $scale = 2$
- Level of refinement - $ref = 2$
- Refinement criterion - phase-field
- Critical factor for refinement - $factor = 0.4$
- Smoothing domain and T-schemes - Edge-based T3 scheme

4.1 Substitution criterion

The first parameter to be analyzed is the substitution criterion. Four different criteria were considered: Shao, phase-field, strain energy and equivalent strain. Fig. 3 illustrates the equilibrium paths obtained for the all substitution criteria. In general, all the methods presented similar equilibrium paths. The curve of strain criterion presented jumps referents to substitution step. The Shao and strain energy criteria substitute and refine the mesh in the first step of the analysis. The contour plots of the phase-field are depicted in Fig. 4. It is possible to observe that strain energy presented a larger substitution area including the region of application of the load.

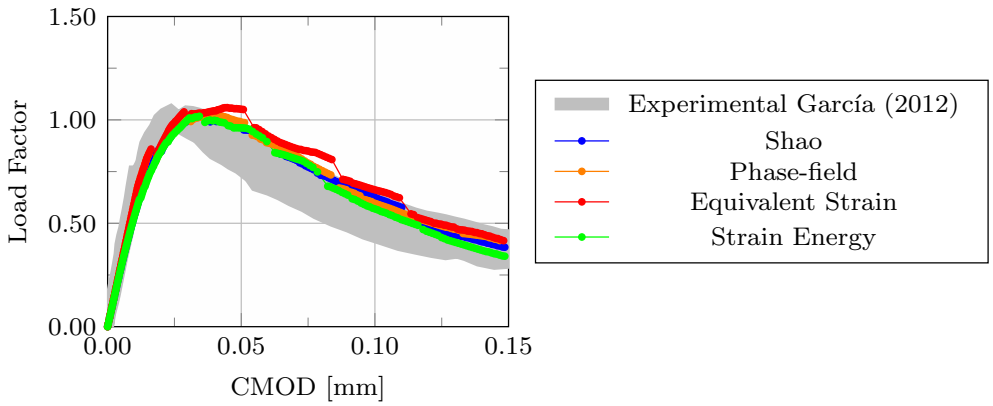
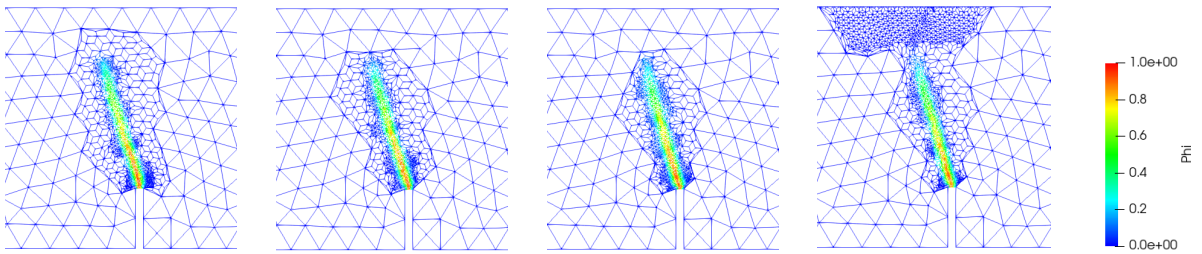


Figure 3. Equilibrium paths: Substitution criteria.

4.2 Critical factor for substitution

The second parameter refers to the critical factor for the substitution. The substitution criterion used is the Shao, the value of the $factor = 0.2$ is constant and the phase-field critical is modified ($PF_{critical}$). Fig. 5 show the equilibrium paths obtained for the three phase-field critical: 0.01, 0.05 and 0.1. It can be observed that smaller values of the factor generate fewer jumps in the curves. This is because the substitution stage occurs in regions of the domain where the phase-field value is still small, meaning the crack has not yet propagated.



(a) Shao (b) Phase-Field (c) Equivalent strain (d) Strain Energy

Figure 4. Phase-field contour plots: Substitution criteria.

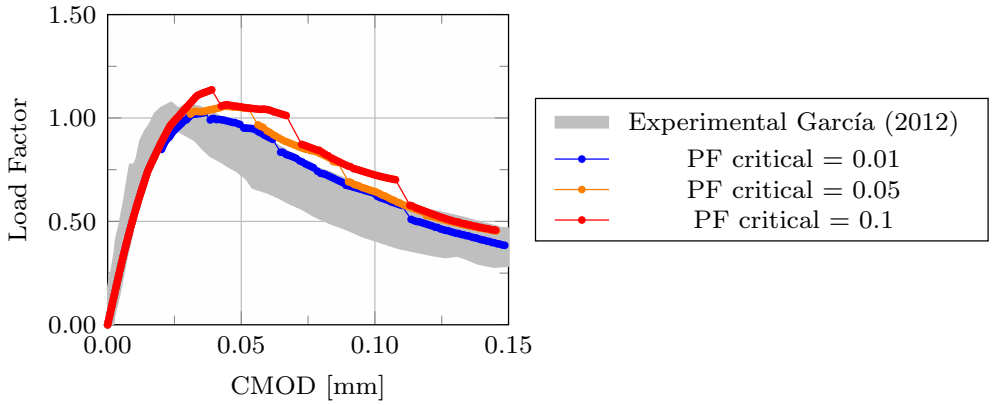


Figure 5. Equilibrium paths: Phase-field critical.

4.3 Scale of substitution

Three scale parameters were adopted in the analysis: $scale = 1, 2$ and 3. The results obtained for the equilibrium paths were similar for the three scale factors and not be presented. The similarity of the curves is attributed to the fact that the substitution factor is small and all replacement scales encompass the crack region.

4.4 Level of refinement

For this analysis, three levels of refinements are performed: $ref = 1$, $ref = 2$ and $ref = 3$. The equilibrium paths are plotted in Fig. 6. Other three analyses are presented for comparison: FEM with the original mesh, only substitution without refinement and substitution with the convert of the integration point for new node. It is possible to observe that the higher the refinement leads to a lower the peak of load. The phase-field contour plots are shown in Fig. 7.

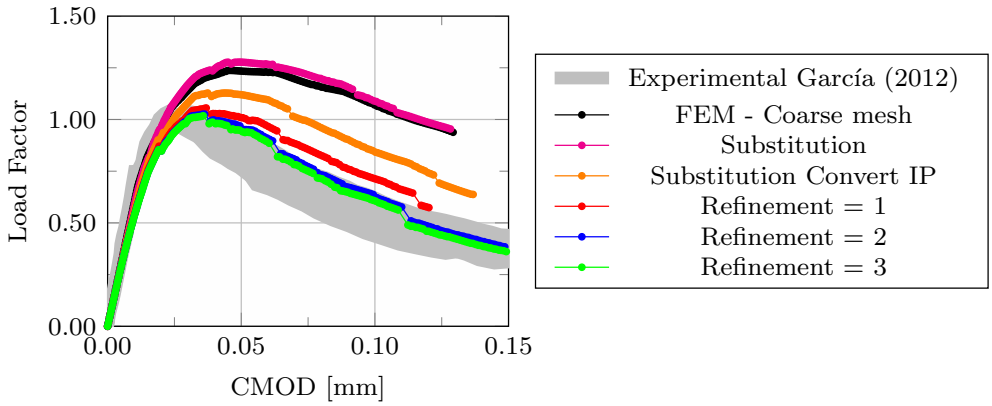
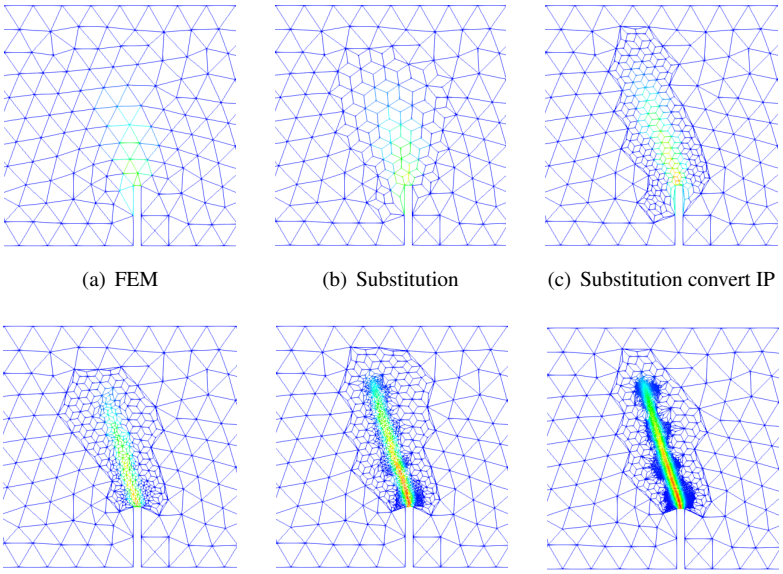


Figure 6. Equilibrium paths: Level of refinement.



(a) FEM (b) Substitution (c) Substitution convert IP

(d) Ref=1 (e) Ref=2 (f) Ref=3

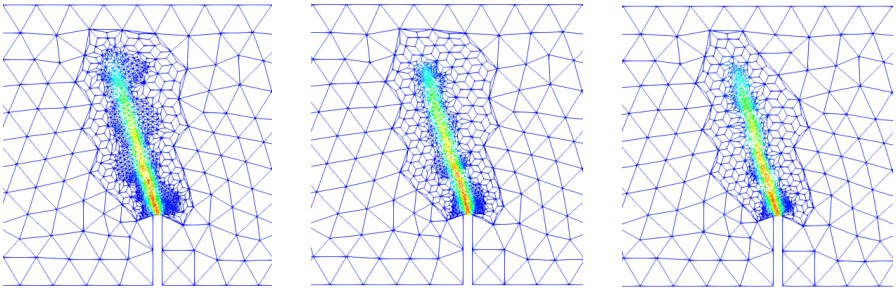
Figure 7. Phase-field contour plots: Level of refinement.

4.5 Refinement criterion

The equilibrium paths obtained for de different refinement criteria are similar and will not be presented. In relation to the phase field contour plots, considering the same refinement factor, the phase field criterion presents a smaller refinement concentration range compared to equivalent strain and strain energy.

4.6 Critical factor for refinement

Again, three critical factors are analyzed: $factor = 0.1, 0.4$ and 0.9. The equilibrium paths are similar and will not be presented. Fig. 8 illustrate the phase-field contour plots for the different values of the factor. It is possible to observe that the refined region is larger for the smallest factor.



(a) Factor Ref=0.1 (b) Factor Ref=0.4 (c) Factor Ref=0.9

Figure 8. Phase-field contour plots: Critical factor for refinement.

4.7 Smoothing domain and T-schemes

This parameter refers to the discretization of the meshfree domain. For this discretization, either a cell-based or edge-based smoothing domain can be used, and the selection of the support nodes is carried out using T-schemes. T4 schemes is used for cell-based and T3 or T6/3 is used for edge-based. The equilibrium paths for these smoothing domains are present in Fig. 9. The results obtained are similar for three strategies.

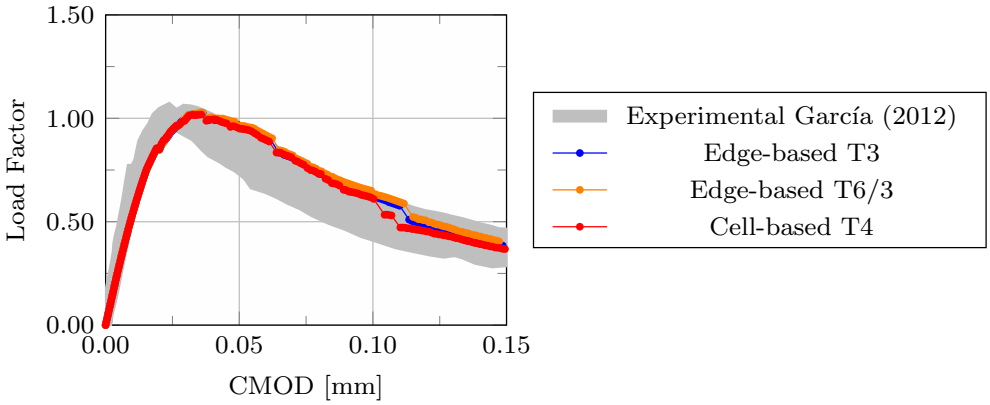


Figure 9. Equilibrium paths: Smoothing domain.

5 Conclusions

This paper presented an analysis of the influence of the adaptive strategy parameters proposed by Novelli et al. [6] on crack propagation. The strategy refers to coupled FEM-SPIM for phase-field modeling. The parameters are related to the substitution and refinement criteria, the value of the refinement factors, the level of refinement and the type of smoothing domains. Numerical simulations were performed analyzing each parameter individually. The equilibrium paths and contour plots for phase-field were shown.

In general, all curves showed good agreement with the experimental range. All criteria were able to correctly detect the crack propagation region. It was observed that the lower the values of the substitution factors, the smaller the jumps in the curves. The level of mesh refinement has a great influence on the equilibrium trajectory, emphasizing the importance of using an adequate mesh size in the crack region.

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