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On the influence of waves and seismic loads on granular seabed stability

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Abstract. A stability study of granular seabed subjected to cyclic wave loading and seismic actions based on limit analysis in terms of effective stress, was conducted. Seismic loading is considered using the pseudo-static method, while wave loading is modeled by the first-order Stokes theory. The adopted formulation incorporates the effects of seepage forces arising from pore pressure gradients induced both by wave pressure oscillations and by soil densification due to cyclic shearing of the granular material. The analysis is conducted using the kinematic approach, based on representative failure mechanisms, and the Mohr-Coulomb strength criterion is adopted. Numerical results highlight the significant destabilizing influence of seismic loading, especially when combined with wave-induced cyclic loading.

Keywords: water waves, stability of submarine slopes, pseudo-static method.

1 Introduction

The increasing development of offshore infrastructure, particularly for energy production, has intensified the need to understand seabed responses to wave loading. In shallow waters or under severe wave conditions, pressure variations induced by surface waves can cause significant soil displacements, compromising the stability of structures such as pipelines and platforms (He et al. [1]).

Madsen [2] and Yamamoto et al. [3] formulated analytical poroelastic solutions based on linear wave theory, and Dormieux and Delage [4] extended this analysis to more general wave loading. More recent studies have highlighted the need to consider irreversible soil densification and the associated excess pore pressure resulting from cyclic shear loading (Rahman and Jaber [5]).

Building on the simplified framework proposed by Dormieux et al. [6], this study aims to extend the evaluation of seepage forces in granular seabeds by incorporating the effects of finite layer thickness and pseudo-static seismic loading. The analysis is developed using a kinematic approach within the effective stress framework, assuming a cohesionless Mohr–Coulomb yield criterion. The adopted methodology combines analytical modeling of the wave-induced poroelastic response with an upper-bound limit analysis formulated in terms of effective stresses, in which seismic effects are represented through a pseudo-static formulation and wave loading follows the first-order Stokes theory. Representative kinematic failure mechanisms are employed to evaluate the combined influence of hydrodynamic and inertial loads on seabed stability, providing a consistent framework for assessing how geometric and loading parameters affect the overall stability of granular seabeds subjected to simultaneous wave and seismic actions.

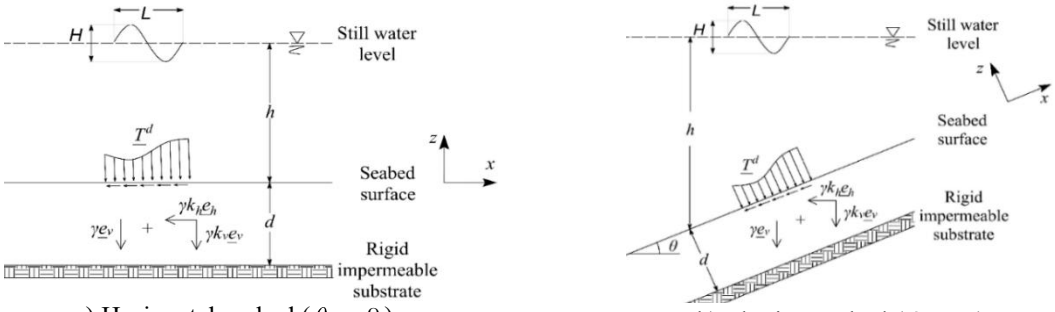
2 Statement of the stability problem

The seabed is modeled as a half-space material domain Ω whose upper boundary surface $z = 0$ is inclined at an angle $\theta \neq 0$ with respect to the horizontal plane (Fig. 1b). The inclination angle is assumed to be notably small $\theta \ll 1$. The soil layer of thickness d is delimited at the top by the water-soil interface located at $z = 0$, where the positive axis points toward the sea surface, and at the bottom by a rigid and impermeable substrate

located at $z = -d$. The particular case of a horizontal seabed where $\theta = 0$ is also sketched in Fig. 1a.

The first components of the loading are defined by the gravity forces $\gamma \underline{e}_v$ in which γ denote the specific weight of porous material, $\underline{e}_v$ and $\underline{e}_h$ are defined as:

$$\underline{e}_v = -(\sin\theta \underline{e}_x + \cos\theta \underline{e}_z) \qquad \underline{e}_h = -\cos\theta \underline{e}_x + \sin\theta \underline{e}_z \,. \tag{1}$$



a) Horizontal seabed ($\theta = 0$) b) Sloping seabed ($\theta \neq 0$)

Figure 1 - Seabed geometry under wave and seismic loading.

The pseudo-static forces are the horizontal $k_h \gamma \underline{e}_h$ and vertical $k_v \gamma \underline{e}_v$ components, where k_h and k_v denote, respectively, the average horizontal and vertical seismic coefficients that stand approximately for the intensity of the distributed earthquake-induced inertia forces. The surface forces $\underline{T}^d$ acting along the upper boundary are:

$$\underline{T}^d = \underline{T}^d_s = T^d_s \underline{e}_z = -(p_w(x,t) + \gamma_w h(x))\underline{e}_z \qquad (z = 0) \tag{2}$$

where $h = h(x)$ denotes the local water depth calculated from the horizontal still sea water level. It may be observed that the value of h is constant for the horizontal plane seabed (Fig. 1a), whereas $h(x) = h_0 - x\sin\theta$ decreases linearly with coordinate x in the case of sloping seabed (Fig. 1b), and h_0 being the water depth at the origin of x -axis. In the above expression, the term $p_w(x,t)$ refers to the overpressure induced by the wave. The wave loading is modeled through an overpressure $p_w(x,t)$ acting on the planar sea floor $z = 0$, with maximum amplitude p_0 . In the context of first order (linear) Stokes theory, which is usually used in engineering practice, the expression of cyclic overpressure is given as (Rahman and Jaber [5]):

$$p_w(x,t) = p_0 \sin\psi = \frac{\gamma_w H \,/\, 2}{\cosh \lambda h} \sin\psi \quad \text{with} \quad \psi = \lambda x - \omega t = \frac{2\pi}{L} x - \frac{2\pi}{T} t \tag{3}$$

where h is the local water depth at rest, γ_w is the unit weight of water, λ and ω is the wave number and wave frequency, respective, T is the wave period and $L = L(h)$ is the wavelength.

The formulation of the stability analysis problem within the framework of limit analysis theory requires a previously defined introduction to the strength capacity of the saturated porous material. The latter is expressed under a general yield condition form $f(\underline{\underline{\sigma}}, p) \leq 0$, where $\underline{\underline{\sigma}}$ and p denote respectively the stress and pore pressure fields in the porous seabed medium. For a given time t within the time interval considered for the study, the stability condition of the seabed structure under applied loading mode stems from compatibility between the equilibrium of the considered seabed structure and the resistance of its constituent porous material:

$$\text{Stabiliy} \quad \Leftrightarrow \quad \exists\, \underline{\underline{\sigma}} \text{ such that } \begin{cases} \text{div}\underline{\underline{\sigma}} + \gamma \underline{e}_v + \gamma \underline{e}_a = 0 & \text{on } \Omega \\ \underline{T} = \underline{T}^d & \text{along } z = 0 \\ f(\underline{\underline{\sigma}}, p) \leq 0 & \text{on } \Omega \end{cases} \tag{4}$$

where $\underline{T} = \underline{\underline{\sigma}} \,.\, \underline{e}_z$ is the stress vector acting upon the upper surface of the seabed, while the prescribed surface loading is given by eq. (2). In seabed soils composed of granular materials, the repeated action of waves leads to an increase in pore water pressure, which can greatly reduce stability. Therefore, using total stress analysis is not appropriate; instead, stability should be evaluated based on effective stress. The excess pore pressure u in the soil is defined as the difference between the actual pore pressure and the hydrostatic pressure:

$$u(x,\, z,\, t) = p(x,\, z,\, t) - \gamma_w \Big[\, h(x) - z\cos\theta \,\Big]. \tag{5}$$

It must therefore satisfy the following boundary condition:

$$u(x,\, z=0,\, t) = p_w(x,\, t) = p_0 \sin\psi \;\text{ with }\; \psi = \lambda x - \omega t\,. \tag{6}$$

This method assumes that the strength of saturated soils is controlled by Terzaghi's effective stress tensor $\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} + p\,\underline{\underline{1}}$. As a result, the seabed stability under external loads is described using effective stress parameters:

$$\textit{Stabiliy} \quad \Leftrightarrow \quad \exists\, \underline{\underline{\sigma}}' \;\; \textit{such that} \quad \begin{cases} \text{div}\underline{\underline{\sigma}}' + \gamma'\underline{e}_v + \gamma\underline{e}_a + \gamma_w\,\underline{i} = 0 & \text{in } \Omega \\ \underline{T}' = 0 & \text{along } z = 0 \\ f(\underline{\underline{\sigma}}') \le 0 & \text{in } \Omega \end{cases} \tag{7}$$

where $\gamma' = \gamma - \gamma_w$ is the buoyant unit weight, $\underline{e}_a = k_h\underline{e}_h + k_v\underline{e}_v$ is the loading related to pseudo-static forces and vector $\underline{i}$ refers to the hydraulic gradient field:

$$\underline{i} = -\frac{1}{\gamma_w}\,\text{grad}\; u\,. \tag{8}$$

The formulation of the stability condition using effective stresses shows that seepage forces can be treated as external body forces given by $\gamma_w\,\underline{i}$. These forces, which drive the hydraulic flow within the soil, originate from the gradient of excess pore pressure $-\,\text{grad}\; u$ and are linked to the filtration velocity according to Darcy's law. Within the effective stress framework, the loading can be symbolically represented by the vector $\underline{Q} = \;\gamma'\underline{e}_v\,,\, \gamma\underline{e}_a,\, \gamma_w\,\underline{i}\;$, with the effective surface forces on the seabed's top boundary $\underline{T}'^{d} = 0$ considered to be zero. For the strength characterization of the granular seabed soil, considered as a homogeneous and isotropic material, the cohesionless Mohr-Coulomb yield criterion will be used in the following effective stress analysis:

$$f(\underline{\underline{\sigma}}') = \sup\,\{\sigma'_i\,(1+\sin\phi) - \sigma'_j\,(1-\sin\phi) \quad | \quad i,\, j = 1,\, 2,\, 3\} \le 0 \tag{9}$$

where σ'_i with $i \in \{1,2,3\}$ are the effective principal stresses, and the strength parameter ϕ is the internal friction angle.

Dormieux et al. [4] showed that the excess pore pressure u can be separated into two independent components:

$$u = u^{el} + U \tag{10}$$

where u^{el} is the periodic poroelastic fluctuation with average equal to zero over the wave period T and U is the drift term corresponding to the mean value of u over one cycle of period T. It is convenient to introduce the elastic and irreversible parts of the hydraulic gradient $\underline{i} = \underline{i}^{el} + \underline{i}^{ir}$, where:

$$\underline{i}^{el} = -\frac{1}{\gamma_w}\,\text{grad}\; u^{el} \qquad \text{and} \qquad \underline{i}^{ir} = -\frac{1}{\gamma_w}\,\text{grad}\; U\,. \tag{11}$$

The periodic pore pressure fluctuation u^{el} and stress variation $\delta\underline{\underline{\sigma}}^{el}$ are solutions to a poroelastic problem stated on the seabed subjected to wave cyclic loading $p_w(x,\, t)$.

2.1 Formulation of the wave-induced poroelastic response

As regards the hydraulic gradient $\underline{i}^{el} = -\text{grad}\; u^{el}\;/\;\gamma_w$ associated with the poroelastic fluctuation, which represents the periodic component of seepage forces in the seabed stability analysis, its given by (Quiuqui et al. [7]):

$$\underline{i}^{el} = -\frac{p_0}{\gamma_w(1-2\nu)}\Big[\,\lambda\,(-\mathsf{H}_u(z)\cos\psi + \mathsf{J}_u(z)\sin\psi)\,\underline{e}_x - (\mathsf{H}_u{}'(z)\sin\psi + \mathsf{J}_u{}'(z)\cos\psi)\,\underline{e}_z\,\Big] \tag{12}$$

where expressions of the non-dimensional functions $\mathsf{H}_u(z)$ and $\mathsf{J}_u(z)$ are also provided in (Quiuqui et al. [7]).

To illustrate the results, the following set of parameters was considered: coefficient of permeability of the seabed soil $k = 10^{-4}$ m / s , shear modulus of the soil skeleton $G = 10$ MPa , Poisson's ratio of the soil $\nu = 1/3$, effective unit weight of the soil (equal to the unit weight of water) $\gamma' = \gamma_w = 10$ kN / m^3 , wave period $T = 10$ s , wave steepness, defined as the ratio of wave height to wavelength $H_0 / L_0 = 0.06$, wave-induced overpressure amplitude at the seabed surface $p_0 = 31.3$ kPa , water depth $h = 15$ m and dimensionless seabed thickness, expressed as the ratio between the soil layer thickness d and the wavelength L $d / L = 0.25,\ 0.5,\ 1,\ \infty$.

Figure 2 illustrates the excess pore pressure fluctuation profiles u^{el} within the soil for different layer thicknesses $d / L \in \{0.25,\ 0.5,\ 1, \infty\}$. The configuration labeled as $d / L \to \infty$ corresponds to the case of an infinitely deep seabed, as examined by Dormieux and Coussy [8]. The profiles shown represent the maximum excess pore pressure beneath the wave crest (i.e., $\psi = \pi / 2$). As expected, the vertical distribution of u^{el} is notably influenced by the normalized seabed thickness when $d / L \leq 1$ is small. For larger thickness values, typically around $d / L > 1$, the pore pressure profile tends to that of the infinite-depth solution. Across all considered thickness ratios, the excess pore pressure decreases continuously with depth from the upper surface, with a change in the curvature of the vertical profile occurring near the impermeable bottom boundary.

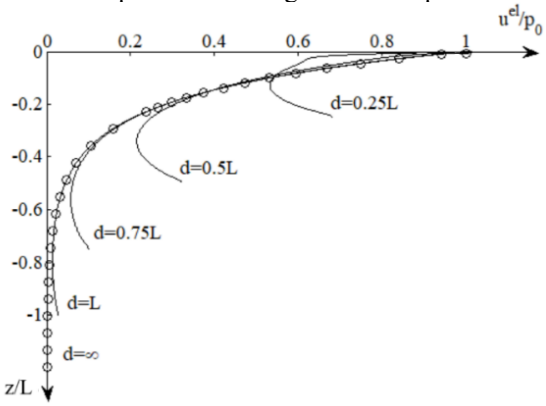


Figure 2– Profiles of normalized wave-induced pore pressure in the soil mass.

2.2 Evaluation of inelastic hydraulic gradient

The calculation of seepage forces $\gamma_w \underline{i}^{ir} = -\text{grad}\, U$ related to the drift component of the excess pore pressure, as defined in eq. (11), is theoretically derived from the solution of a poromechanical boundary value problem. This problem is formulated for the seabed material system using the average quantities and over a single loading cycle. The diffusion equation of the pore pressure U takes the following form (Dormieux et al. [4]):

$$\frac{1}{T_c}\frac{\partial U}{\partial \bar{t}} = c_m \nabla^2 U - K \Delta\varepsilon_v^{ir} / T \tag{13}$$

where T_c is the characteristic time of consolidation, c_m is the fluid diffusivity coefficient, K is the drained elastic bulk and $\Delta\varepsilon_v^{ir}$ is the variation of the contractive irreversible volumetric strain. In addition, the pore pressure U is subjected to the following boundary conditions:

$$U = 0 \quad \text{at } z = 0; \qquad \frac{\partial U}{\partial z} = 0 \quad \text{at } z = -d \qquad \forall\, \bar{t} \tag{14}$$

together with zero initial condition

$$U = 0 \quad \text{for } \bar{t} = 0\,. \tag{15}$$

Figure 3 presents the results obtained for different values of the ratio d / L. Following an initial increase with the number of loading cycles N, the hydraulic gradient $i^{ir}(0)$ reaches a peak and then gradually declines. This decreasing phase is likely associated with the experimental model in eq. (13), which accounts for a reduction in $\Delta\varepsilon_v^{ir}$ as the soil becomes denser.

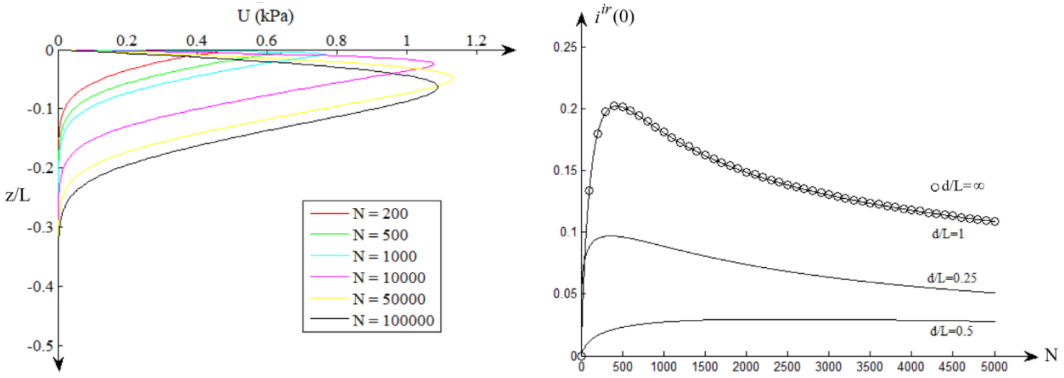


Figure 3– Pore pressure U for $d / L = 1$ and irreversible hydraulic gradient at the seabed surface $i^{ir}(0)$.

2.3 Effective stress-based upper bound kinematic approach

The failure mechanism analyzed in this study involves a triangular wedge undergoing uniform translational motion, while the rest of the seabed remains stationary. As illustrated in Fig. 4, this mechanism is characterized by geometric parameters ϖ, $l = AB$ and the inclination χ of $\underline{U}$ with respect to sides MA. For the wedge MAB geometry to be valid, these angular parameters must satisfy the following conditions:

$$-\pi / 2 + \theta < \varpi < \theta . \tag{16}$$

The vertex M of the translating triangular block is positioned near the upper surface, such that its coordinate z_M falls within the region $z_* \leq z_M \leq 0$. This failure mechanism features velocity discontinuities $[\underline{U}] = \underline{U}$ inclined at an angle χ relative to lines MA and MB. The translational velocity of the wedge is expressed by:

$$\underline{U} = U\left[-\cos(\chi + \theta - \varpi)\underline{e}_x + \sin(\chi + \theta - \varpi)\underline{e}_z\right] \tag{17}$$

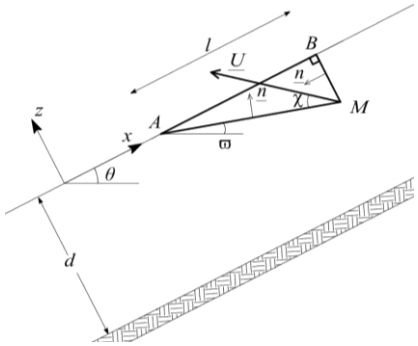


Figure 4 – Geometry of the translational failure mechanism.

The necessary stability condition of the seabed under applied loading is derived by applying the fundamental kinematic inequality $P_{ext}(\underline{U}) \leq P_{mr}(\underline{U})$ to the above failure mechanism. For this purpose, it is necessary to compute the expressions of maximum resisting work rate $P_{mr}(\underline{U})$ and external work rate $P_{ext}(\underline{U})$ developed in the failure mechanism. In that context, the rate of maximum resisting work reduces to $P_{mr}(\underline{U}) = 0$ and the kinematic theorem yields the following necessary condition for stability:

$$P_{ext}(\underline{U}) \leq P_{mr}(\underline{U}) = 0 \tag{18}$$

that should be satisfied for any set of parameters ϖ, l, χ complying with conditions (16). The following restriction on the slope of the speed jump must be respected $\phi \leq \chi \leq \pi/2 - \phi + \varpi - \theta$.

In the framework of effective stress stability formulation (7), the rate of external work $P_{ext}(\underline{U})$ includes the contributions of gravity forces $\gamma' \underline{e}_v$ (gravity minus buoyancy), seismic components associated with the gravity forces $\gamma \underline{e}_a$ and seepage forces $\gamma_w \underline{i} = \gamma_w \underline{i}^{el} + \gamma_w \underline{i}^{ir}$:

$$P_{ext}(\underline{U}) = \int_{MAB} \gamma' \underline{e}_v \cdot \underline{U}\, dS + \int_{MAB} \gamma \underline{e}_a \cdot \underline{U}\, dS + \int_{MAB} \gamma_w \underline{i}^{el} \cdot \underline{U}\, dS + \int_{MAB} \gamma_w \underline{i}^{ir} \cdot \underline{U}\, dS \,. \quad (19)$$

Substituting the above expressions into (19), inequality (18) reads

$$\begin{aligned} -\gamma' \sin(\chi - \varpi) + \gamma \left[k_h \cos(\chi - \varpi) - k_v \sin(\chi - \varpi) \right] + \gamma_w \sin(\chi + \theta - \varpi) \langle i^{ir} \rangle \\ - \frac{p_0}{1 - 2\nu} \cos(\chi + \theta - \varpi) \langle \mathsf{A}_u \rangle + \sin(\chi + \theta - \varpi) \langle \mathsf{B}_u \rangle \leq 0 \end{aligned} \quad (20)$$

in which symbol $\langle . \rangle = 1/S \int_{MAB} . \, dS$ denotes the average value over the area MAB, and $\langle \mathsf{A}_u \rangle, \langle \mathsf{B}_u \rangle$ are given as a function of $\mathsf{H}_u(z)$ and $\mathsf{J}_u(z)$. The most critical stability condition derived from considered class of failure mechanisms is obtained by maximizing the left-hand side in the above inequality with respect to the set of parameters ϖ, l, χ complying with condition (16), $\phi \leq \chi \leq \pi/2 - \phi + \varpi - \theta$ and to $\psi \in \left[0, 2\pi\right]$. Thus, the necessary stability condition is given by:

$$\gamma_w \, i^{ir}(0) \leq \gamma' \sin(\phi - \theta) - \Lambda \, \lambda \, p_0 - \gamma \, k^* \,) / \sin\phi \quad (21)$$

where coefficients Λ and k^* are defined as:

$$\Lambda = \sqrt{\left(-\Upsilon_1 \cos\phi + \Upsilon_4 / \lambda \sin\phi \right)^2 + \left(\Upsilon_2 \cos\phi + \Upsilon_3 / \lambda \sin\phi \right)^2} \,; \quad k^* = k_h \cos(\phi - \theta) - k_v \sin(\phi - \theta) \quad (22)$$

and the coefficients $\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4$ are given as a function of $\mathsf{H}_u(z)$ and $\mathsf{J}_u(z)$.

This extends the findings of Dormieux et al. [4] for the case of an infinitely deep seabed (i.e., $d \to \infty$) under static loading conditions (i.e., $k_v = k_h = 0$). In this scenario, it can be demonstrated that $\lim_{d \to \infty} \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4 = 1, 0, \lambda, 0$, and the stability condition given in eq. (21) simplifies to:

$$\gamma_w \, i^{ir}(0) \leq \gamma' \sin(\phi - \theta) - \lambda \, p_0 \,) / \sin\phi \quad (23)$$

The stability condition expressed in eq. (22), provides an upper bound K^c estimate for the stability domain K. For a given set of seismic coefficients k_h, k_v and a fixed soil-to-water unit weight ratio γ / γ', it is useful—particularly for graphical purposes—to describe the loading mode using two dimensionless components $\underline{Q} = Q_1 = \lambda p_0 / \gamma', Q_2 = i^{ir}(0)\gamma_w / \gamma'$. With this representation, inequality (21) can be reformulated as:

$$\frac{\Lambda}{\sin(\phi - \theta)} Q_1 + \frac{\sin\phi}{\sin(\phi - \theta)} Q_2 \leq 1 - \frac{\gamma / \gamma'}{\sin(\phi - \theta)} k^* \quad (24)$$

and associated upper bound domain K^c is

$$K \subseteq K^c = \left\{ \underline{Q} \,; \quad \frac{\Lambda}{\sin(\phi - \theta)} Q_1 + \frac{\sin\phi}{\sin(\phi - \theta)} Q_2 \leq 1 - \frac{\gamma / \gamma'}{\sin(\phi - \theta)} k^* \leq 0 \right\}. \quad (25)$$

The geometrical domain describing the set K^c in the plane Q_1, Q_2 is sketched in Fig. 5 and the bounds are $Q_1^+ = [\sin(\phi - \theta) - \gamma / \gamma' k^*] / \Lambda$ and $Q_2^+ = [\sin(\phi - \theta) - \gamma / \gamma' k^*] / \sin\phi$. Referring to the plane Q_1, Q_2, K^c represents the set of loads that are potentially withstand by the seabed, in the sense any loading lying outside this domain will certainly cause seabed instability.

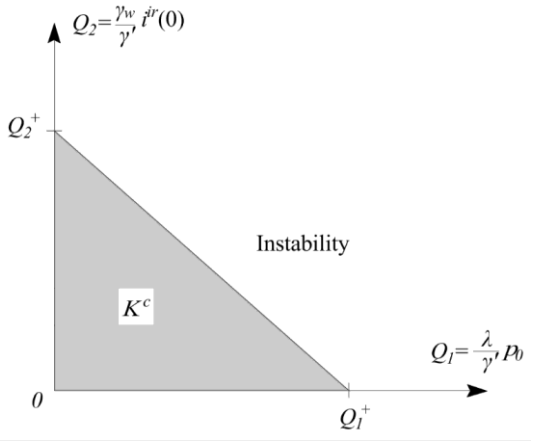


Figure 5 – Representation of the kinematic approximation K^c of stability domain in the plane Q_1, Q_2 .

3 Conclusion

This study has presented a simplified rigorous framework for assessing the stability of granular seabeds under wave-induced loading, based on limit analysis in effective stress terms. By extending the formulation of Dormieux et al. [4], the combined effects of poroelastic fluctuations and irreversible soil densification were incorporated into the evaluation of seepage forces. The results demonstrate that seabed thickness plays a critical role when it is smaller than the incident wavelength but becomes negligible for sufficiently large depths. The implementation of the kinematic approach using virtual failure mechanisms enabled upper bound predictions that clearly demonstrate the influence of the irreversible hydraulic gradient along the seabed surface. Overall, the proposed methodology offers a robust and practical tool for analyzing seabed stability, accounting for key parameters such as geometry, loading conditions, and soil behavior.

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