XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGENEERING

Comparison between 3D Finite Element and analytical models in railway track analysis

Guilherme N. Bassegio[1], Paulo A. M. Pereira[1], Alfredo G. Neto[1]

[1]*Dept. of Structural and Geotechnical Engineering, Polytechnic School at University of São Paulo*
Av. Prof. Luciano Gualberto, 380 - Butantã, 05508-010, São Paulo, Brazil
guilherme.bassegio@usp.br, paulompereira@usp.br, alfredo.gay@usp.br

Abstract. The satisfactory performance of a railway track throughout its service life depends on an adequate design. In this process, analytical models are often employed which, despite their practicality, are based on assumptions that may not accurately represent real site conditions. A more enhanced representation of these conditions can be provided by higher-hierarchy models, such as ones employing the finite element method. A reliable finite element analysis requires a suitable constitutive model and properly defined parameters, which can be obtained from on-site data, laboratory tests, or from the literature referring to similar conditions. In this paper, the analytical model of the Winkler foundation is compared to a finite element analysis for the problem of a statically loaded railway track section, characterized by literature data, to assess the main discrepancies that may arise between the two methods. The comparison between both models is made by evaluating the track modulus, which is an important parameter for analytical models. In the high hierarchy model, stress fields are visualized along layers of under-rail materials, helping in understanding the physical behavior of the system.

Keywords: Railway, Track Modulus, FEM, Analytical model, Structural Analysis

1 Introduction

Railway track design requires accurate prediction of structural behavior. Traditional analytical approaches, such as the Winkler foundation model, simplify the system to a beam resting on independent linear springs, as described by Pantelidis [1]. While practical for initial calculations, this model assumes no interaction between adjacent springs and uniform foundation properties, which neglects shear interaction, diverging from real-world conditions where stress distribution depends on layered material interface behavior and requires the definition of an equivalent modulus of elasticity. Yet, this method is still widely used in railway track design due to the simplicity of inputs and large academic research.

The finite element method (FEM) is frequently employed in railway engineering to evaluate track deformation. However, due to the high computational cost of detailed modeling, simplifications are often necessary when simulating large railway lengths. In this context, most studies represent the superstructure using beam elements and approximate the behavior of non-linear components, such as the ballast, using continuous approaches, since the modeling of individual particles through approaches like the Discrete Element Method (DEM) can be prohibitively expensive. While FEM overcomes the limitations of the Winkler method by explicitly modeling multi-layered systems and complex boundary conditions, its predictive reliability critically depends on appropriate constitutive models and rigorous parameter calibration.

One of the most widely used methods for assessing the mechanical performance of railway track structures is the concept of track modulus. Introduced by Talbot et al. [2] as the vertical displacement under a unit load applied by the vehicle wheel, the track modulus represents the combined effect of the mechanical properties across the entire analyzed section during loading. Although it does not account for the track as a layered system, this parameter has played a key role in structural evaluations, such as the works of Paderno [3], Costa et al. [4], Khordehbinan [5], Zakeri and Sadeghi [6], Priest and Powrie [7], Pereira et al. [8].

The present study bridges analytical and numerical approaches by conducting a comparative analysis of the Winkler model and FEM for a statically loaded railway section. The objective is to quantify discrepancies in track modulus predictions between the two methods while visualizing stress fields in under-rail materials via FEM to interpret physical behavior. Two case studies are conducted, the first being a static point load analysis on a railway section, while the second simulating the bogie load induced by a Dash-9 locomotive, validating the result with

published data from the same locomotive configuration.

2 Literature Review

2.1 Track Modulus

Initially devised by Talbot et al. [2], the Track modulus is a representation of the vertical stiffness of the track structure. It is obtained by measuring the resulting rail displacement due to vertical wheel loads from railway vehicles, representing the global track stiffness (encompassing rails, sleepers, fixations, ballast, subballast and subgrade). When conducting field experiments, the track modulus can be obtained by eq. 1:

$$u = \frac{(P/w_0)^{4/3}}{(64EI)^{1/3}}, \tag{1}$$

where:
- $u =$ Track modulus (MPa),
- $P =$ Wheel Load (kN)
- $w_0 =$ Maximum vertical displacement (m)
- $EI =$ Bending stiffness of the rail (MPa.mm^4)

According to Cai et al. [9], it is possible to obtain this coefficient analytically by employing a Winkler foundation method.

The Winkler foundation represents the rail as an Euler-Bernoulli beam resting on a series of springs with stiffness k. The governing equation is represented by eq. 2:

$$EI \cdot \frac{d^4w}{dx^4} + k \cdot w(x) = q(x), \tag{2}$$

where
- $w =$ displacement in the vertical direction,
- $x =$ coordinate parallel to the rail,
- $k =$ support reaction per unit width of the beam
- $q(x) =$ distributed load equivalent to the wheel loads.

For a point load P on the rail, the displacement w is given by eq. 3:

$$w(x) = \frac{P\beta}{2k} e^{-\beta x}(cos\beta x + sin\beta x), \tag{3}$$

where

$$\beta = (\frac{k}{4EI})^{\frac{1}{4}}. \tag{4}$$

Checking for $x = 0$, where the function $w(x)$ reaches the maximum absolute value (w_0), it can be found that $k \equiv u$.

This method assumes:
- Linear-elastic foundation response
- No shear transfer between springs
- Homogeneous soil properties

2.2 Finite element method

According to Jagota et al. [10], modelling a problem with FEM gives an approximation to the governing equations. The basic premise being that a solution region can be analytically modeled or approximated by replacing

it with an assemblage of discrete elements. Transforming the continuous problem into a system of algebraic equations.

In this study, the geometry is adapted from the cross-sectional design described in Pereira et al. [8] with 16.5 m of track, as seen in Table 1, the mechanical properties of each layer were based on the same paper and complemented with the work of Paixão et al. [11].

Table 1. Simulation Parameters

Geometry	Element Type	Thickness (m)	Density $(\frac{kg}{m^3})$	Elasticity Modulus (MPa)	Poisson Ratio
Ballast	Solid	0.32	1530	130	0.2
Subballast	Solid	0.3	2326.5	216	0.3
Subgrade	Solid	6	2040	80	0.3
Rail (RE 136)	Beam	-	7850	210000	0.3
Sleepers	Solid	-	2604.4	30000	0.25
Rail Pads	Solid	-	1200	1000	0.49

The wheel load is applied via solid elements, each assigned a mass equal to half the axle load (32 tons per axle on the analyzed track), and sized sufficiently small to be modeled as a point load.

In Ansys Mechanical™[12], the FEM software used in this study, simulating the contact between the rail (modeled as a beam) and the wheel (modeled as a solid) requires the introduction of an intermediate layer. In this case, a shell element is used, with a width equal to that of the rail head and negligible weight and thickness.

The contact between the beam and the shell is defined using a Multi-Point Constraint (MPC) contact, while the contact between the shell and the solid is modeled using beam-to-surface contact.This modelling strategy was done in order to correctly promote the load transference. The shell thus acts as an interface to enable simultaneous contact interactions.

3 Methodology

In Fig. 1, the methodology flowchart is presented.

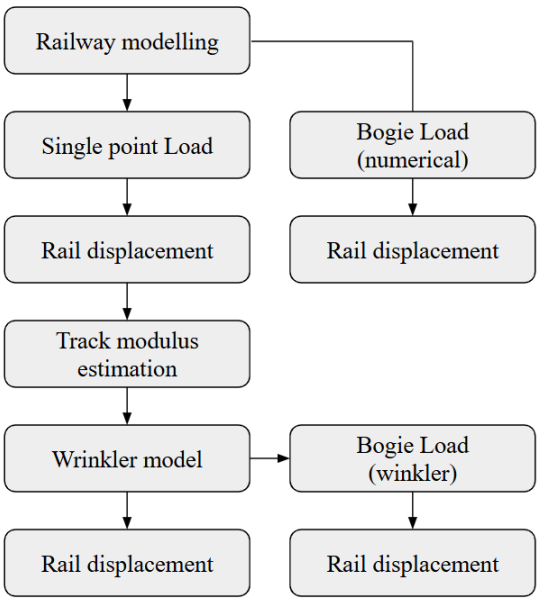


Figure 1. Methodology Flowchart

Both models simulate a 16.5 m track section with RE-136 rail on concrete sleepers, distanced by 60 cm, and

ballasted foundation. The RE136 rail, which geometry can be found at ArcelorMittal [13], has a moment of inertia of $3.95 \cdot 10^7 mm^4$, and foot width of 136 mm.

In order to numerically solve through FEM, a 3D model is implemented in Ansys according to the geometry described. The body is fully constrained in the lower part of the subgrade solid, and movement is permitted on the lateral surfaces. The specifics of each component are presented in Table 2:

Table 2. Simulation Parameters

Geometry	Ansys Element ID	Number of elements
Ballast	SOLID185	51150
Subballast	SOLID185	57090
Subgrade	SOLID185	286440
Rail (RE 136)	BEAM188	330 (each)
Sleepers	SOLID185	486 (each)
Rail Pads	SOLID186	9 (each)

To solve the Winkler model, we need to first estimate the value of the track modulus, which, in order to compare with the numerical approach, will be done according to the results from the FEM simulation and eq.1:

4 Results

4.1 Point load analysis

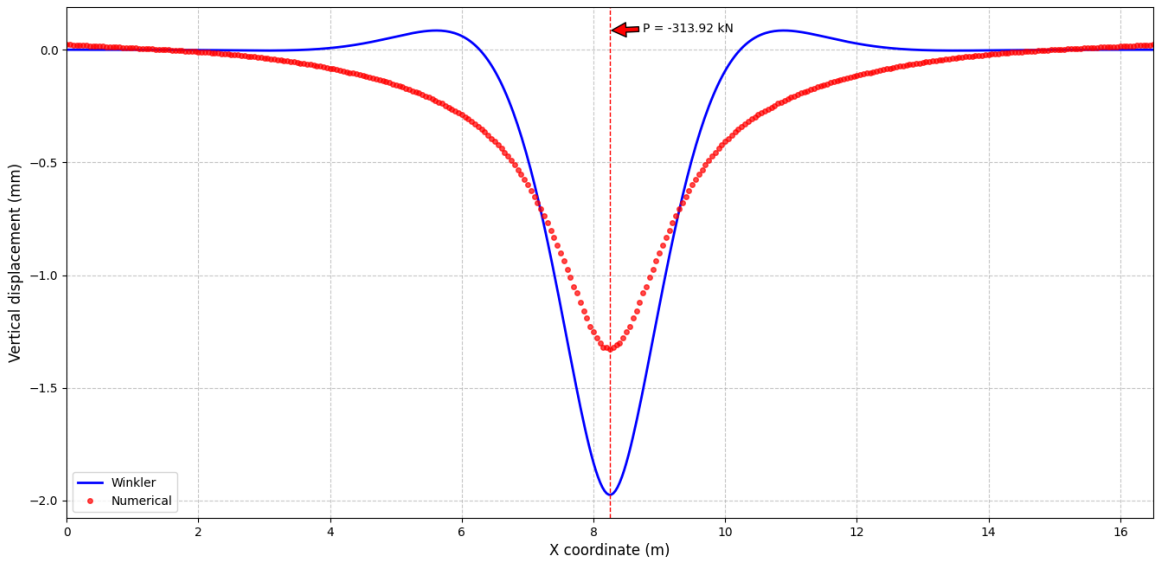


Figure 2. Vertical displacement according to horizontal (X) coordinate in numerical and Winkler analysis

The numerical simulation exhibited a displacement at the center load location of 1.33 mm, which translates to a track modulus of 71.49 MPa, this value was then inserted in the Winkler foundation analysis.

The maximum displacement predicted by the Winkler for a model with a point load of 32 tf exceeded the numerical result by 48.47% as shown in Fig. 2.

It can be observed that the displacement in the Winkler model decays faster with increasing distance from the load, indicating a lack of load transfer beyond the immediate load vicinity, which can be partially explained by the absence of shear interaction between springs.

4.2 Bogie Load on a Railway section

The results for multiple point loads of 32 tf applied to the rail, simulating the load of a Dash-9 locomotive, are presented in Fig. 3. The maximum displacement shown by the numerical simulation surpassed the result of the Winkler method by 1.27%. It is noticeable that the maximum displacement in the numerical method increased by 50.4% while the Winkler analysis did not show variation.

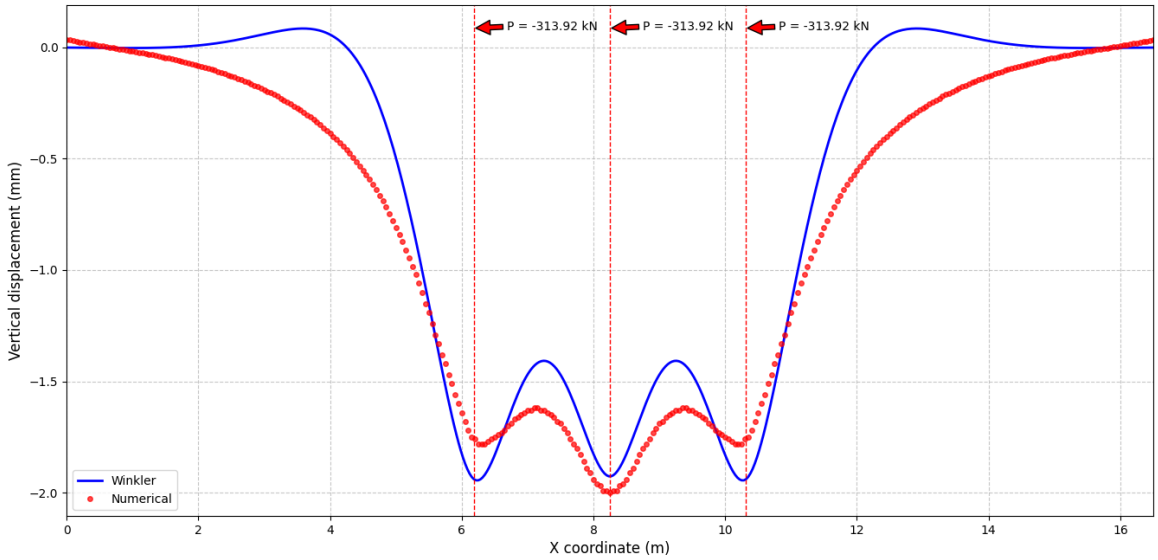


Figure 3. Vertical displacement according to horizontal coordinate in numerical and Winkler analysis

The variation observed between single and multiple load applications highlights the importance of considering the lateral stiffness influence when analyzing the track modulus. In the examples presented, applying a series of loads does not necessarily increase the maximum displacement predicted by the Winkler foundation model. In the single-point load case, the numerical simulation indicates negative displacement effects extending from approximately 1.5 m to 15 m from the beginning of the railway section, whereas the Winkler model predicts such effects only from 6.25 m to 10.25 m. This corresponds to elastic influence lengths of about 13.5 m for the numerical model and 4 m for the Winkler model. Therefore, the lack of maximum displacement variation in the Winkler model can be explained due to the spacing of axles in a dash 9 locomotive (2.064 m according to Bachand [14]). In the analytical model, this distance is sufficient for the rail to return a stabilized state (for a single load). However, in the numerical model, this condition is not fully met.

Displacement surfaces under bogie load

The displacement distribution for the bogie load simulation is shown in Fig. 4:

It is noted that the displacement surfaces remain nearly parallel near the base of the railway. This pattern is repeated across half the section and remains uniform up to a height of approximately 3 m. This indicates that the stress bulb might reach a depth of about 3.6 m. Stress analysis along the pavement with the defined boundary conditions, such as the fixed support beneath the subgrade, is still preliminary. However, alternatives like infinite elements are being explored to better represent the model conditions.

5 Conclusions

We compared an analytical solution of the railway with a FEM numerical approach, obtaining different results. The analytical solution, although simpler to employ, showed an overestimation of the maximum displacement provoked by each load, while showing less contribution of the pavement outside of the central point of the applied load. It is important to notice that when designing with Winkler foundation based methods, the lateral contribution may be underestimated. In light of this, provided that the designer has a good enough parametrization of the railway components, better representing the physical behavior, the finite element analysis of the pavement may prove valuable in cases of an isolated statical load force, as it considers 3D effects in a more detailed way.

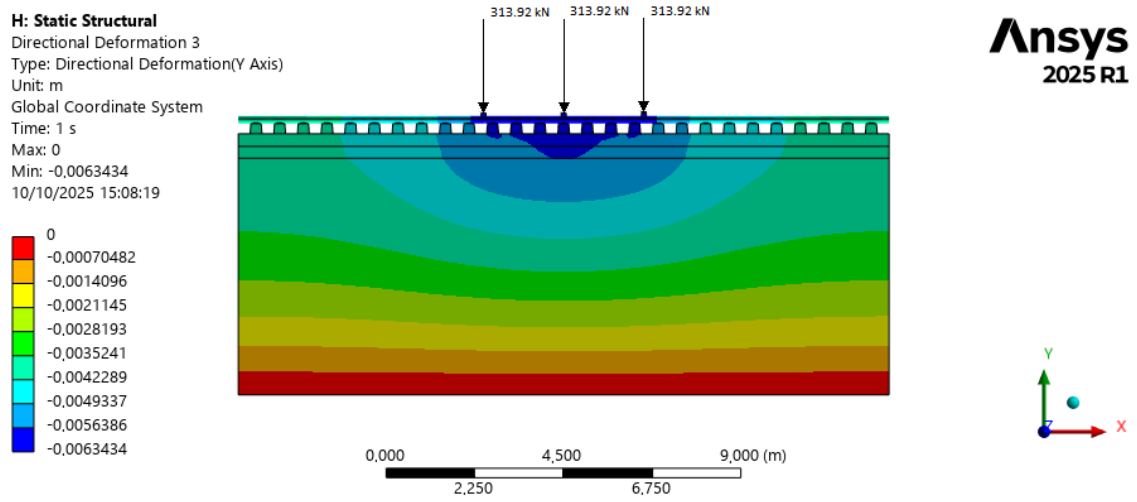


Figure 4. Vertical displacement across railway section

6 Next Steps

Future improvements may involve increasing the model length to evaluate boundary effects on the results, as well as performing dynamic analysis of the pavement with modified boundary conditions to avoid problems such as mechanical wave reflection.

Acknowledgements. The authors acknowledge CNPq for supporting co-author Alfredo Gay Neto under the grant number 304321/2021-4.

Authorship statement. The authors hereby confirm that they are the solely liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] L. Pantelidis. The equivalent modulus of elasticity of layered soil mediums for designing shallow foundations with the winkler spring hypothesis: A critical review. Engineering Structures, vol. 201, pp. 109452, 2019.
[2] A. Talbot and others. Second progress report of special committee to report on stresses in railroad track. Proceeding of the AREA, vol. 21, pp. 645–814, 1920.
[3] C. Paderno. Comportement du ballast sous l'action du bourrage et du trafic ferroviaire. Ecole Polytechnique de Lausanne, 2010.
[4] R. C. d. Costa, R. d. S. Motta, E. d. Moura, J. A. P. d. Oliveira Filho, and L. L. B. Bernucci. Measuring device for in situ determination of the track modulus in a heavy haul track. Proceedings of the IHHA 2017: Advancing Heavy Haul Technologies and Operations in a Changing World, 2017.
[5] M. W. Khordehbinan. Investigation on the effect of railway track support system characteristics on the values of track modulus. Proceedings of AREMA, 2010.
[6] J. A. Zakeri and J. Sadeghi. Field investigation on load distribution and deflections of railway track sleepers. Journal of Mechanical Science and Technology, vol. 21, n. 12, pp. 1948–1956, 2007.
[7] J. Priest and W. Powrie. Determination of dynamic track modulus from measurement of track velocity during train passage. Journal of geotechnical and geoenvironmental engineering, vol. 135, n. 11, pp. 1732–1740, 2009.
[8] P. Pereira, R. Motta, A. G. Neto, L. Bernucci, E. Moura, R. Silva, and L. Oliveira. Field assessment of different tamping operations for newly constructed or renovated railway tracks. International Journal of Rail Transportation, vol. 13, n. 2, pp. 402–424, 2025.
[9] Z. Cai, G. Raymond, and R. Bathurst. Estimate of static track modulus using elastic foundation models. Transportation Research Record, vol. 1470, pp. 65, 1994.

[10] V. Jagota, A. P. S. Sethi, and K. Kumar. Finite element method: an overview. Walailak Journal of Science and Technology (WJST), vol. 10, n. 1, pp. 1–8, 2013.
[11] A. Paixão, E. Fortunato, and R. Calçada. Transition zones to railway bridges: track measurements and numerical modelling. Engineering structures, vol. 80, pp. 435–443, 2014.
[12] ANSYS, Inc. Mechanical User's Guide, version 2025 R1, 2025.
[13] ArcelorMittal. ArcelorMittal Transport Rails, 2023.
[14] J. Bachand. GE C44-9W Data Sheet, 2010.